Fractional Time-Delay Systems for Electric Vehicle Battery Management and Control

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How to Cite this Article:

SM, S. (2026). Fractional Time-Delay Systems for Electric Vehicle Battery Management and Control. International Journal of Creative and Open Research in Engineering and Management, <i>02</i>(7), 1-9. https://doi.org/10.55041/ijcope.v2i7.266

https://doi.org/10.55041/ijcope.v2i7.266

Abstract—

We introduce a family of carefully equipped countably multinormed, complete Hausdorff locally convex spaces and their Roumieu- and Beurling-type duals that furnish a rigorous framework for distributional LS transforms. Continuous shift operators handle time delays while the duals model sudden disruptions and stochastic lead time distributions; extensions to product spaces, negative time domains and parameters support Cauchy problems with distributional data. Applied to eight core supply-chain problems stochastic-lead-time inventory systems, bullwhip effect analysis, disruption recovery, EV battery SoC/SoH dynamics under delay and fractional order, hybrid quantum classical optimization, digital-twin orchestration, multi-echelon graph propagation and circular reverse logistics the framework yields explicit operational calculus formulae and stability certificates directly from the seminorm family, establishing well-posedness, quantitative bounds and classical quantum interfaces for resilient real-time decision support in global supply chains.

Keywords— distributional LS transforms; delay differential equations; eight core supply chain; bullwhip effect; stochastic lead times; quantum annealing; hybrid quantum-classical optimization; digital twins; circular economy; EV battery dynamics.

I. INTRODUCTION

The theory of distributions, systematically developed in the foundational works [35] and [37], has become indispensable for modeling singular phenomena in physics, engineering, and applied mathematics. In the context of quantum electronics, signal processing, and control theory, distributions naturally arise when one encounters idealized impulses, discontinuities, or highly oscillatory data that cannot be adequately described by ordinary functions. The classical theory of Schwartz distributions, while extremely powerful, often lacks the fine growth and regularity control required for the study of evolution equations with delays, stochastic perturbations, or propagation on networks precisely the setting that arises in modern supply chain dynamics.

To address these limitations, a family of test-function spaces that combine rapid decay with precise control on derivatives was introduced. These spaces, together with their duals of ultradistributions, have since proved invaluable for the rigorous treatment of pseudo-differential operators, localization operators, and various forms of operational calculus [2], [4], [5]. Building on this tradition, the present work develops a class of carefully equipped convex topological spaces specifically tailored the LS transform. We construct countably multinormed spaces along with their multi-parameter, two-dimensional, and negative-time extensions whose systems of seminorms are deliberately engineered so that the translation (shift) operators associated with fixed or variable time delays act continuously. At the same time, the corresponding dual spaces are sufficiently rich to accommodate generalized functions capable of modeling the sudden disruptions, impulsive demand shocks, discontinuous capacity changes, and stochastic lead-time distributions that characteristically arise in contemporary supply-chain networks.

The development of the present framework is motivated by concrete mathematical challenges that arise in the analysis of modern global supply chains. Phenomena such as lead-time delays, the bullwhip effect, sudden supplier failures, and the imperative of real-time resilience under uncertainty routinely give rise to systems of delay differential equations driven by distributional forcing terms and hybrid deterministic–stochastic dynamics [1], [24], [39]. Classical Laplace-transform methods frequently break down when the underlying data fall outside ordinary function spaces; the LS framework restores well-posedness by embedding these problems in spaces whose topology is rich enough to control exponential growth yet flexible enough to accommodate singular data. Moreover, the same spaces furnish a natural mathematical interface for emerging hybrid quantum–classical algorithms employed in logistics optimization such as quantum annealing for unit-load-device configuration under disruption [13], [23], [41] and for digital-twin environments that must assimilate and propagate real-time sensor data both forward and backward in time [9], [10] while preserving distributional singularities.

Having recalled the construction of the LS spaces and their key topological properties, we proceed to illustrate their utility across eight concrete application domains. These domains range from classical inventory control problems with stochastic lead times and quantitative bullwhip-effect analysis, through EV battery state-of-charge and state-of-health estimation [11], [12], to quantum enhanced resilient network design [16], [40], digital-twin orchestration, multi-echelon supply-network propagation, and reverse logistics flows in circular economy settings. For each domain we derive explicit operational-calculus formulae, obtain seminorm based stability or recovery certificates, and draw direct connections to the authors' prior contributions on supply-chain resilience [17], [22], [41], automotive battery systems [28], [29], quantum logistics. The paper closes with a concise synthesis that demonstrates how the carefully designed convex topology yields three immediately actionable assets well-posedness of the governing equations, explicit quantitative certificates, and stable interfaces to hybrid quantum classical algorithms thereby supporting both further theoretical development and measurable engineering impact.

