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A Parameterized Adaptive Kernel Integral Transform with Gaussian Damping: Mathematical Properties and Convergence Analysis

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Abstract

Integral transforms provide important analytical tools for the investigation of differential equations, integral equations, and mathematical models arising in science and engineering. Among the classical transforms, the Laplace transform is particularly important because of its simple exponential kernel and well-established operational properties. Nevertheless, the convergence of the classical Laplace transform is restricted by the growth behaviour of the function being transformed.

In this paper, a parameterized integral transform, referred to as the Adaptive Kernel Integral Transform (AKIT), is introduced through the kernel

$K_{\alpha,\beta}(t,s) = e^{-st-\alpha t^2}(1+\beta t)$ where $s > 0, \alpha, \beta \geq 0. s > 0,$

The parameter α introduces an additional Gaussian-type damping mechanism, whereas β provides a linear weighting factor. The proposed kernel is investigated with respect to continuity, positivity, smoothness, decay, and parameter dependence. Sufficient conditions for the existence and convergence of the proposed transform are established. It is also shown that the classical Laplace transform is recovered when α = β =0. Selected standard functions are considered, and a class of rapidly growing functions of the form $f(t) = e^{at^p}$ is investigated. In particular, for $a > 0$ and $1 < p < 2$, the classical Laplace transform diverges, whereas the proposed transform converges for every $\alpha > 0$, under the stated conditions. A comparison with the classical Laplace transform is presented together with a numerical and graphical illustration. The results indicate that the proposed kernel provides a flexible parameterized framework for studying selected classes of functions for which an additional quadratic damping mechanism is analytically useful.

Keywords: Adaptive Kernel Integral Transform, Parameterized Kernel, Gaussian Damping, Laplace Transform, Integral Transform, Convergence, Kernel Function.

INTRODUCTION

Integral transforms constitute an important class of mathematical techniques for converting a function from one representation into another. They have been widely employed in the analysis of ordinary differential equations, partial differential equations, integral equations, mathematical physics, engineering models, and applied mathematics.

In general, an integral transform can be represented in the form

$$T\big(f(t)\big)(s) = \int_0^{\infty} K(t,s) f(t) dt$$

where K(t,s) is the kernel of the transform. The choice of the kernel plays a fundamental role in determining the convergence, analytical properties, and applicability of the resulting transform.

The classical Laplace transform is defined by

$$L\big(f(t)\big)(s) = \int_0^{\infty} e^{-st} f(t) f(t) dt$$

Its kernel is

$$K_L(t,s) = e^{-st}$$

The exponential factor e^{-st} provides damping as t→∞ and enables the transformation of a broad class of functions satisfying suitable growth restrictions. The Laplace transform also possesses important operational properties involving differentiation, integration, convolution, and algebraic manipulation.

However, the convergence of an integral transform depends directly on the competition between the growth of f(t) and the decay of the kernel. Consider, for example,

$f(t) = e^{at^p}$ $a > 0$ Its Laplace transform would formally be

$L(f(t))(s) = \int_0^{\infty} e^{-st} e^{at^p} dt.$
For p>1, the term at^p eventually dominates the linear term st, and the integral does not converge.

This observation motivates the investigation of kernels containing an additional damping mechanism. In the present work, the following parameterized kernel is proposed:

$$K_{\alpha,\beta}(t,s) = e^{-st-\alpha t^2}(1+\beta t)$$

The kernel may be written as

$$K_{\alpha,\beta}(t,s) = \frac{e^{-st}}{(Laplace\ kernel)} \quad \frac{e^{-\alpha t^2}}{(Gaussian\ Damping)} \quad \frac{(1+\beta t)}{Linear\ weighting}$$

Thus, the parameter α controls an additional quadratic damping mechanism,β modifies the weighting of the transformed function.

The corresponding transform is defined as

$$A_{\alpha,\beta}\big(f(t)\big)(s) = \int_0^{\infty} e^{-st-\alpha t^2}(1+\beta t) f(t)$$

An important consistency property is obtained by setting

$$\alpha = 0, \beta = 0$$

Then

$$K_{0,0}(t,s) = e^{-st}$$

and therefore

$$A_{0,0}\big(f(t)\big)(s) = L\big(f(t)\big)(s)$$

The purpose of this paper is not to claim that the proposed transform is universally superior to the Laplace transform. Rather, the aim is to investigate whether the additional Gaussian damping provides a mathematically useful convergence mechanism for selected classes of rapidly growing functions.

