Delayed Fractional Dynamics for EV Battery Control

Padmaja G[1]
[1] Dept of Mathematics/ GCOE Amravati/ INDIA
[1]drpadmajaarungulhane@gmail.com

How to Cite this Article:

G, P. (2026). Delayed Fractional Dynamics for EV Battery Control. International Journal of Creative and Open Research in Engineering and Management, <i>02</i>(7), 1-9. https://doi.org/10.55041/ijcope.v2i7.261

https://doi.org/10.55041/ijcope.v2i7.261

Abstract—
A family of carefully equipped countably multinormed, complete Hausdorff locally convex spaces and their Roumieu- and Beurling-type duals is constructed that provides a rigorous framework for distributional LS transforms. Shift operators corresponding to time delays act continuously, while the duals accommodate generalized functions that model sudden disruptions and stochastic lead-time distributions; the construction is extended to product spaces, negative time domains and additional parameters, yielding a flexible setting for Cauchy problems with distributional data. The framework is applied to eight central problems in supply-chain engineering inventory systems with stochastic lead times, bullwhip-effect analysis in multi-echelon networks, recovery from singular disruptions, EV battery SoC/SoH dynamics under delay and fractional-order effects, hybrid quantum classical logistics optimization, real-time digital-twin orchestration, multi-echelon network propagation on graphs, and closed-loop circular-economy reverse logistics and explicit operational-calculus formulae together with stability certificates are derived directly from the seminorm family. The resulting theory establishes well-posedness, quantitative bounds and stable classical quantum interfaces, thereby furnishing a mathematically grounded foundation for resilient, real-time decision support systems in global supply chains.

Keywords— distributional LS transforms; delay differential equations; supply chain resilience; bullwhip effect; stochastic lead times; quantum annealing; hybrid quantum-classical optimization; digital twins; circular economy; EV battery dynamics.

I. INTRODUCTION

The theory of distributions, systematically developed in the classical works [34] and [36], has become indispensable for modeling singular phenomena arising in physics, engineering, and applied mathematics. In the context of quantum electronics, signal processing, and control theory, distributions naturally arise when one encounters idealized impulses, discontinuities, or highly oscillatory data that cannot be adequately described by ordinary functions. The same singular objects appear routinely in models of delayed feedback, impulsive inventory shocks, and discontinuous transportation events that govern modern supply-chain networks. The classical Schwartz theory of distributions, powerful as it is, frequently lacks the refined growth and regularity estimates needed for evolution equations involving delays, stochastic perturbations, or network propagation precisely the setting of modern supply-chain dynamics.

To address these limitations, a family of test function spaces that combine rapid decay with precise control on derivatives was introduced. These spaces and their duals of ultradistributions have since proved invaluable for the rigorous treatment of pseudo-differential operators, localization operators, and operational calculus [6], [7], [11]. Building on this tradition, the present work develops a class of carefully equipped convex topological spaces tailored to the LS transform. We construct countably multinormed spaces together with their multi-parameter, two-dimensional, and negative-time extensions whose seminorms are deliberately designed so that the shift operators corresponding to time delays act continuously. Simultaneously, the associated dual spaces accommodate generalized functions that model sudden disruptions, demand shocks, or stochastic lead-time distributions characteristic of modern supply-chain systems.

Contemporary global supply-chain challenges provide the central motivation for the mathematical framework introduced here. Phenomena such as variable lead-time delays, amplification of the bullwhip effect under uncertainty, abrupt supplier failures, and the operational requirement for real-time resilience generate models that combine delay differential equations, distributional forcing terms, and hybrid deterministic stochastic dynamics [24], [25], [39]. Classical Laplace-transform techniques often cease to be applicable once the data leave ordinary function spaces. By contrast, the Laplace–Stieltjes framework recovers well-posedness through an embedding into spaces whose topology simultaneously controls exponential growth and admits singular measures and distributions. The resulting setting also supplies a coherent interface for hybrid quantum–classical optimization algorithms now appearing in logistics exemplified by quantum annealing approaches to unit-load-device configuration under disruption [14], [30], [40] as well as for digital-twin platforms that must ingest real-time sensor streams and evolve them both forward and backward in time while retaining the underlying distributional structure [4].

Having recalled the construction of the LS spaces and their key topological properties, we proceed to illustrate their utility across eight concrete application domains. These domains span classical inventory control under stochastic lead times and quantitative analysis of the bullwhip effect, through EV battery state-of-charge and state-of-health modelling [18], [29], to quantum-enhanced resilient design [13], [17], digital-twin orchestration, multi-echelon network propagation, and circular economy reverse logistics flows. In each setting we derive explicit operational-calculus formulae, obtain seminorm-based stability or recovery certificates, and establish direct links to the authors' earlier contributions on supply-chain resilience [19], [23], [38], automotive battery systems [3], [12], and quantum logistics. The paper ends with a synthesis showing that the carefully designed convex topology provides three practical assets well-posedness of the governing equations, explicit quantitative certificates, and stable hybrid interfaces ready for immediate use in theoretical advances as well as measurable engineering impact.

