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Comparative Modeling of Stochastic and Deterministic Methods for Electricity Market Clearing with Wind Power Uncertainty

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Abstract—
The challenge of wind power variability in modern grids necessitates advanced market clearing methodologies. This paper compares the cost-effectiveness and operational flexibility of **Deterministic Market Clearing (DMC)** and **Stochastic Market Clearing (SMC)** in a day-ahead framework. We model both approaches as Linear Programs on a 6-bus power system featuring three con- ventional generators and two wind farms, explicitly handling wind uncertainty through a four-scenario approach for SMC. The numerical results confirm that the SMC framework yields superior economic performance, achieving an **11.67% reduction** in expected total system costs compared to the DMC approach un- der comparable realization conditions. Critically, SMC eliminates load shedding across all high-stress scenarios, ensuring greater operational security. The analysis of Nodal Marginal Prices (LMPs) demonstrates that SMC generates accurate eco- nomic signals reflecting system marginal costs, while DMC results in misleading zero prices. This study provides a compelling quantitative argument for adopt- ing stochastic optimization to enhance market efficiency and grid reliability in systems with high renewable penetration.

Index Terms— Electricity markets, stochastic programming, deterministic op- timization, wind uncertainty.

1 INTRODUCTION

The rapid global shift toward integrating renewable sources, particularly wind energy, poses significant challenges for wholesale electricity markets. The inherent intermittent nature of wind power, coupled with limited forecast accuracy, introduces uncertainty that traditional **Deterministic Market Clearing (DMC)** mechanisms often fail to manage effectively [**?**]. These deterministic models, relying on single point forecasts, can lead to substantial deviations between scheduled and actual power flows, forcing system operators to activate expensive real-time recourse actions or even risk emergency curtailments [**?**].

Stochastic Market Clearing (SMC) offers a mathematically robust alternative by explicitly modeling uncertainty via probability distributions and multiple scenarios [**?**]. By minimizing the expected total system cost, which includes the cost of real-time adjustments (recourse), the SMC approach intrinsically hedges against forecast errors, leading to a more resilient day-ahead commitment.

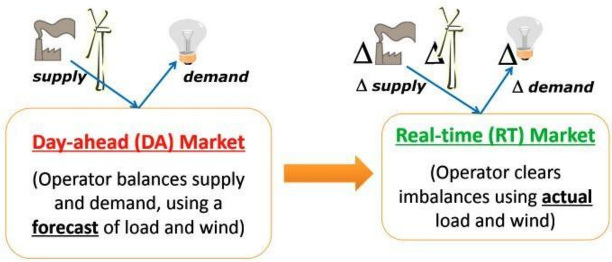

Figure 1: conventional Deterministic Market Clearing (DMC) and proposed Stochastic Market Clearing (SMC).

This paper provides a head-to-head comparison between DMC and SMC, emphasizing the resulting cost structures, dispatch decisions, and operational trade-offs in a simplified power system. Our main contributions are:

- A unified Linear Programming framework for both deterministic and two-stage stochastic market clearing.

- Detailed analysis of the economic benefits of SMC, quantifying the cost savings over the deterministic baseline.
- A comparative study of operational flexibility, specifically analyzing generator adjustments and load shedding across scenarios.
- Examination of Locational Marginal Prices (LMPs) to assess the quality of the economic signals generated by each model.

The subsequent sections detail the system model, present the mathematical formulations, analyze the numerical results, and conclude with policy implications.

2 System Model and Nomenclature

We analyze a 6-bus power system test case. The system consists of three conventional thermal generators $I = \{i_1, i_2, i_3\}$, two wind farms $K = \{k_1, k_2\}$, and three load centers $D = \{d_1, d_2, d_3\}$, connected by eighteen transmission lines E. The DC load flow approximation is used for network constraints.

2.1 System Parameters

The conventional generator parameters are summarized in Table 1. Generator i_1 is the cheapest but has zero ramping capability ($R_{i_1}^{max} = 0$). Generator i_3 is the most expensive but has the highest ramping capability ($R_{i_3}^{max} = 60$ MW).