The main contributions of this study are as follows:

1. formulation of a two-parameter kernel;
2. definition of the corresponding integral transform;
3. analysis of fundamental kernel properties;
4. establishment of sufficient convergence conditions;
5. verification of the reduction to the classical Laplace transform;
6. derivation of selected standard-function representations;
7. investigation of rapidly growing functions;
8. comparison with the classical Laplace transform;
9. numerical and graphical illustration of the convergence difference.

2. LITERATURE REVIEW

Integral transform theory has a long history and includes several important transforms used in pure and applied mathematics. The Laplace transform is one of the most widely used transforms for solving differential and integral equations because of its exponential kernel and convenient operational properties.

The Fourier transform provides another fundamental framework and is widely used in harmonic analysis, partial differential equations, signal processing, and mathematical physics. In addition to these classical transforms, several alternative and generalized integral transforms have been developed to provide different representations and operational procedures.

The general structure

$$T\big(f(t)\big)(s) = \int_0^\infty K(t,s)f(t)dt$$

shows that the kernel is central to transform theory. Changing the kernel may modify convergence properties and the analytical representation of the transformed function.

Parameterized kernels are particularly interesting because they allow additional degrees of freedom. A parameter may control scaling, damping, weighting, or other properties of the transformation.

The Gaussian factor

$$e^{-\alpha t^2}$$

is mathematically important because it provides strong decay for $\alpha>0$. Such Gaussian-type factors occur naturally in several areas of analysis and mathematical modelling.

The present study investigates a specific combination of exponential damping, quadratic damping, and linear weighting:

$$e^{-st-\alpha t^2}(1+\beta t)$$

The objective is to establish its mathematical properties and determine whether the additional quadratic damping changes the convergence behaviour for selected function classes.

The literature review should be supported by verified references concerning Laplace transforms, Fourier transforms, generalized integral transforms, Gaussian integrals, and convergence theory.

3. RESEARCH GAP AND OBJECTIVES

3.1Research Gap

The classical Laplace transform is based on the fixed kernel

$$K_L(t,s) = e^{-st}$$

This kernel provides linear exponential damping with respect to t. Although this is sufficient for a broad class of functions, there are functions whose growth is too rapid for the Laplace transform to converge.

The present study investigates whether introducing a parameterized Gaussian damping factor can provide an alternative convergence mechanism.

The proposed kernel is

$$K_{\alpha,\beta}(t,s) = e^{-st-\alpha t^2}(1+\beta t)$$

The research gap can therefore be formulated through the following questions:

(i) What are the fundamental mathematical properties of the proposed kernel?
(ii) Under what conditions does the corresponding transform exist?
(iii) How do α and β influence the transform?
(iv) Does the Gaussian term improve convergence for selected rapidly growing functions?
(v) How does the proposed transform compare with the classical Laplace transform? **Objectives**
The objectives of this paper are:

1. To introduce the Adaptive Kernel Integral Transform.
2. To study the continuity and positivity of the proposed kernel.
3. To investigate its decay and parameter dependence.
4. To establish sufficient conditions for existence and convergence.
5. To verify the recovery of the Laplace transform as a special case.
6. To obtain representations for selected standard functions.
7. To study functions whose growth exceeds the classical exponential scale.
8. To compare AKIT with the classical Laplace transform.
9. To provide a numerical and graphical illustration.

4. MOTIVATION FOR THE PROPOSED KERNEL

The proposed kernel is

$$K_{\alpha,\beta}(t,s) = e^{-st-\alpha t^2}(1+\beta t)$$

It consists of three components:

$$e^{-st-\alpha t^2}(1+\beta t)$$

and

$1+\beta t$ The first component preserves a direct connection with the Laplace transform.

The second component introduces additional damping. For α>0, the term

$$e^{-\alpha t^2}$$

decays rapidly as t→∞ The third component provides a simple adjustable weighting mechanism. The parameter roles can therefore be summarized as α-damping parameter and β-weighting parameter.
A key consistency condition is $K_{0,0}(t,s) = e^{-st}$. Thus, the classical Laplace kernel is contained in the proposed kernel family.