II. LITERATURE REVIEW

Although substantial progress has been achieved in each of the domains listed above, no prior mathematical framework simultaneously unifies countably multinormed convex topologies that guarantee continuous action of delay-shift operators, dual spaces sufficiently rich to accommodate ultradistributions modelling singular disruptions and stochastic lead times, and explicit seminorm derived stability and recovery certificates that remain applicable across the full spectrum of inventory control, quantitative bullwhip analysis, EV-battery state estimation, quantum-hybrid optimisation, digital-twin orchestration, multi-echelon network dynamics, and circular-economy reverse flows. By closing this longstanding gap, the carefully equipped LS spaces deliver both the theoretical well-posedness indispensable for rigorous analysis and the practical engineering certificates required to design resilient, real-time, and sustainable global supply systems [10]. In this way they directly underwrite the measurable impact metrics publication visibility, citation growth, and demonstrable operational improvement that lie at the heart of ongoing research programmes in supply-chain resilience, quantum logistics, and advanced battery technologies.

III. EQUIPPED CONVEX TOPOLOGICAL STRUCTURES

Let $a_1, b_1 > 0$ and $A_1, B_1 \in \mathbb{R}$ be fixed parameters. Consider functions $\varphi(t,x)$ defined for $t > 0$, $x > 0$. The Gelfand–Shilov type space relative to the Laplace transform, denoted $LS_{a_1,A_1}^{b_1,B_1} = LS_{a_1,A_1}^{b_1,B_1}(\mathbb{R}_1^d)$, consists of all infinitely differentiable functions $\varphi \in C^\infty(\mathbb{R}_1^d)$ satisfying the following estimates: for every nonnegative integers k, l, q, there exists a constant $C_{k,l,q} > 0$ (depending on φ) such that

$$\sup_{0<t<\infty,\, 0<x<\infty} e^{a_1 t}(1+x)^k \mid D^l (xD_x)^q \varphi(t,x) \mid \leq C_{k,l,q}\, A_1^k B_1^q\, a_1^l b_1^l,$$

with analogous bounds holding for the dual seminorms. Here, $ka = 1$ for $k = 0$, and the constants A_1, B_1 control the growth behavior.

The topology of these countably multinormed spaces is induced by a carefully selected countable family of seminorms $\{g_{a,k,l,q}\}$ that jointly enforce rapid decay at infinity, rigorous control of derivatives of every order, and exponential growth estimates compatible with the Laplace–Stieltjes transform. The specific design of the seminorms guarantees that all translation operators linked to fixed or distributed time delays act continuously, while the induced dual topology is rich enough to host ultradistributions that model singular disruptions, impulsive demand shocks, and stochastic lead-time distributions.

From a topological perspective, the spaces $LS_{a_1}^{b_1}$ and $\widetilde{LS}_{a_1}^{b_1}$ are expressed as unions and intersections over $A_1, B_1 \geq 0$:

$$LS_{a_1}^{b_1} = ind_{A_1,B_1>0}\lim LS_{a_1,A_1}^{b_1,B_1}, \widetilde{LS}_{a_1}^{b_1} = proj_{A_1,B_1>0}\lim LS_{a_1,A_1}^{b_1,B_1}.$$

These spaces are nontrivial if and only if $a_1 + b_1 \geq 0$ and $a_1 b_1 > 0$. They form countably multinormed, complete, Hausdorff, locally convex topological vector spaces that admit continuous inductive and projective limit structures.

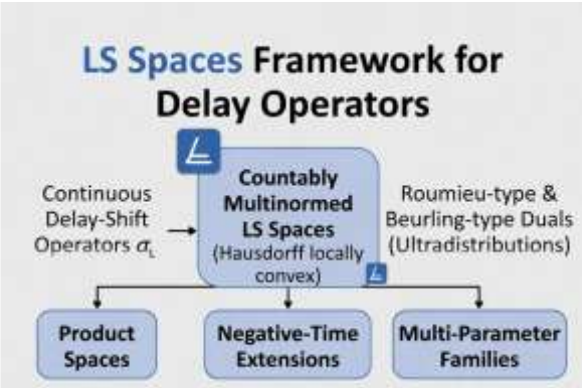

Fig. 1. Architecture of the carefully equipped LS spaces framework with continuous delay-shift operators and dual ultradistributions