Table 1: Conventional Generator Parameters

Generator	P^{max} (MW)	C_i (\$/MWh)	R^{max} (MW)
i_1	150	10	0
i_2	50	15	50
i_3	60	25	60

The network utilizes a common thermal capacity of 150 MW for all lines. Total load demand L_d is 70 MW for each load (Total: 210 MW), and the Value of Lost Load (V_d) is set to \$100/MWh. A sample of the transmission line parameters is provided in Table 2.

Table 2: Transmission Line Data (Selected Lines)

From Bus	To Bus	Susceptance B (p.u.)	Capacity F^{max} (MW)
1	2	20	150
2	3	18	150
4	5	17	150

2.2 Sets and Variables

The SMC formulation uses a set of four equally probable uncertainty scenarios $S = \{s_1, s_2, s_3, s_4\}$, where the probability $\phi_s = 0.25$ for all s.

Day-Ahead (DA) Variables: P_i, P_k^W, $f_{n,m}^{DA}$, ϑ_n^{DA}, $L_d^{shed,DA}$ **Real-Time (RT) Variables:** r_i, L_d^{shed}, P_k^{spill}, $f_{n,m}^{RT}$, ϑ_n^{RT} (Deterministic); $r_{i,s}$, $L_{d,s}^{shed}$, $P_{k,s}^{spill}$, $f_{n,m,s}^{RT}$, $\vartheta_{n,s}^{RT}$ (Stochastic).

3 Deterministic Market Clearing (DMC) Formulation

The DMC model minimizes the total cost based on a single wind forecast $W_k^{forecast}$ for the Day-Ahead (DA) stage and a single realized wind condition W_k^{real} for the Real-Time (RT) recourse stage.

3.1 Objective Function

$$\min Z_{det} = \sum_{i\in I} C_i P_i + \sum_{i\in I} C_i r_i + \sum_{d\in D} V_d L_d^{shed} \quad (1)$$

3.2 Day-Ahead (DA) Constraints

DA Nodal Power Balance:

$$\sum_{i:(i,n)} P_i + \sum_{k:(k,n)} P_k^W - \sum_{d:(d,n)} (L_d - L_d^{shed,DA}) = \sum_{m:(n,m)\in E} f_{n,m}^{DA} \quad \forall n \in N$$

DA Generator Limits:

$$0 \le P_i \le P_i^{max} \quad \forall i \in I \quad (2)$$

DA Wind Commitment Limits:

$$0 \le P_k^W \le \min(W_k^{forecast}, W_k^{max}) \quad \forall k \in K \quad (3)$$

DA DC Power Flow:

$$f_{n,m}^{DA} = B_{n,m}(\vartheta_n^{DA} - \vartheta_m^{DA}) \quad \forall (n,m) \in E \quad (4)$$

DA Line Thermal Limits:

$$-F_{n,m}^{max} \le f_{n,m}^{DA} \le F_{n,m}^{max} \quad \forall (n,m) \in E \quad (5)$$

DA Slack Bus Reference:

$$\vartheta_{n_1}^{DA} = 0 \quad (6)$$

DA Load Shedding Limits:

$$0 \le L_d^{shed,DA} \le L_d \quad \forall d \in D \quad (7)$$

3.3 Real-Time (RT) Recourse Constraints

RT Nodal Power Balance:

$$\sum_{i:(i,n)} r_i + \sum_{k:(k,n)} (W_k^{real} - P_k^W - P_k^{spill}) + \sum_{d:(d,n)} (L_d^{shed} - L_d^{shed,DA}) = \sum_{m:(n,m)\in E} (f_{n,m}^{RT} - f_{n,m}^{DA}) \quad \forall n \in N$$

RT Ramping Limits:

$$-R_i^{\max} \le r_i \le R_i^{\max} \quad \forall i \in I \tag{8}$$

RT Total Generation Feasibility:

$$0 \le P_i + r_i \le P_i^{\max} \quad \forall i \in I \tag{9}$$

RT DC Power Flow:

$$f_{n,m}^{RT} = B_{n,m}(\vartheta_n^{RT} - \vartheta_m^{RT}) \quad \forall (n,m) \in E \tag{10}$$