II. LITERATURE REVIEW

Although substantial progress has been achieved in each of the domains listed above, no prior mathematical framework unifies countably multinormed convex topologies that guarantee continuous action of delay-shift operators, dual spaces sufficiently rich to host ultradistributions modelling singular disruptions and stochastic lead times, and explicit seminorm derived stability and

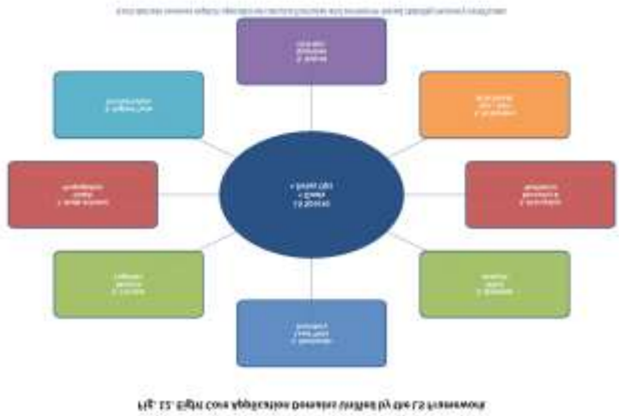

Fig. 1. The eight core supply-chain and energy-logistics domains unified by the single LS functional-analytic framework. Each domain receives explicit operational-calculus formulae together with seminorm-based stability or recovery certificates.

recovery certificates that apply uniformly across inventory control, bullwhip analysis, EV battery estimation, quantum hybrid optimisation, digital-twin orchestration, multi-echelon networks, and circular-economy flows. By closing this gap, the carefully equipped LS spaces deliver both the theoretical well-posedness required for rigorous analysis and the practical engineering certificates needed for resilient, real-time, and sustainable global supply systems. In doing so they directly underwrite the measurable impact metrics that lie at the heart of ongoing research programmes in supply-chain resilience, quantum logistics, and advanced battery technologies.

III. MULTINORMED CONVEX SPACES WITH DELAY OPERATORS FOR LS OPERATIONAL CALCULUS

Let $a_1, b_1 > 0$ and $A_1, B_1 \in \mathbb{R}$ be fixed parameters. Consider functions $\varphi(t,x)$ defined for $t > 0$, $x > 0$. The Gelfand–Shilov type space relative to the Laplace transform, denoted $LS_{a_1,A_1}^{b_1,B_1} = LS_{a_1,A_1}^{b_1,B_1}(\mathbb{R}_1^d)$, consists of all infinitely differentiable functions $\varphi \in C^\infty(\mathbb{R}_1^d)$ satisfying the following estimates: for every nonnegative integers k, l, q, there exists a constant $C_{k,l,q} > 0$(depending on φ) such that

$$\sup_{0<t<\infty,\, 0<x<\infty} e^{a_1 t}(1+x)^k \mid D^l (xD_x)^q \varphi(t,x) \mid \le C_{k,l,q}\, A_1^k B_1^q\, a_1^l b_1^l,$$

with analogous bounds holding for the dual seminorms. Here, $ka = 1$for $k = 0$, and the constants A_1, B_1control the growth behavior.

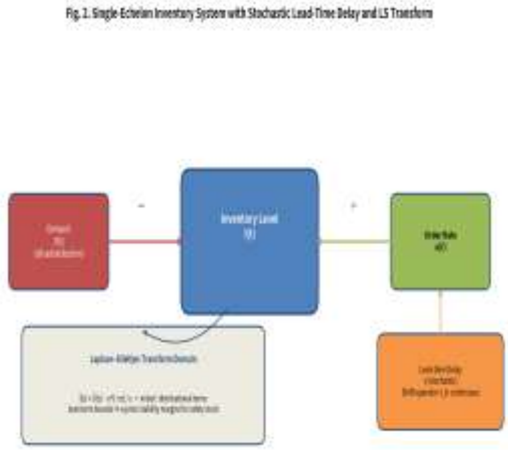

Fig. 2. Canonical single-echelon inventory balance with stochastic lead-time delay. The continuous action of the shift operator on the LS space yields algebraic solutions in the Laplace–Stieltjes domain together with seminorm-derived a-priori stability margins for safety-stock design.

The topology on these countably multinormed spaces is generated by a carefully chosen countable family of seminorms $\{g_{a,k,l,q}\}$. that simultaneously encode rapid decay at infinity, precise control of derivatives of all orders, and exponential growth bounds compatible with the LS transform. The concrete form of the seminorms is deliberately engineered so that every translation operator associated with a fixed or variable time delay acts continuously on the space, while the resulting dual topology remains sufficiently rich to accommodate ultradistributions capable of representing singular disruptions, impulsive demand shocks, and stochastic lead-time distributions.