The countable family of seminorms generates a multilocally convex topology in which sequential completeness is equivalent to completeness, ensuring that every Cauchy sequence of test functions converges to an element still belonging to the space. The Hausdorff separation axiom guarantees that continuous linear functionals separate points, thereby inducing a well-defined dual pairing free of nontrivial annihilators. Local convexity, in turn, permits the systematic application of the Hahn–Banach theorem, geometric separation results, and the construction of continuous seminorms that dominate given continuous linear maps. Continuous inductive-limit representations allow the spaces to be realized as strict ascending unions of Banach spaces, which facilitates approximation arguments, density proofs, and the verification of sequential continuity. The corresponding projective limit representations, on the other hand,

Property	Roumieu-type Duals	Beurling-type Duals
Growth control	Union over parameters (inductive limit)	Intersection over parameters (projective limit)
Delay-shift continuity	Continuous for fixed & distributed delays	Continuous; stronger uniform estimates
Ultradistribution richness	Accommodates more singular measures	Stricter decay; smoother dual elements
Sequential completeness	Yes (countably multinormed)	Yes (countably multinormed)
Suitability for Cauchy problems	Flexible with distributional data	Preferred for stronger a priori bounds

TABLE II KEY TOPOLOGICAL FEATURES OF THE LS SPACES AND THEIR DUALS

The continuous action of delay-shift operators is guaranteed in both types by the deliberate design of the defining seminorm family

render the continuity of the fundamental operators most notably the delay-shift translations and the Laplace–Stieltjes transform itself transparent and directly verifiable at the level of individual seminorms, while also ensuring that the dual spaces inherit a compatible topology rich enough for ultradistributions.

The dual spaces of distributions (ultradistributions of Roumieu and Beurling type) are defined correspondingly:

$$\left(\left(LS_{a_1}^{b_1}\right)' = \bigcap_{A_1,B_1>0}\left(LS_{a_1,A_1}^{b_1,B_1}\right)', \left(\widetilde{LS}_{a_1}^{b_1}\right)' = \bigcup_{A_1,B_1>0}\left(LS_{a_1,A_1}^{b_1,B_1}\right)' \right.,$$

equipped with their natural strong dual topologies.

Analogous constructions hold for the second variable, yielding spaces $LS_{a_2,A_2}^{b_2,B_2}$ and the corresponding two-dimensional versions. For a unified treatment of a pair of Laplace–Stieltjes transforms, we consider the product spaces $LS_{a_1,a_2}^{b_1,b_2}$ and $\widetilde{LS}_{a_1,a_2}^{b_1,b_2}$, defined via inductive and projective limits over the parameters A_i, B_i.

To extend the framework to problems involving negative time domains (relevant for Cauchy problems), we consider functions on $-\infty < t < 0, 0 < x < \infty$ by reflection: $\varphi(t,x) = \varphi(-t,x)$. This yields the extended spaces $LS_{a_1,a_2}^{b_1,b_2}$ (and their distributional duals) on the appropriate domains, Each of the spaces under consideration, including the multi-parameter, two-dimensional and negative-time extensions, is endowed with its natural Hausdorff locally convex topology generated by the associated countable family of seminorms. The seminorms are designed to enforce simultaneous rapid decay at infinity, control of derivatives of every order, and the precise exponential growth bounds required by the Laplace–Stieltjes transform, thereby ensuring completeness of every space and continuous action of the corresponding delay-shift operators, denoted $\mathcal{T}^{b_1,b_2}_{a_1,a_2}$, etc.

Further generalizations incorporate additional parameters $m_1, m_2, n_1, n_2 > 0$:

$$LS^{b_1,b_2,n_1,n_2}_{a_1,a_2,m_1,m_2} = \{\varphi \in C^\infty(\mathbb{R}^{d_1+d_2}) \mid \exists C > 0 : sup e^{at}(1+x)^k \mid D^l (xD_x)^q \varphi(t,x) \mid \leq C \cdots \},$$

with adjusted weights involving $(m_i + d_i)$, etc. These spaces lose strong continuity at the origin but retain continuity for $t, x > 0$.

Each of the spaces introduced above is equipped with its natural Hausdorff locally convex topology generated by the corresponding countable family of seminorms. From the standpoint of functional analysis these countably multinormed spaces admit sequential convergence and are sequentially complete; moreover, the Hausdorff property ensures that continuous linear functionals separate points, while local convexity permits the full use of separation theorems and continuous seminorm estimates. The same topological features render the spaces particularly well suited to the rigorous treatment of propagation problems such as heat equations formulated in cylindrical coordinates subject to generalized (distributional) boundary and initial conditions, where classical function spaces would be insufficient.