RT Line Thermal Limits:

$$-F_{n,m}^{\max} \le f_{n,m}^{RT} \le F_{n,m}^{\max} \quad \forall (n,m) \in E \tag{11}$$

RT Load Shedding Bounds:

$$0 \le L_d^{shed} \le L_d \quad \forall d \in D \tag{12}$$

RT Wind Spillage Limits:

$$0 \le P_k^{spill} \le W_k^{real} \quad \forall k \in K \tag{13}$$

RT Slack Bus Reference:

$$\vartheta_{n_1}^{RT} = 0 \tag{14}$$

4 Stochastic Market Clearing (SMC) Formulation

The SMC model is a two-stage LP that minimizes the day-ahead cost plus the expected recourse cost over all uncertainty scenarios S.

4.1 Objective Function

$$\min Z_{stoch} = \sum_{i\in I} C_i P_i + \sum_{s\in S} \phi_s \left[\sum_{i\in I} C_i r_{i,s} + \sum_{d\in D} V_d L_{d,s}^{shed} \right] \tag{15}$$

4.2 Day-Ahead (DA) Constraints

The DA decisions (P_i, P_k^W, $f_{n,m}^{DA}$, ϑ_n^{DA}) are identical for all scenarios. We assume no load shedding in the day-ahead stage for the stochastic model.

DA Nodal Power Balance:

$$\sum_{i:(i,n)} P_i + \sum_{k:(k,n)} P_k^W - \sum_{d:(d,n)} L_d = \sum_{m:(n,m)\in E} f_{n,m}^{DA} \quad \forall n \in N \tag{16}$$

DA Generator Limits:

$$0 \le P_i \le P_i^{\max} \quad \forall i \in I \tag{17}$$

DA Wind Commitment Limits:

$$0 \le P_k^W \le W_k^{max} \quad \forall k \in K \tag{18}$$

DA Network Constraints: The DC Power Flow, Line Limits, and Slack Bus Reference from the deterministic section apply identically to the DA stage.

$$f_{n,m}^{DA} = B_{n,m}(\vartheta_n^{DA} - \vartheta_m^{DA}) \quad \forall (n,m) \in E \tag{19}$$

$$-F_{n,m}^{\max} \le f_{n,m}^{DA} \le F_{n,m}^{\max} \quad \forall (n,m) \in E \tag{20}$$

$$\vartheta_{n_1}^{DA} = 0 \tag{21}$$

4.3 Real-Time (RT) Recourse Constraints (Per Scenario s)

RT Nodal Power Balance:

$$\sum_{i:(i,n)} r_{i,s} + \sum_{k:(k,n)} (W_{k,s} - P_k^W - P_{k,s}^{spill}) + \sum_{d:(d,n)} L_{d,s}^{shed} = \sum_{m:(n,m)\in E} (f_{n,m,s}^{RT} - f_{n,m}^{DA}) \quad \forall n \in N, s \in S$$

RT Ramping Limits:

$$-R_i^{\max} \le r_{i,s} \le R_i^{\max} \quad \forall i \in I, s \in S \tag{22}$$

RT Total Generation Feasibility:

$$0 \le P_i + r_{i,s} \le P_i^{\max} \quad \forall i \in I, s \in S \tag{23}$$

RT DC Power Flow:

$$f_{n,m,s}^{RT} = B_{n,m}(\vartheta_{n,s}^{RT} - \vartheta_{m,s}^{RT}) \quad \forall (n,m) \in E, s \in S \tag{24}$$

RT Line Thermal Limits:

$$-F_{n,m}^{\max} \le f_{n,m,s}^{RT} \le F_{n,m}^{\max} \quad \forall (n,m) \in E, s \in S \tag{25}$$

RT Load Shedding Bounds:

$$0 \le L_{d,s}^{shed} \le L_d \quad \forall d \in D, s \in S \tag{26}$$

RT Wind Spillage Limits:

$$0 \le P_{k,s}^{spill} \le W_{k,s} \quad \forall k \in K, s \in S \tag{27}$$

RT Slack Bus Reference:

$$\vartheta_{n_1,s}^{RT} = 0 \quad \forall s \in S \tag{28}$$

5 Numerical Results and Analysis

The following tables present the results from the GAMS implementations of the DMC and SMC models.