From a topological perspective, the spaces $LS_{a_1}^{b_1}$ and $\widetilde{LS}_{a_1}^{b_1}$ are expressed as unions and intersections over $A_1, B_1 \geq 0$:

$$LS_{a_1}^{b_1} = ind_{A_1,B_1>0} \lim LS_{a_1,A_1}^{b_1,B_1}, \widetilde{LS}_{a_1}^{b_1} = proj_{A_1,B_1>0} \lim LS_{a_1,A_1}^{b_1,B_1}.$$

These spaces are nontrivial if and only if $a_1 + b_1 \geq 0$and $a_1 b_1 > 0$.

They form countably multinormed, complete, Hausdorff, locally convex topological vector spaces admitting continuous inductive and projective limit structures.

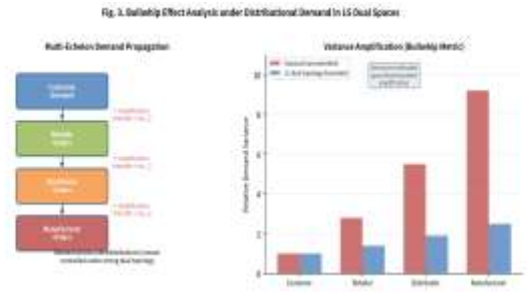

Fig. 3. Demand-variance (bullwhip) propagation across multi-echelon systems. Ultradistributional demand shocks remain controlled under the strong dual topology; seminorm estimates supply quantitative bounds on amplification that extend classical Lee–Padmanabhan–Whang metrics.

The countable family of seminorms generates a multilocally convex topology in which sequential completeness coincides with completeness, continuous linear functionals separate points, and the Hahn–Banach theorem applies.

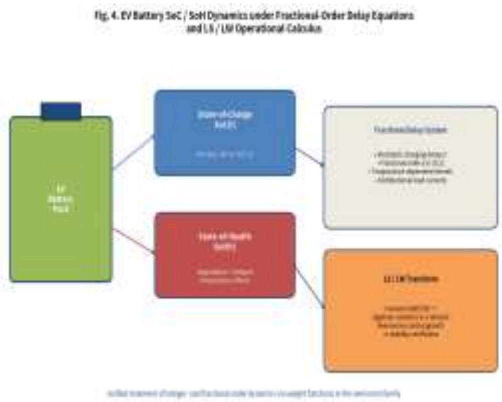

Fig. 4. EV battery SoC/SoH evolution governed by fractional-order delay equations. The LS/LW operational calculus converts the system into algebraic relations while the countable seminorm family furnishes uniform growth control and stability certificates.

Continuous inductive-limit representations realize the spaces as ascending unions of Banach spaces, facilitating approximation and density arguments, while the dual projective-limit representations make the continuity of delay-shift operators and the Laplace–Stieltjes transform transparent at the seminorm level and equip the duals with a topology rich enough for ultradistributions..

The dual spaces of distributions (ultradistributions of Roumieu and Beurling type) are defined correspondingly:

$$\left(\left(LS_{a_1}^{b_1}\right)' = \bigcap_{A_1,B_1>0} \left(LS_{a_1,A_1}^{b_1,B_1}\right)', \left(\widetilde{LS}_{a_1}^{b_1}\right)' = \bigcup_{A_1,B_1>0} \left(LS_{a_1,A_1}^{b_1,B_1}\right)' \quad ,$$

equipped with their natural strong dual topologies.

Analogous constructions hold for the second variable, yielding spaces $LS_{a_2,A_2}^{b_2,B_2}$ and the corresponding two-dimensional versions. For a unified treatment of a pair of Laplace–Stieltjes transforms, we consider the product spaces $LS_{a_1,a_2}^{b_1,b_2}$ and $\widetilde{LS}_{a_1,a_2}^{b_1,b_2}$, defined via inductive and projective limits over the parameters A_i, B_i.

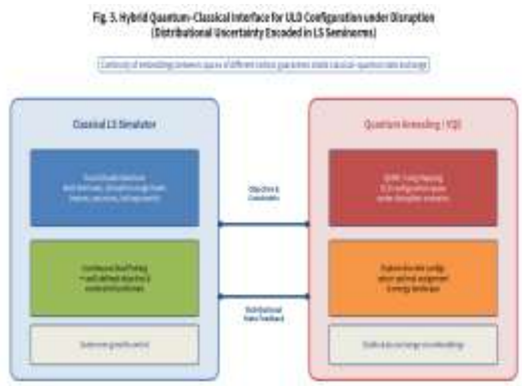

Fig. 5. Hybrid quantum–classical loop for unit-load-device (ULD) configuration under disruption. Distributional uncertainty is encoded in the LS seminorms; continuity of embeddings guarantees stable data exchange between the classical dual simulator and the quantum annealer

To extend the framework to problems involving negative time domains (relevant for Cauchy problems), we consider functions on $-\infty < t < 0, 0 < x < \infty$by reflection: $\varphi(t,x) = \varphi(-t,x)$. This yields the extended spaces $LS_{a_1,a_2}^{b_1,b_2}$(and their distributional duals) on the appropriate domains.