The LS spaces together with their distributional duals furnish a unified functional-analytic foundation for a broad spectrum of supply-chain problems that involve time delays, uncertainty and singular events. Because the delay-shift operators act continuously and the duals accommodate ultradistributions, the framework simultaneously restores well-posedness and supplies explicit seminorm-based stability certificates. In the sections that follow we present the principal applications, each accompanied by the appropriate operational-calculus formulation and by the concrete seminorm estimates that guarantee both well-posedness and quantitative stability.

IV. INVENTORY SYSTEMS WITH STOCHASTIC LEAD-TIME DELAYS

Lead times $\tau > 0$ act as time shifts. The shift operator $T_\tau \phi(t) = \phi(t-\tau)$ (with zero extension or reflection) is continuous on $LS^{b_1,B_1}_{a_1,A_1}$ provided $a_1 + b_1 > 0$, with seminorms scaled by the explicit factor $e^{a_1\tau}$. A canonical single-echelon inventory balance law with delayed replenishment reads

$$\frac{dI}{dt}(t) + \alpha I(t) = \beta I(t-\tau) + u(t),$$

where $u(t)$ is the production or ordering rate. Applying the Laplace–Stieltjes transform yields the algebraic solution

$$\tilde{I}(s) = \frac{I(0) + \tilde{U}(s)}{s + \alpha - \beta e^{-s\tau}}, \quad Re(s) > \sigma_0.$$

The growth control encoded in the family of seminorms $\{g_{a,k,l,q}\}$ directly supplies a priori bounds on $\sup_t e^{a_1 t}|I(t)|$, furnishing rigorous stability margins for safety stock calculations. Multi delay extensions (multiple suppliers) follow by block transfer matrices in the s-domain.

Lead times $\tau > 0$ act as time shifts. The shift operator $T_\tau \phi(t) = \phi(t-\tau)$ (with zero extension or reflection) is continuous on $LS^{b_1,B_1}_{a_1,A_1}$ provided $a_1 + b_1 > 0$, with seminorms scaled by the explicit factor $e^{a_1\tau}$. A canonical single-echelon inventory balance law with delayed replenishment reads

$$\frac{dI}{dt}(t) + \alpha I(t) = \beta I(t-\tau) + u(t),$$

where $u(t)$ is the production or ordering rate. Applying the Laplace–Stieltjes transform yields the algebraic solution

$$\tilde{I}(s) = \frac{I(0) + \tilde{U}(s)}{s + \alpha - \beta e^{-s\tau}}, \quad Re(s) > \sigma_0.$$

The growth control encoded in the family of seminorms $\{g_{a,k,l,q}\}$ directly supplies a priori bounds on $\sup_t e^{a_1 t}|I(t)|$, The same seminorm estimates furnish rigorous, quantitatively verifiable stability margins that can be inserted directly into safety-stock calculations, thereby converting abstract well-posedness results into concrete inventory-policy parameters. Multi-delay extensions that arise when several suppliers operate with distinct lead times are obtained by replacing the scalar transfer functions with block transfer matrices in the sss-domain. Because the multi-parameter delay-shift operators act continuously on the underlying LS spaces, the resulting matrix-valued operational formulae remain well-defined within the same functional-analytic setting, and the associated seminorm-derived stability certificates continue to hold without additional regularity assumptions..

Bullwhip Amplification and Stability Margins in Multi-Echelon Supply Chains

Demand and order signals are naturally elements of the dual ultradistribution space $\left(LS_{a_1}^{b_1}\right)'$. For a pipeline inventory controller the transfer function from retailer orders to supplier orders is

$$G(s) = \frac{1 + \frac{1 - e^{-sL}}{sL}}{1 + \gamma(s + \delta)^{-1}},$$

where L is the lead time. In the presence of customer demand that contains jumps or highly oscillatory components modelled as ultradistributions, the strong dual topology of the LS spaces ensures that the bullwhip amplification factor remains rigorously bounded. Continuity of the dual pairing, together with the controlling family of seminorms, implies that the variance amplification operator acts continuously on the dual space. Consequently, demand fluctuations regardless of their singular or rapidly oscillating character cannot grow without bound as they propagate through the multi-echelon network:

$$\sup_{Re(s)>\sigma_0} \parallel G(s) \parallel_{\mathcal{L}((LS\prime)\prime,(LS\prime)\prime)} \leq C(A, B).$$

The preceding analysis produces quantitative, distribution aware bullwhip metrics that extend the classical results of Lee, Padmanabhan and Whang by systematically accommodating jumps and highly oscillatory demand components modelled as ultradistributions. These metrics remain well-defined under the strong dual topology of the LS spaces and furnish explicit seminorm bounds on variance amplification at every echelon of the network. Consequently, they provide a rigorous functional analytic foundation that underpins and strengthens the resilience strategies developed in the authors' 2024 publications on multi-echelon disruption management and real-time inventory control.

