



A Mathematical Fuzzy Cognitive Framework for Contextual Emotion and Ethical Reasoning in Sangam Literature

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Abstract

Classical literature contains emotional experiences that are ambiguous and conditional based on context. Emotional experiences cannot be well represented by traditional methods of binary logic or discrete sentiment analysis. The purpose of this paper is to present a new Fuzzy Cognitive Framework that mathematically models the dynamical evolution of emotional states (emotions) and the ethical basis for these emotions in Sangam Literature—using fuzzy set theory, fuzzy relational structures, fuzzy cognitive maps, nonlinear dynamic systems, and contextually- dependent aggregation operators. The proposed Fuzzy Cognitive Framework differs from existing fuzzy sentiment analysis models because it introduces contextually-dependent fuzzy emotional spatial domains; the use of fuzzy operators and properties for establishing ethical bases for emotions; and the use of mathematically stable models for emotional evolution. This work is new, rigorous, and consistent with recent developments in fuzzy natural language processing (NLP) and cognitive models.

Keywords: Fuzzy Sets, Fuzzy Cognitive Maps, Emotion Dynamics, Ethical Dependency, Sangam Literature, Mathematical Modeling, Digital Humanities

AMS Classification: 03E72, 91C99, 68T37, 37N99

1. Introduction

Sangam literature represents one of the most sophisticated classical systems of emotional and ethical expression, where human experiences are described through complex interrelations between emotions, actions, and cultural norms. Unlike modern analytical systems, emotions are not discrete entities but continuous and overlapping states that evolve under contextual influence. Standard binary logic fails to account for the complexity of emotional behaviors. Fuzzy Logic is a natural fit for modeling degrees of truth and the intermediary steps toward certain outcomes. Addition of cognitive interaction systems allows for the effective modeling of relationships between several different variables that change over time. This research presents a unifying fuzzy cognitive mathematical model that accounts for emotional dynamic behavior, context- based variability, and processes of ethical reasoning simultaneously, thereby allowing for a mathematical and conceptual understanding comparable to that of a literary work.

2. Literature Review

This Section will provide a review of literature concerning fuzzy set theory and its evolution from original theorist Lotfi A. Zadeh (1965) in creating a viable mathematical means of representing uncertainty and containing multiple degrees of truth; Bart Kosko's (1986) use of fuzzy cognitive maps in this theory to define interactions between multiple variables over time; and the current application of these models used by Nápoles et al. (2026), Wang et al. (2024), and Do et al. (2024) and the use of fuzzy semantics as defined by Rokhva et al. (2025) used in defining vagueness and uncertainty in a linguistic context; as well as by Apostolopoulos et al. (2024), who applied these principles to fuzzy reasoning in explainable artificial intelligence.

Thus far there has been little application of this research to Sangam Literature, providing the impetus for this study.



3. Mathematical Preliminaries

3.1 Fuzzy Sets

Let X be a universe of discourse.

A fuzzy set $A \subseteq X$ is defined by a membership function: $\mu_A : X \rightarrow [0,1]$

In literary analysis, emotions are fuzzy sets rather than crisp categories.

3.2 Fuzzy Relations

A fuzzy relation $R \subseteq X \times Y$ is defined as: $\mu_R(x, y) \in [0,1]$

This is used to represent emotional influence, ethical dependency and contextual relevance.

3.3 T-Norms and T-Conorms

We use: T-norms \otimes for conjunction (AND), T-conorms \oplus for disjunction (OR). Example: $a \otimes b = ab, a \oplus b = a + b - ab$

These operations satisfy Closure in $[0,1]$, Monotonicity, Associativity.

3.4 Activation Function

To ensure bounded output, we use $\sigma(x) = \frac{x}{1+x}$

This function is continuous, increasing, and bounded in $[0,1]$, making it suitable for dynamical systems.

4. Fuzzy Emotion Space

Definition 4.1 (Emotion Space)

We can view the emotional construct as continuous and that emotions can develop at the same time and interact with one another. This allows for an increased complexity of literature where multiple emotion states are expressed at the same time and interact with one another to affect all.

Let $\mathcal{E} = \{e_1, e_2, \dots, e_n\}$ be the set of core emotional concepts.

A fuzzy emotion vector is defined as $\vec{\mu} = (\mu_{e_1}, \mu_{e_2}, \dots, \mu_{e_n}) \in [0,1]^n$. Each component of the emotion vector represents the intensity level of that emotion.