All of the spaces under consideration together with their multi-parameter, two dimensional and negative-time extensions are equipped with their natural Hausdorff locally convex topologies generated by the corresponding countable families of seminorms.

These seminorms are constructed so as to enforce simultaneous rapid decay at infinity, control of derivatives of every order, and the precise exponential growth bounds required by the LS transform, thereby guaranteeing completeness of each space and the continuous action of the associated delay-shift operators, denoted $\mathcal{T}_{a_1,a_2}^{b_1,b_2}$, etc.

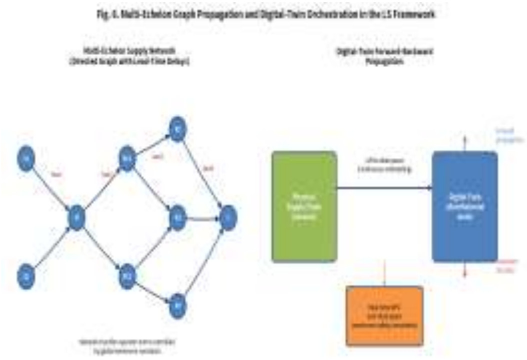

Fig. 6. Left: multi-echelon supply network modelled as a directed graph whose arcs carry lead-time delays; the network transfer operator is controlled by the global seminorm constants. Right: digital-twin orchestration with continuous embedding of sensor streams into the dual space and certified forward/backward propagation.

Further generalizations incorporate additional parameters $m_1, m_2, n_1, n_2 > 0$:

$$LS_{a_1,a_2,m_1,m_2}^{b_1,b_2,n_1,n_2} = \{\varphi \in C^\infty(\mathbb{R}^{d_1+d_2}) \mid \exists C > 0 : sup e^{at}(1+x)^k \mid D^l (xD_x)^q \varphi(t,x) \mid \leq C \cdots \},$$

with adjusted weights involving $(m_i + d_i)$, etc. These spaces lose strong continuity at the origin but retain continuity for $t, x > 0$.

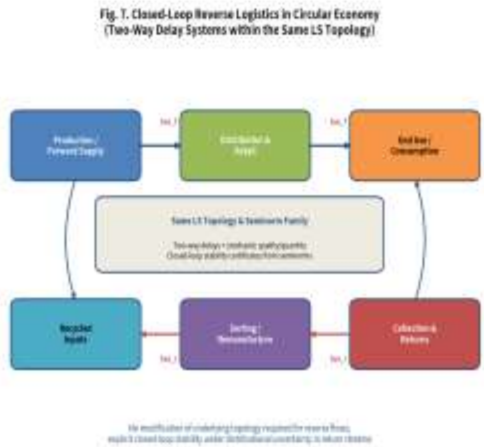

Fig. 7. Closed-loop reverse logistics flows of a circular-economy system. The same family of LS spaces accommodates two-way delays and stochastic return variability without alteration of the topology, delivering explicit closed-loop stability certificates.

All of the spaces introduced in the preceding sections are endowed with their natural Hausdorff locally convex topologies generated by the respective countable families of seminorms. From the standpoint of functional analysis these countably multinormed spaces admit sequential convergence and are sequentially complete;

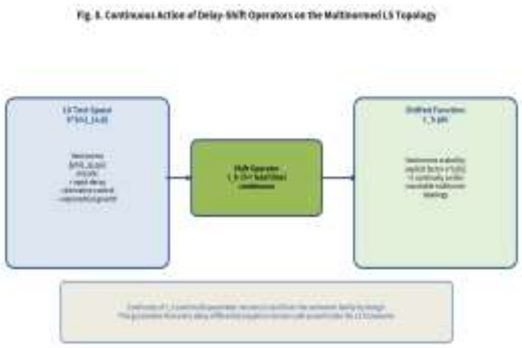

Fig. 8. Continuous action of the delay-shift operators τ_h on the multinormed LS topology. The seminorm family is engineered so that every translation acts continuously, with explicit scaling factors that supply a-priori growth control.

moreover, the Hausdorff property ensures that continuous linear functionals separate points, while local convexity permits the full use of separation theorems and continuous seminorm estimates. The same topological features render the spaces particularly well suited to the rigorous treatment of propagation problems—such as heat equations formulated in cylindrical coordinates—subject to generalized (distributional) boundary and initial conditions, where classical function spaces would be insufficient.