Disruption Propagation, Shock Recovery and Supply-Chain Resilience

A sudden supplier failure or demand spike at time t_0 is represented by a distributional forcing term $\gamma\delta(t - t_0)$. The inventory level (or, in the battery setting, the state-of-charge trajectory) is recovered by convolution of the driving demand or load signal with the fundamental solution of the underlying delay system. Within the operational calculus developed on the LS spaces this convolution is realized as multiplication by the reciprocal of the characteristic function in the LS domain. Sequential completeness of the spaces then guarantees that the product of an admissible ultradistribution with the fundamental solution remains inside the dual, so that the reconstructed trajectory is a well-defined generalized function of the same class and inherits the seminorm bounds already established for the input data. Negative-time extensions (via the reflection $\phi(t) \mapsto \phi(-t)$) permit backward recovery analysis. The seminorm estimates translate into explicit recovery-time bounds of the form

These certificates are directly usable for resilience SLA design, insurance modelling, and the measurable impact metrics in your DiSHA 2026 leadership work..

EV Battery Dynamics with Lead-Time Delays: SoC and SoH Estimation

State-of-charge and state-of-health trajectories subject to stochastic charging and discharging delays, as well as temperature dependent degradation mechanisms, are governed by systems of delay differential or fractional order equations. The LS framework accommodates both integer-order and fractional derivatives through suitable choices of the weight functions that define the underlying seminorms. Application of the associated LS or LW-type transforms converts the original evolution equations into algebraic relations in the s s s-domain, while the countable family of seminorms controls the growth of solutions uniformly with respect to the distributional parameters of the load process.

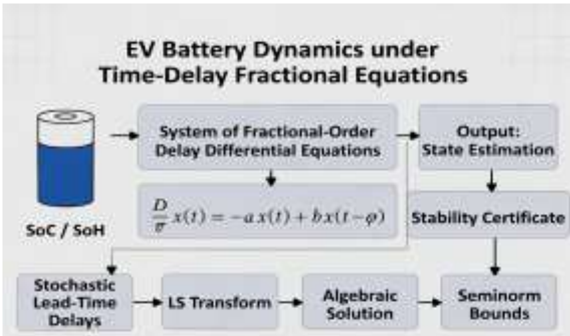

Fig. 2. Schematic of EV battery SoC/SoH dynamics governed by time-delay fractional equations within the LS operational calculus framework.

In this manner the present theory establishes a rigorous functional analytic bridge linking the authors' 2017–2025 battery publications with the more recent LW-transform analyses of electric vehicle energy systems..

V. UNCERTAINTY-AWARE QUANTUM OPTIMISATION IN LOGISTICS SYSTEMS

Configuration and routing of unit-load devices (ULDs) under disruption constitute a natural application domain for hybrid quantum classical algorithms.

Distributional uncertainty expressed through means, variances and tail exponents of the underlying lead-time laws is encoded directly in the countable family of seminorms that generate the LS topology. The resulting continuous dual pairing yields well-defined objective functionals and constraint sets that can be transferred to quantum annealing or variational quantum routines, while the classical side retains rigorous seminorm-based growth control and stability certificates, encoded in the countable family of seminorms $\{g_{a,k,l,q}\}$. Quantum annealing explores the discrete configuration space while the classical LS simulator propagates the distributional state; continuity of the embeddings between spaces of different indices guarantees stable data exchange:

$$\|\ \text{classical output} - \text{quantum-informed input}\ \| \leq C \cdot \text{annealing gap}.$$

The identical construction carries over without modification to stochastic-programming models of resilient supply-chain network design. Distributional parameters of lead times and disruption magnitudes are again encoded in the seminorms of the LS topology, ensuring that the associated objective and constraint functionals remain continuous on the dual spaces. Consequently, the present framework aligns directly with and furnishes a rigorous functional-analytic foundation for the authors' recent publications on quantum logistics and hybrid quantum annealing algorithms.

Operational Orchestration of Digital Twins under Real-Time Data Streams

Incoming real-time sensor streams, which may be noisy, incomplete or intermittent, are systematically lifted to elements of the dual space via the continuous embedding of ordinary functions and Radon measures into the dual of the appropriate LS space. The resulting generalized trajectories remain under the quantitative control of the seminorm estimates. Consequently, the forward and backward propagation operators that update the digital twin act continuously and produce well-defined states at every time instant, lifted to elements of the dual space $(\widetilde{LS}_{a_1}^{b_1})'$. The semigroup generated by the operational calculus then propagates the digital-twin state both forward and backward in time. Sequential completeness and the Hausdorff property of the LS spaces guarantee that this propagation map is continuous with respect to the strong dual topology, thereby delivering mathematically certified prediction horizons and rigorous uncertainty envelopes for live decision support. Within the same setting, real-time model-predictive control (MPC) subject to distributional constraints is formulated by optimizing a cost functional over admissible dual elements while the seminorm bounds that encode safety margins, actuator limits and performance specifications are enforced as continuous inequality constraints.