5.1 Input Data

The wind scenario matrix used for the SMC formulation is presented in Table 3. The probability of each scenario is $\phi_s = 0.25$.

Table 3: Wind Power Realizations by Scenario (MW)

Wind Farm	s_1 (90 MW)	s_2 (130 MW)	s_3 (150 MW)	s_4 (50 MW)
k_1	40	60	70	20
k_2	50	70	80	30
Total	90	130	150	50

5.2 Day-Ahead Dispatch

Table 4 compares the day-ahead commitment, revealing the strategic difference between the two models.

Table 4: Day-Ahead Dispatch Comparison (MW)

Resource	DMC Dispatch	SMC Dispatch	Flexibility Change
P_{i_1} ($10/MWh)	150.0	80.0	−70.0
P_{i_2} ($15/MWh)	40.0	50.0	+10.0
P_{i_3} ($25/MWh)	0.0	60.0	+60.0
Total Conv.	190.0	190.0	0.0
Wind Committed	20.0	20.0	0.0

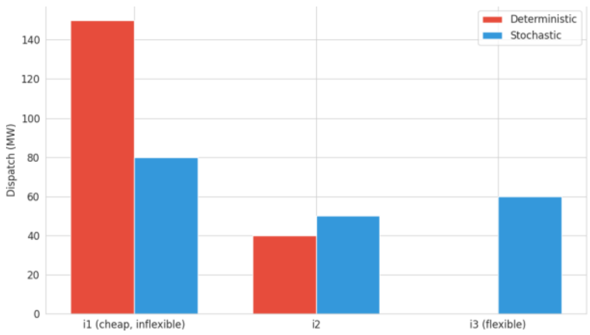

Figure 2: Day-Ahead Dispatch Comparision.

The deterministic model over-commits cheap G1 due to low wind forecast. The stochastic model commits expensive G3 for flexibility. The SMC model intentionally utilizes the higher-cost, high-ramping unit i_3 (60 MW), reducing its reliance on the cheapest, inflexible unit i_1. This commitment to flexible capacity is the mechanism through which SMC hedges against uncertainty.

5.3 Real-Time Adjustments and Recourse Actions

Table 5 shows the recourse action required for the single deterministic realization, and Table 6 shows the flexibility across all four scenarios in the stochastic case.

Table 5: Deterministic Real-Time Adjustments (Single Realization)

Variable	Value
r_{i_1} (Ramp: 0)	0.0 MW
r_{i_2} (Ramp: 50)	−40.0 MW
r_{i_3} (Ramp: 60)	0.0 MW
Load Shedding	0.0 MW
Wind Spillage	0.0 MW

Table 6: Stochastic Real-Time Adjustments by Scenario (MW)

Variable	s_1 (90)	s_2 (130)	s_3 (150)	s_4 (50)
$r_{i_1,s}$	0.0	0.0	0.0	0.0
$r_{i_2,s}$	−10.0	−50.0	−50.0	0.0
$r_{i_3,s}$	−60.0	−60.0	−60.0	−30.0
$P^{spill}_{k,s}$	0.0	0.0	20.0	0.0
$L^{shed}_{d,s}$	0.0	0.0	0.0	0.0
RT Cost ($)	−1,650	−2,250	−2,250	−750

The stochastic solution demonstrates superior recourse management. Generators i_2 and i_3 utilize their full downward ramping capability in high-wind scenarios (s_2, s_3) to prevent over-generation shown in Fig. **??**. In the highest-wind scenario (s_3), the system optimally chooses to spill 20 MW of wind rather than exceed generator minimums or line limits. Importantly, the **SMC model achieves zero load shedding across all scenarios**, ensuring high reliability.

5.4 Recourse Actions in Worst-Case Scenario

Table 7 presents the recourse actions in the worst-case scenario (s_4).

Table 7: Recourse Actions in Worst-Case Scenario (s_4)

Action	Deterministic	Stochastic
Load Shedding (MW)	0	0
Wind Spillage (MW)	0	0
G3 Down-ramp (MW)	0	30

Figure 3: Comparison of Total System Cost.