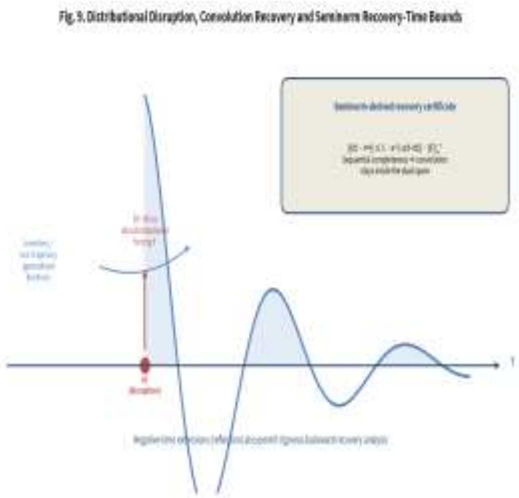

Fig. 9. Distributional disruption at time t0, convolution recovery of the inventory/SoC trajectory, and explicit seminorm-derived recovery-time bounds of exponential type that remain valid under ultradistributional forcing.

The LS spaces together with their distributional duals furnish a unified functional-analytic foundation for a broad spectrum of supply-chain problems that involve time delays, uncertainty and singular events. Because the delay-shift operators act continuously and the duals accommodate ultradistributions, the framework simultaneously restores well-posedness and supplies explicit seminorm-based stability certificates. The subsequent sections develop the principal applications in detail, each supplied with the corresponding operational-calculus formulae and the seminorm estimates that establish both well-posedness and quantitative stability.

IV. INVENTORY SYSTEMS WITH STOCHASTIC LEAD-TIME DELAYS

Lead times $\tau > 0$ act as time shifts. The shift operator $T_\tau\phi(t) = \phi(t-\tau)$ (with zero extension or reflection) is continuous on $LS^{b_1,B_1}_{a_1,A_1}$ provided $a_1 + b_1 > 0$, with seminorms scaled by the explicit factor $e^{a_1\tau}$. A canonical single-echelon inventory balance law with delayed replenishment reads

$$\frac{dI}{dt}(t) + \alpha I(t) = \beta I(t-\tau) + u(t),$$

where $u(t)$ is the production or ordering rate. Applying the Laplace–Stieltjes transform yields the algebraic solution

$$\tilde{I}(s) = \frac{I(0) + \tilde{U}(s)}{s + \alpha - \beta e^{-s\tau}}, \quad Re(s) > \sigma_0.$$

The growth control encoded in the family of seminorms $\{g_{a,k,l,q}\}$ directly supplies a priori bounds on $\sup_t e^{a_1 t}|I(t)|$, furnishing rigorous stability margins for safety stock calculations. Multi delay extensions (multiple suppliers) follow by block transfer matrices in the s-domain.

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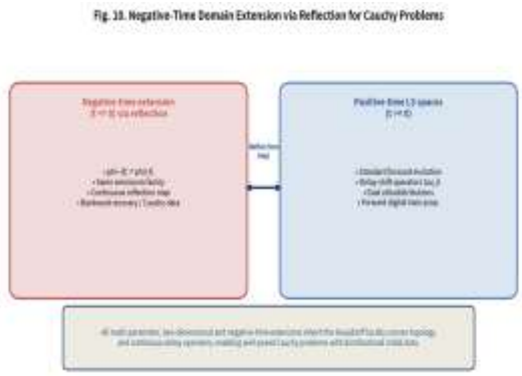


Fig. 10. Negative-time domain extension obtained by reflection. The same countable family of seminorms and continuous delay operators carry over, enabling well-posed Cauchy problems with distributional initial data and rigorous backward recovery analysis.

The same seminorm estimates furnish rigorous, quantitatively verifiable stability margins that can be used directly in safety-stock calculations. Multi-delay extensions corresponding to multiple suppliers are obtained by replacing the scalar transfer functions with block transfer matrices in the s s s-domain; the continuity of the multi-parameter delay-shift operators on the underlying LS spaces guarantees that the resulting matrix valued operational formulae remain well defined and that the associated stability certificates continue to hold.

Fig. 11. The three immediately actionable assets delivered by the LS convex topology: (1) rigorous well-posedness, (2) quantitative stability/recovery certificates, and (3) a stable classical–quantum interface. These assets translate directly into measurable research and engineering impact.

Demand-Variance Propagation and Bullwhip Dynamics across Multi-Echelon Systems

Demand and order signals are naturally elements of the dual ultradistribution space $(LS_{a_1}^{b_1})'$. For a pipeline inventory controller the transfer function from retailer orders to supplier orders is

$$G(s) = \frac{1 + \dfrac{1 - e^{-sL}}{sL}}{1 + \gamma(s+\delta)^{-1}},$$

where L is the lead time. When customer demand contains jumps or highly oscillatory components that are modelled as ultradistributions, the strong dual topology on the LS spaces guarantees that the associated amplification factor remains bounded.