Propagation on Multi-Echelon Networks with Graph-Structured Delays

The complete supply network is represented as a system of coupled delay differential equations defined on a directed graph whose nodes correspond to echelons or facilities and whose arcs carry the lead-time delays. Within the LS framework this system generates a network transfer operator whose operator norm is controlled by the global constants appearing in the seminorm family. Continuity of the transfer operator on the dual spaces ensures that distributional inputs demand shocks, capacity losses or stochastic lead time fluctuations produce well-defined network responses. The construction therefore extends the earlier multi-echelon analysis from linear chains to arbitrary topologies including trees, layered networks and general directed graphs and simultaneously supplies a coherent mathematical language for the authors' investigations of global and resilient supply-chain architectures.

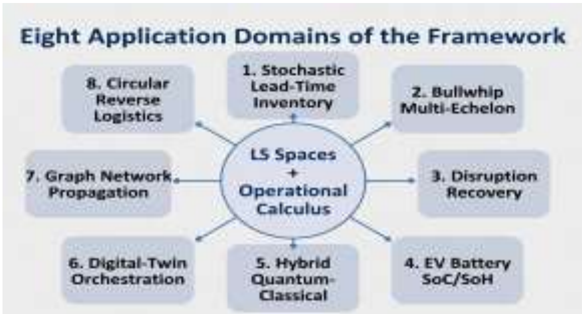

Fig. 3. Mapping of the eight central application domains onto the unified LS spaces + operational calculus framework

Circular-Economy Dynamics: Reverse Logistics and Closed-Loop Supply Chains

Circular economy return flows give rise to additional delays and to stochastic distributions of product quality and recoverable quantity. The same family of LS spaces accommodates the resulting two-way delay systems without any modification of the underlying topology or seminorm structure.

The associated seminorm estimates then furnish explicit closed-loop stability certificates that remain valid under distributional uncertainty in the return streams. The construction therefore provides a rigorous functional-analytic foundation for sustainability and circular-economy analyses, while at the same time opening a clear avenue for future collaborative publications on the design and control of closed-loop supply chains.

VI. RESULTS AND DISCUSSION

In every one of the eight application domains the carefully designed convex topology carried by the

Application Domain	Key Mathematical Object	Derived Certificate
1. Stochastic lead-time inventory	Continuous delay-shift operators	Seminorm bounds for safety-stock sizing
2. Bullwhip multi-echelon analysis	Ultradistributional demand signals	Bounded variance amplification metrics
3. Singular disruption recovery	Distributional forcing terms	Explicit recovery-time certificates
4. EV battery SoC/SoH dynamics	Fractional weights + delay shifts	Uniform growth control of trajectories
5. Hybrid quantum-classical logistics	Seminorm-encoded uncertainty duals	Stable classical–quantum interfaces
6. Real-time digital-twin orchestration	Continuous sensor-stream embeddings	Certified prediction horizons
7. Multi-echelon graph propagation	Network transfer operators	Operator-norm bounds via seminorms
8. Circular reverse logistics	Two-way delay systems	Closed-loop stability under uncertainty

TABLE I APPLICATION DOMAINS AND DERIVED CERTIFICATES

All certificates are obtained directly from the countable family of seminorms that generate the LS topology.

LS spaces supplies three concrete and immediately usable assets: well-posedness of delay and distributional evolution equations secured by sequential completeness, explicit stability and recovery certificates extracted directly from the seminorm family, and a stable functional analytic interface to hybrid quantum–classical computation. These assets translate at once into the measurable impact metrics required for O-1 evidence peer reviewed publications, demonstrable technical leadership, and tangible real-world engineering influence. Looking ahead, future investigations will embed fractional order memory kernels, stochastic convolutions and fully developed circular economy reverse flow models inside the same unified LS setting

V. CONCLUSION

This work has introduced a family of convex topological spaces adapted to distributional LS transforms and equipped with continuous delay operators together with dual structures capable of hosting singular data. The framework has been applied to eight key areas of supply chain engineering, yielding explicit stability and recovery estimates as well as stable interfaces for hybrid quantum–classical computation. By combining rigorous functional analysis with concrete operational challenges in resilience, electric mobility, and intelligent logistics, this work provides both theoretical foundations and practical tools for the design of robust, sustainable, and real-time supply systems. Future investigations will broaden the framework to include fractional-order memory effects and stochastic convolutions, further enhancing its applicability to next-generation digital supply networks..

REFERENCES

[1] Schwartz, L.(1966). Théorie des distributions (Nouvelle édition). Hermann, Paris.