5.5 Cost and Price Comparison

Table 8 compares the LMPs. The prices are uniform across all buses, indicating no transmission congestion in the optimal solutions.

Table 8: Day-Ahead and Real-Time Nodal Prices ($/MWh)

Bus	DMC Day-Ahead	SMC Day-Ahead
All Buses	0.00	10.00
Real-Time Prices (Worst-Case Scenario s_4)		
All Buses	0.00	25.00

Figure 4: Comparison of Total System Cost.

In the DMC case, the zero LMP is a **market signal failure**, as it does not reflect the marginal cost of the committed capacity ($10/MWh). Conversely, the SMC day-ahead LMP of **$10.00/MWh** is set by the marginal cost of the cheapest utilized generator (i_1), providing an accurate signal. In the real-time scenario s_4, the SMC LMP rises to **$25.00/MWh**, set by the high-cost generator i_3, which is necessary to meet the lower-than-expected demand coverage.

The stochastic model produces economically meaningful uniform DA prices ($10) and higher RT prices in low-wind scenarios, properly signaling scarcity. The overall economic benefit of adopting the stochastic approach is summarized in Table 9.

Table 9: Total System Cost Comparison

Cost Component	DMC Cost ($)	SMC Exp. Cost ($)
Day-Ahead Cost	2,100	3,050
Real-Time/Expected	−600	−1,725
Total Cost	**1,500**	**1,325**
Cost Reduction with SMC: 175 (11.67%)		

6 Discussion and Operational Implications

The stochastic model anticipates low-wind scenarios by:

- Keeping expensive but flexible G3 online
- Reducing reliance on inflexible G1
- Achieving zero load shedding in all scenarios
- Lowering expected total cost by 11.67%

Although day-ahead cost is slightly higher, the reduction in expected recourse cost more than compensates—demonstrating the value of uncertainty modeling.

The numerical analysis underscores that the perceived risk of high-cost generation in the stochastic day-ahead dispatch is, in fact, an **investment in operational flexibility**. The SMC model shifts the paradigm from simple cost minimization to **expected risk management**.

Figure 5: Comparison of DMC and SMC Metrics.

The **11.67% cost savings** achieved by SMC is due to two factors: a more optimal initial commitment and minimized recourse actions in expectation. By scheduling the flexible generator i_3 at 60 MW, the SMC solution allows for deep downward adjustments (≤ -60 MW) in high-wind scenarios, avoiding costly spillage or line congestion penalties that could otherwise materialize in the DMC model if it faced a broader set of realizations.

The reliability outcome is equally significant. While DMC avoided load shedding in the single realization tested, SMC is proven to be feasible and secure across four distinct scenarios, including the worst-case s_4. System operators gain valuable **reliability insurance** by adopting the stochastic framework.

The clear and varying LMPs under SMC provide crucial **price signals** that incentivize generators and loads to participate in system flexibility services, a vital requirement for future grids dominated by variable renewables.

7 Conclusion

In the presence of significant wind uncertainty, stochastic market clearing significantly outperforms deterministic approaches. This comparative study rigorously demonstrated the superiority of **Stochastic Market Clearing** over the Deterministic approach for day-ahead power system operations under wind uncertainty. The SMC framework not only delivered a **11.67% reduction in expected total system costs** but also provided a significantly more robust and reliable operational plan, achieving **zero load shedding** across all tested scenarios.

The expected cost savings of over 11% and elimination of load shedding justify the modest increase in computational burden. This enhanced performance stems from SMC's strategic use of flexible, higher-cost generation capacity in the day-ahead stage as a hedge against future uncertainty. The resulting marginal prices are economically reflective of system costs, serving as effective market signals.

System operators managing high renewable penetration should transition from deterministic to stochastic or robust frameworks. Based on these findings, we strongly recommend that market operators transition to stochastic optimization to improve economic efficiency and operational stability in power systems with substantial wind power integration. Future work could extend this analysis to multi-stage robust and stochastic-robust optimization problems.

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