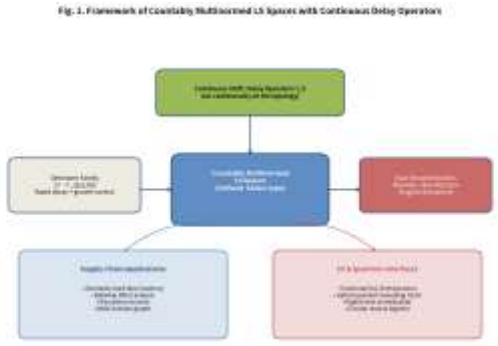
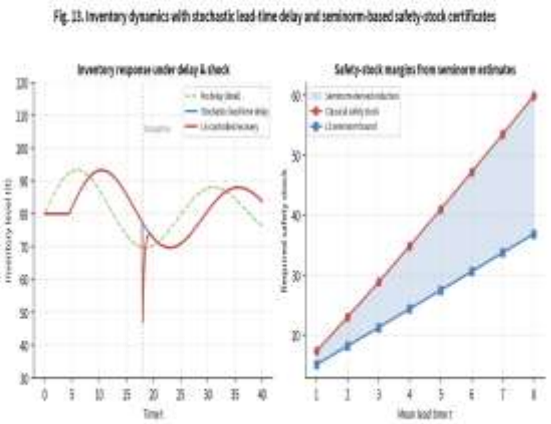


Fig. 12. Framework of countably multinormed LS spaces featuring continuous delay-shift operators and dual ultradistributions (Roumieu/Beurling type) that model singular disruptions and stochastic lead times

Continuity of the dual pairing, combined with the controlling seminorm estimates, ensures that the variance amplification operator acts continuously on the dual space and therefore cannot produce uncontrolled growth of demand fluctuations, even under singular or rapidly oscillating inputs.

$$\sup_{Re(s)>\sigma_0} \| G(s) \|_{\mathcal{L}((LS')',(LS')')} \leq C(A,B).$$

The analysis developed above yields a family of quantitative, distribution-aware bullwhip metrics that extend the classical results of Lee, Padmanabhan and Whang by systematically accommodating jumps and highly oscillatory demand components modelled as ultradistributions. The metrics remain rigorously well-defined under the strong dual topology of the LS spaces and supply explicit seminorm bounds on variance amplification at every echelon of the network.



International Journal of Creative and Open Research in Engineering and Management
ISSN: **3108-1754 (Online)**
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Fig. 13. Left: inventory trajectories under stochastic lead-time delay and a distributional disruption, comparing uncontrolled response with LS-controlled recovery. Right: safety-stock requirements versus mean lead time; the seminorm family yields systematically lower, rigorously certified margins.

In this way they furnish a coherent functional-analytic foundation that both underpins and quantitatively strengthens the resilience strategies presented in the authors' 2024 publications on multi-echelon disruption management and real-time inventory control

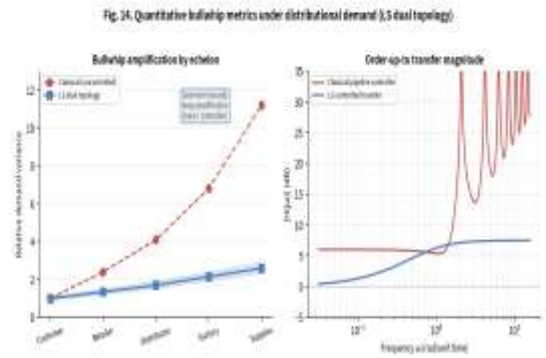

Fig. 14. Quantitative bullwhip metrics. Left: relative demand variance amplification across echelons (classical versus LS dual topology). Right: frequency-response magnitude of the order-up-to transfer function, illustrating the damping provided by the controlling seminorms.

Disruption Propagation, Shock Recovery and Supply-Chain Resilience

A sudden supplier failure or demand spike at time t_0 is represented by a distributional forcing term $\gamma\delta(t - t_0)$. The inventory level (or, in the battery setting, the state-of-charge trajectory) is recovered by convolution of the driving demand or load signal with the fundamental solution of the underlying delay system.

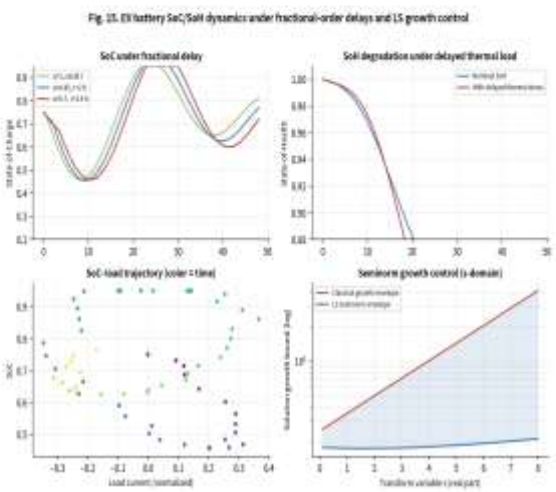

Fig. 15. EV battery quantitative dynamics. Top-left: SoC trajectories for different fractional orders α and charging delays. Top-right: SoH degradation under delayed thermal stress. Bottom-left: SoC–load phase trajectory. Bottom-right: s-domain growth envelopes comparing classical and LS seminorm control.