Each poem is mapped to a point in the n -dimensional fuzzy emotion space. The interpretation of the mapping is:

- Emotions co-exist (multiple non-zero memberships).
- Intensities vary continuously.
- Emotional ambiguity is preserved mathematically.

Traditional systems tend to view emotions as mutually exclusive, whereas in everyday life and literature, emotions can co-exist and can affect one another. For instance, one can love another person while at the same time feeling sadness due to the loss of that person, or one may have joy but that joy may diminish during times when they are separated from the one they love. The emotional fuzzy space allows for every emotion to take on values between zero and one allowing for emotions to be represented at the same time. This creates a multi-dimensional space where emotional states evolve dynamically.

5. Context-Conditioned Emotion Modeling Definition 5.1 (Context Vector)

Let $C = (c_1, c_2, c_3)$ Where c_1 = cultural setting, c_2 = narrative role, c_3 = ethical background and Each $c_i \in [0,1]$.

Context-Adjusted Emotion Function

For each emotion e_i : $\mu_{e_i}^c = \mu_{e_i} \otimes \phi_i(C)$ where ϕ_i is a context modulation function.

This ensures the same emotion is interpreted differently under different contexts, a property absent in classical sentiment models.

6. Fuzzy Cognitive Emotion Network Definition 6.1 (Emotion Interaction Matrix)

The emotional interaction matrix (M) demonstrates how emotions impact one another. Conceptual basis: Row = how emotion influences other emotions

Column = how emotion is influenced by other emotions

Let: $M = [m_{ij}], m_{ij} \in [-1,1]$

where: $m_{ij} > 0.5$: reinforcement,

$m_{ij} < 0.5$: suppression,

$m_{ij} = 0.5$: no interaction.

Emotion Evolution Equation

Define a nonlinear fuzzy dynamical system:

$E(t+1) = \sigma(E(t)M \oplus C(t))$ where σ is a bounded activation function. This E represents internal



interaction (via M), External influence (via $C(t)$), Nonlinear normalization (via σ).

Proposition 6.1 (Bounded Stability)

If σ is Lipschitz continuous and bounded in $[0,1]$, then: $E(t) \in [0,1]^n \forall t$

Thus, emotional evolution remains mathematically stable.

7. Ethical Dependency Modeling Definition 7.1 (Ethical Dimensions)

Let: Intent I , Social Impact S , and Cultural Alignment A , for all values in $[0,1]$.

Ethical Dependency Operator (Novel)

Formulate: $\Psi(I, S, A) = (I \otimes S)^\alpha \otimes A^\beta, \quad \alpha, \beta > 1$

Key Property: If any ethical dimension is weak, the overall ethical strength decreases non-linearly, reflecting moral reasoning in Sangam texts.

α : controls importance of intent + impact

β : controls importance of cultural conformity

These parameters act as **ethical weighting exponents**. Because they are exponents, their effect is nonlinear:

- Increasing α sharply increases sensitivity to intent
- Increasing β amplifies cultural dependence

Sensitivity Behavior

If $\alpha > \beta$ then even small changes in intent significantly affect ethical output. If $\beta > \alpha$ then ethical judgment becomes rigid and culture-dominated.

Proposition 7.1 (Ethical Sensitivity)

$\frac{\partial \Psi}{\partial I} > \frac{\partial \Psi}{\partial S}$ when $\alpha > \beta$

This allows mathematically tunable ethical priorities.

8. Integrated Framework

The full system is represented as: $(\mathcal{E}, C, M, \Psi)$ where \mathcal{E} = fuzzy emotion space, C = context vector, M = emotion interaction matrix, Ψ = ethical evaluation operator.

The system: (\mathcal{E}, C, M, T) forms a feedback loop:

$$\mathcal{E} \rightarrow T \rightarrow C \rightarrow \mathcal{E}$$

This represents:

- Emotions influence ethical decisions
- Ethical decisions modify context
- Context affects future emotions

This creates a nonlinear dynamical cognitive system.

9. Illustrative Example & Theorems

Initial state: $E(0) = (0.8, 0.6, 0.5, 0.3)$

Step 1: $E(0)M = (1.23, 1.16, 1.03, 1.10)$

Step 2: $E(1) = \sigma(x) = x \quad E(1) \approx (0.55, 0.54, 0.51, 0.52)$

Interpretation

The system stabilizes toward equilibrium (~ 0.5), indicating emotional balance.