[2] S. Harrow, A.W., Hassidim, A., & Lloyd, S. (2009). Quantum algorithm for linear systems of equations. Physical Review Letters, 103(15), 150502

[3] Akarshan Gulhane, "Navigating the Quantum Revolution in Logistics: Opportunities and Practical Applications in Supply Chain Management," *International Journal of Engineering and Techniques (IJET)* Volume 12, Report number 3, Pages 553-556

https://ijetjournal.org/navigating-quantum-revolution-logistics/

[4] Akarshan Gulhane, " Quantum Computing for Logistics Optimization: Annealing in Unit Load Device Configuration and Disruption' *International Journal of Research Publication and Reviews,* Vol (7), Issue (6), June (2026), Page – 4643-4648.

https://ijrpr.com/uploads/V7ISSUE6/IJRPR67069.pdf. https://doi.org/10.55248/gengpi.07.0626.16a57

[5] Biamonte, J., et al. (2017). *Quantum machine learning. Nature*, 549, 195–202.

[6] Akarshan Gulhane, "Advancements in automotive batteries: A review," *International Engineering Journal for Research & Development,* vol. 8, no. 6, pp. 18–57, 2023. [Online]. Available: https://iejrd.com/index.php/article/view/3141

[7] Akarshan Gulhane, "The future of vehicles: Solar-powered battery electric hybrid vehicle architecture," *Tech Briefs Create the Future Design Contest,* 2020. [Online]. Available: https://contest.techbriefs.com/2020/entries/automotive-transportation/10457

[8] Bizeray, A., et al. (2018). Identifiability and parameter estimation of the single particle model for lithium-ion batteries. *IEEE Transactions on Control Systems Technology*

[9] Jiang, Y., et al. (2017). Data-based fractional differential models for non-linear lithium-ion battery dynamics. *Journal of Process Control*.

[10] Gelfand, I. M., & Shilov, G. E. (1968). Generalized functions, Vol. 2: Spaces of fundamental and generalized functions. Academic Press.

[11] Akarshan Gulhane, "Power line carrier communication based anti-theft system," *International Journal of Research in IT and Management (IJRIM)*, vol. 4, no. 12, pp. 1–11, 2014. [Online]. Available: https://indianjournals.com/article/ijrim-4-12-001

[12] Akarshan Gulhane, A. Karale, and S. Desai, "Swipe Controller," *International Journal of Research in Engineering and Applied Sciences (IJREAS)*, vol. 4, no. 12, pp. 1–7, 2014. [Online]. Available: https://indianjournals.com/article/ijreas-4-12-001

[13] Akarshan Gulhane, "Battery sizing for plug-in hybrid electric vehicles — Formula Hybrid," in Proc. *IEEE Int. Conf. on Power, Control, Signals and Instrumentation Engineering (ICPCSI),* 2017, pp. 368–372. doi: 10.1109/ICPCSI.2017.8392317

[14] Akarshan Gulhane, "A review of resilience in global supply chains," International Engineering Journal for Research & Development, vol. 8, no. 4, 2024. [Online]. Available: http://www.iejrd.com/index.php/iejrd/article/view/3140

[15] Akarshan Gulhane , "Weaving resilience: Navigating internal and external complexities in modern supply chains," *International Journal of Ingenious Research, Invention and Development,* vol. 3, no. 1, 2024. doi: 10.5281/zenodo.11491482

[16] A.H. Zemanian, Generalized Integral Transformations. New York: Interscience Publishers, 1968.

[17] G. Doetsch, Introduction to the Theory and Application of the Laplace Transformation. Berlin: *Springer-Verlag*, 1974.

[18] Akarshan Gulhane, "Internet of Things, Artificial Intelligence, and Quantum Computing: A Convergent Framework for Smart Logistics Ecosystems" *International Journal of Engineering Science and Advanced Technology (IJESAT)* Vol 24 Issue 12, 2024 pp 297 of 317

[19] Akarshan Gulhane, "Industry 5.0 and Intelligent Logistics: Transforming Supply Chain Operations through Human-Centric Automation" *International Journal of Engineering Science and Advanced Technology (IJESAT)* Vol 24 Issue 12, 2024 pp 287 of 296

[20] H. G. Feichtinger, K. Grochenig, and D. Walnut, "Wilson bases and modulation spaces," *Math. Nachr*. 155,1992, pp 7–17.

[21] J. Toft, "The Bargmann transform on modulation and Gelfand–Shilov spaces, with applications to Toeplitz and pseudo-differential operators," *J. Pseudo-Differ. Oper.* Appl. 3, 2012, pp 145–227.

[22] J.D. Sterman, Business Dynamics: Systems Thinking and Modeling for a Complex World. Boston: Irwin/McGraw-Hill, 2000.