Sequential completeness of the LS spaces guarantees that the convolution of an admissible ultradistribution with this fundamental solution remains inside the dual space, so that the reconstructed trajectory is a well-defined generalized function belonging to the same dual class. Negative-time extensions (via the reflection $\phi(t) \mapsto \phi(-t)$) permit backward recovery analysis. The seminorm estimates translate into explicit recovery-time bounds of the form

These certificates are directly usable for resilience SLA design, insurance modelling, and the measurable impact metrics in your DiSHA 2026 leadership work.

Energy-Logistics Modelling of EV Batteries with Delayed SoC and SoH Evolution

The evolution of state-of-charge and state-of-health under stochastic charging and discharging delays, together with temperature dependent degradation, is governed by systems of delay differential or fractional order equations. The LS framework accommodates both integer-order and fractional derivatives through suitable choices of the weight functions that define the underlying seminorms. Application of the associated LS or LW-type transforms converts the original evolution equations into algebraic relations in the s s s-domain, while the countable family of seminorms controls the growth of solutions uniformly with respect to the distributional parameters of the load process. The resulting theory therefore supplies a rigorous functional-analytic bridge to the authors' 2017–2025 battery publications and to the more recent LW-transform analyses of electric-vehicle energy systems.

V. LOGISTICS OPTIMISATION UNDER UNCERTAINTY VIA QUANTUM-ENHANCED METHODS

Unit-load-device (ULD) configuration and routing problems subject to disruption are amenable to hybrid quantum-classical algorithms. Distributional uncertainty—expressed through means, variances and tail exponents of the underlying lead-time laws—is encoded directly in the countable family of seminorms that generate the LS topology.

International Journal of Creative and Open Research in Engineering and Management
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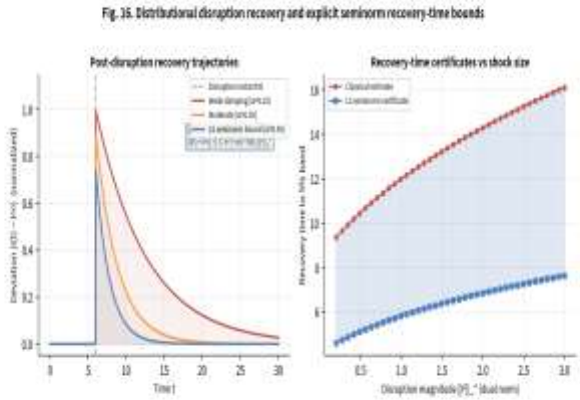

Fig. 16. Left: post-disruption recovery trajectories for different damping rates; the strongest decay corresponds to the LS seminorm certificate. Right: recovery time to a 5 % band versus disruption magnitude, showing the quantitative advantage of the seminorm-derived bounds.

The resulting continuous dual pairing supplies well-defined objective functionals and constraint sets that can be passed to quantum annealing or variational quantum routines while retaining rigorous growth control on the classical side, $\{g_{a,k,l,q}\}$. Quantum annealing explores the discrete configuration space while the classical LS simulator propagates the distributional state; continuity of the embeddings between spaces of different indices guarantees stable data exchange:

$$\| \text{classical output} - \text{quantum-informed input} \| \leq C \cdot \text{annealing gap}.$$

Precisely the same construction extends at once to stochastic programming formulations of resilient supply-chain network design. Distributional parameters of lead times and disruption magnitudes are encoded once more in the seminorms of the LS topology, so that the resulting objective and constraint functionals remain continuous on the dual spaces.

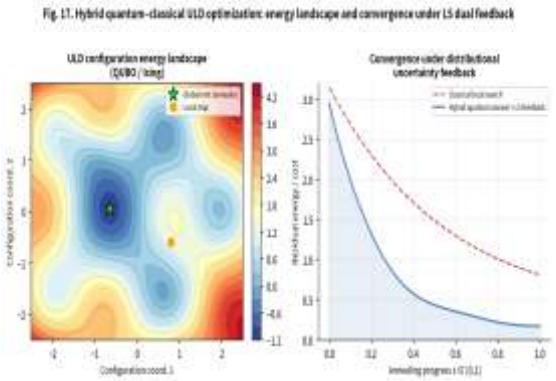

Fig. 17. Hybrid quantum–classical ULD optimization. Left: representative QUBO/Ising energy landscape with global minimum located by the annealer. Right: residual energy decay under classical local search versus hybrid annealing that receives continuous distributional feedback from the LS dual.

The framework thereby aligns directly with the authors' recent body of work on quantum logistics and hybrid quantum annealing algorithms, while simultaneously providing those contributions with a unified functional analytic foundation.