Theorem 1: Boundedness of Emotional Dynamics

Statement: For all $t \geq 0$, the emotional state vector satisfies: $E(t) \in [0,1]^n$

Proof: Consider: $E(t+1) = \sigma(E(t)M \oplus$

$C(t))E(t+1)$

Each component: $(E(t)M)_j = \sum^n E_i(t)m_{ij}$



Since: $E_i(t), m_{ij} \in [0,1]$

We obtain: $0 \leq E_i(t)m_{ij} \leq 1$. Thus: $0 \leq \sum_n E_i(t)m_{ij} \leq n$

Applying the activation function: $\sigma(x) = \frac{x}{1+x}$

We have: $0 \leq \sigma(x) \leq 1$

Also, since \oplus preserves bounds: $E(t + 1) \in [0,1]^n$. Hence proved.

Theorem 2: Convergence and Stability

Statement: If the activation function σ is Lipschitz continuous with constant $L < 1$, then the system converges to a unique fixed point.

Proof:

Define: $F(E) = \sigma(EM \oplus C)$ Let $E_1, E_2 \in [0,1]^n$.

Then: $\|F(E_1) - F(E_2)\| \leq L \|E_1M - E_2M\|$

Let $k = L \|M\|$. If $k < 1$, then: $\|F(E_1) - F(E_2)\| \leq k \|E_1 - E_2\|$

Hence, F is a contraction mapping. By the Banach Fixed Point Theorem, there exists a unique fixed point E^* , and the sequence $E(t)$ converges to E^* . Thus, the system is stable.

Theorem 3: Ethical Sensitivity and Dominance

Statement: If $\alpha > \beta$, then the ethical system is more sensitive to intent than cultural alignment.

Proof:

Given:

$$T(I, S, A) = (IS)^\alpha A^\beta$$

We obtain $\frac{\partial T}{\partial I} = \alpha (IS)^{\alpha-1} A^\beta$ & $\frac{\partial T}{\partial A} = \beta (IS)^\alpha A^{\beta-1}$. Given that $\alpha > \beta$, and all variables lie in

$[0,1]$, the coefficient of $\frac{\partial T}{\partial I}$ prevails. Consequently:

$\frac{\partial T}{\partial I} > \frac{\partial T}{\partial A}$. Thus, the system exhibits stronger sensitivity to intent. Hence proved.

Theorem 4 (New – Monotonicity of Ethical Function)

Statement: The function $T(I, S, A)$ is monotonic increasing in each variable.

Proof:

Since: $I, S, A \in [0,1], \alpha, \beta > 0$.

All partial derivatives: $\frac{\partial T}{\partial I}, \frac{\partial T}{\partial S}, \frac{\partial T}{\partial A} \geq 0$

Thus, T is increasing in all arguments. Hence proved.

10. Applications & Novel Contributions

The field of digital humanities has produced new trajectories for visualizing emotion, as well as new techniques for extracting ethical patterns from classical texts. The emerging field of explainable artificial intelligence has developed techniques for fuzzy reasoning that allow for interpreting emotions and ethical contexts. The area of low-resource natural language processing has recently introduced methods for creating sentiment models that do not require large amounts of annotated data and that allow for the sensitivity to culture, or cultural and social norms. This body of work proposes a continuous-valued, contextually dependent fuzzy emotion space that can be modulated using a definable, stable fuzzy cognitive dynamic system. The work also introduces an ontology of new, non-linear ethical dependency operators. As far as the author knows, this is the first rigorous mathematical framework using fuzzy set theory and cognitive modeling to specifically conduct research in Sangam literature.

11. Conclusion

This work develops a new mathematical foundation for performing literary analysis that combines fuzzy set theory, cognitive modeling, and ethical reasoning. The research framework adheres to the fact that language has high levels of ambiguity; cultures are complex in their depth and rich in emotional expression; therefore, fuzzy mathematics will provide a strong mathematical basis for continuing cutting-edge research in digital humanities and especially literature. Future research will provide expanding opportunities to broaden the proposed imaginative space by integrating deep learning techniques with fuzzy logic to form hybrid models that can interpret as well as predict. Moreover, additional computational and data analytic techniques to find additional patterns of emotion and ethics through large-scale analysis of literary databases may be developed. Additionally, other mathematical extensions of the proposed framework may include stochastic fuzzy systems, timed-delay models, or high-dimensional cognitive networks. Aside from the languages and cultures used, the proposed framework can also be applied to analyze literature across all languages and cultures of the world.



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