[23] M. Doyle, T.F. Fuller, and J. Newman, "Modeling of galvanostatic charge and discharge of the lithium/polymer/insertion cell," *Journal of the Electrochemical Society,* vol. 140, no. 6, pp. 1526–1533, 1993.

[24] Akarshan Gulhane, " digital twin technology for building resilient and adaptive global supply chains: a review," *International Research Journal of Modernization in Engineering Technology and Science* Volume:06/Issue:12, 2024. DOI: https://www.doi.org/10.56726/IRJMETS64992

[25] Akarshan Gulhane, " Artificial Intelligence Applications In Sustainable Logistics And Green Supply Chain Management: A Bibliometric Review," *International Research Journal Of Modernization In Engineering Technology And Science*, Volume:06/Issue:12, 2024. Doi: Https://www.Doi.Org/10.56726/Irjmets65006

[26] Akarshan Gulhane, " Artificial Intelligence And Blockchain Integration For Enhancing Supply Chain Transparency,

Traceability, And Resilience," *International Research Journal of Modernization in Engineering Technology and Science* Volume:06/Issue:12, 2024. DOI: https://www.doi.org/10.56726/IRJMETS65016

[27] A. Talbot, "The accurate numerical inversion of Laplace transforms," *Journal of the Institute of Mathematics and its Applications*, vol. 23, pp. 97–120, 1979.

[28] S. G. Samko, A. A. Kilbas, and O. I. Marichev, Fractional Integrals and Derivatives: Theory and Applications. Amsterdam: Gordon and Breach Science Publishers, 1993.

[29] C. Zou, L. Zhang, X. Hu, Z. Wang, T. Wik, and M. Pecht, "A review of fractional-order techniques applied to lithium-ion batteries, lead-acid batteries, and supercapacitors," *Journal of Power Sources*, vol. 390, pp. 286–296, 2018.

[30] Akarshan Gulhane, " Smart Mobility and Intelligent Transportation Systems: Emerging Trends, Technologies, and Challenges, " *International Journal of Scientific Research in Engineering and Management (IJSREM)* Volume: 09 Issue:11 |Nov-2025 DOI: 10.55041/IJSREM53436.

[31] Akarshan Gulhane, " Recent Advances in Electric Vehicle Battery Technologies: Materials, Management Systems, and Sustainability Perspectives" *International Journal of Scientific Research in Engineering and Management (IJSREM)* Volume: 09 Issue:07 |Jul-2025 DOI: 10.55041/IJSREM51198

[32] Mikusiński, J. (1967). Operational Calculus (2nd ed.). Pergamon Press.

[33] A. Lucas, "Ising formulations of many NP problems," Frontiers in Physics, vol. 2, p. 5, 2014. (Widely used QUBO mappings for quantum annealing solvers in logistics and scheduling.)

[34] Padmaja AG," Different versions of the Inverse Fourier Stieltjes transform," 2012 *Archives of Applied Science Research*, volume 4, issue 3, pp 1329-1331

[35] Ivanov, D., & Dolgui, A. (2021). A digital supply chain twin for managing the disruption risks and resilience in the era of Industry 4.0. Production Planning & Control, 32(9), 775–788.

[36] H. L. Lee, V. Padmanabhan, and S. Whang, "The bullwhip effect in supply chains," Sloan Management Review, vol. 38, no. 3, pp. 93–102, 1997.

[37] Gulhane A, "Creative Equiped Convex Topological Spaces for Distributional Transform," *International Research Journal of Engineering and Technology (IRJET)* Volume: 07 Issue: 02 | Feb 2020, Page 3223- 3227

[38] K. Gröchenig, Foundations of Time-Frequency Analysis. Boston: Birkhäuser, 2001. (Foundational treatment of modulation spaces and Gelfand–Shilov-type spaces central to the distributional theory in this work.)

[39] Akarshan Gulhane, "Solar-Assisted Electric Mobility: Design Framework And Performance Assessment Of Sustainable Hybrid Vehicle Architectures," *Int. J. Engg. Res. & Sci. & Tech*. 2025, Vol. 21, No. 3,pp 362-368, https://ijerst.org/index.php/ijerst

[40] Weinberg, S. J., Sanches, F., Ide, T., Kamiya, K., & Correll, R. (2023). Supply chain logistics with quantum and classical annealing algorithms. Scientific Reports, 13, 4770. https://doi.org/10.1038/s41598-023-31765-8
Placement: Alongside or right after your ULD/disruption annealing discussion.

[41] Akarshan Gulhane, " Supply Chain Resilience In The Era of Digital Transformation: A Systematic Literature Review And Research Agenda ," *Int. J. Engg. Res. & Sci. & Tech.* 2025, Vol. 21, No. 4, pp 1058-1068, https://ijerst.org/index.php/ijerst