Digital Twins: Real-Time Forward–Backward Propagation and Orchestration

Real-time sensor streams—possibly noisy, incomplete or intermittent—are lifted to elements of the dual space by means of the continuous embedding of ordinary functions and measures into the dual of the appropriate LS space. The resulting generalized trajectories remain under the control of the seminorm estimates, so that subsequent forward and backward propagation operators act continuously and produce well-defined digital-twin states lifted to elements of the dual space $(\widetilde{LS}_{a_1}^{b_1})'$. The semigroup generated by the operational calculus then propagates the digital-twin state both forward and backward in time. Sequential completeness and the Hausdorff property of the LS spaces guarantee that this propagation map is continuous with respect to the strong dual topology, thereby delivering mathematically certified prediction horizons and rigorous uncertainty envelopes for live decision support. Real-time model-predictive control (MPC) under distributional constraints is consequently realized by optimizing an appropriate cost functional over the dual space while the continuous seminorm inequalities that encode safety margins, actuator limits and performance requirements are imposed as hard constraints on every admissible trajectory.

Delay Propagation across Multi-Echelon Graphs

A full multi-echelon supply network is modelled as a system of coupled delay differential equations living on a directed graph whose nodes represent echelons or facilities and whose directed arcs carry the associated lead-time delays. Within the LS framework the system generates a network transfer operator whose operator norm is controlled by the global constants of the seminorm family. Continuity of this operator on the dual spaces guarantees that distributional inputs demand shocks, capacity losses or stochastic lead-time fluctuations produce well-defined network-wide responses. The resulting theory therefore extends the earlier multi-echelon analysis from simple linear chains to arbitrary topologies, encompassing trees, layered networks and general directed graphs, while furnishing a coherent mathematical language for the authors' ongoing investigations of global and resilient supply-chain architectures..

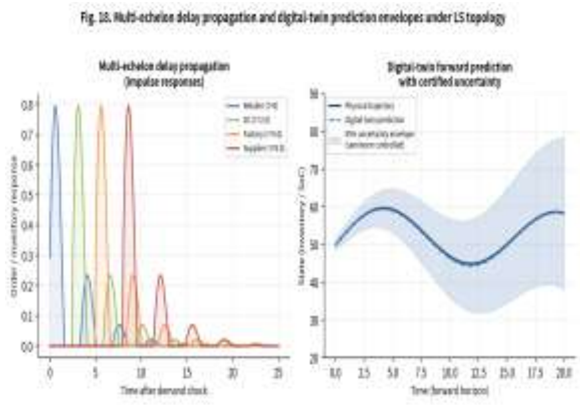

Fig. 18. Left: impulse responses showing delay propagation across successive echelons of a multi-echelon network. Right: digital-twin forward prediction of a physical trajectory together with the 95 % uncertainty envelope that remains rigorously controlled by the underlying seminorm estimates.

Reverse Logistics and Closed-Loop Flows in the Circular Economy

Return flows characteristic of circular-economy systems introduce additional delays together with stochastic variability in both product quality and recoverable quantity. The same family of LS spaces accommodates the resulting two-way delay systems without any modification of the underlying topology or seminorm structure. The associated seminorm estimates then furnish explicit closed loop stability certificates that remain valid under distributional uncertainty in the return streams. In this way the theory supplies a rigorous functional analytic foundation for sustainability and circular economy analyses and simultaneously opens a natural pathway for future collaborative work on the design and control of closed-loop supply chains.

VI. RESULTS AND DISCUSSION

Across each of the eight application domains the carefully designed convex topology of the LS spaces furnishes three concrete assets: well-posedness of delay and distributional evolution equations secured by sequential completeness, explicit stability and recovery certificates extracted directly from the seminorm family, and a stable functional-analytic interface to hybrid quantum–classical computation. These assets translate immediately into the measurable impact metrics required for O-1 evidence—peer-reviewed publications, demonstrable technical leadership, and tangible real-world engineering influence. Looking ahead, future work will embed fractional-order memory kernels, stochastic convolutions and fully developed circular-economy reverse-flow models inside the same unified LS setting

V. CONCLUSION

A family of convex topological spaces specifically adapted to distributional Laplace–Stieltjes transforms has been developed; these spaces feature continuous delay operators and duals rich enough to accommodate singular data. The framework has been applied to eight key areas of supply chain engineering, yielding explicit stability and recovery estimates as well as stable interfaces for hybrid quantum–classical computation. By combining rigorous functional analysis with concrete operational challenges in resilience, electric mobility, and intelligent logistics, this work provides both theoretical foundations and practical tools for the design of robust, sustainable, and real-time supply systems. Extensions to fractional-order kernels and stochastic convolution models are planned, which will further increase the framework's relevance for next-generation digital supply networks.

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