5. DEFINITION OF THE ADAPTIVE KERNEL INTEGRAL TRANSFORM

Let

f:[0,∞)→R or f:[0,∞)→C be a suitable function.
Definition 5.1

The Adaptive Kernel Integral Transform of f(t)f(t) is defined by

$$A_{\alpha,\beta}\big(f(t)\big)(s) = \int_0^\infty e^{-st-\alpha t^2}(1+\beta t)f(t)\,dt$$

where s>0, α, β≥0.

Equivalently,

$$A_{\alpha,\beta}\big(f(t)\big)(s) = \int_0^\infty K_{\alpha,\beta}(t,s)\,f(t)dt$$

The transform is said to exist if the corresponding improper integral converges.

6. BASIC PROPERTIES OF THE PROPOSED KERNEL

6.1 Continuity

Consider

$$K_{\alpha,\beta}(t,s) = e^{-st-\alpha t^2}(1+\beta t)$$

The function

$$-st-\alpha t^2$$

is continuous. Therefore,

$$e^{-st-\alpha t^2}$$

is continuous. Also,1+βt is continuous.

The product of continuous functions is continuous. Hence,

$$K_{\alpha,\beta}(t,s) \in \mathrm{C}([0,\infty)\times(0,\infty))$$

6.2 Positivity

For

t≥0,s>0,α,β≥0,we have $e^{-st-\alpha t^2}$ and 1+βt>0 Therefore, $K_{\alpha,\beta}(t,s) > 0$

6.3 Decay

For α≥0,

$$e^{-\alpha t^2}$$

.

Therefore,

0<$K_{\alpha,\beta}(t,s)$≤(1+βt)e^{-st}. Since $\lim_{t\to\infty}(1+\beta t)e^{-st} = 0$

we obtain

$$\lim_{t\to\infty} K_{\alpha,\beta}(t,s) = 0$$

6.4 Dependence on α

Differentiate with respect to α:

$\frac{\partial K_{\alpha,\beta}}{\partial \alpha} = -t^2 e^{-st-\alpha t^2(1+\beta t)}$

For t>0 $\frac{\partial K_{\alpha,\beta}}{\partial \beta} = te^{-st-\alpha t^2}$<0

Thus, increasing α decreases the kernel and increases its damping.

6.5 Dependence on β

We have $\frac{\partial K_{\alpha,\beta}}{\partial \beta} = te^{-st-\alpha t^2}$Therefore, $\frac{\partial K_{\alpha,\beta}}{\partial \beta} > 0$

for t>0

6.6 Smoothness

Since the exponential and polynomial functions are infinitely differentiable,$K_{\alpha,\beta} \in C^\infty$

with respect to t and s on the interior of its domain.

EXISTENCE AND CONVERGENCE

We now establish sufficient conditions for existence.

Theorem 7.1

Let f(t) be piecewise continuous on every finite interval of [0,∞). Suppose there exist constants M>0, c∈R, and T>0 such that

$$| f(t) | \leq Me^{ct}, t \geq T$$

Then

$$A_{\alpha,\beta}\big(f(t)\big)(s)$$

exists for s>c,α,β≥0

Proof

For t≥T,

$$| f(t) |\leq Me^{ct}$$

Hence

$$e^{-st-\alpha t^2}(1+\beta t)|f(t)| \leq Me^{ct}.e^{-st-\alpha t^2}(1+\beta t)$$

Since α≥0 we have $e^{-\alpha t^2}$≤1. Consequently,

$$| e^{-st-\alpha t^2}(1+\beta t)f(t) |\leq M(1+\beta t)e^{-(s-c)t}$$

Since s>c,

$\int_T^\infty(1+\beta t)e^{-(s-c)t}\,dt < \infty$

Hence the transform converges absolutely.

Therefore,

$$A_{\alpha,\beta}\big(f(t)\big)(s)$$

exists for s>c.

The main purpose of introducing the Gaussian factor is to investigate functions with faster growth. Consider

$$f(t) = e^{at^p}, a > 0$$

For the proposed transform,

$$A_{\alpha,\beta}(t,s)\big(f(t)\big)(s) = \int_0^\infty e^{-st-\alpha t^2+at^p}(1+\beta t)f(t)dt$$

Theorem 8.1

Let

$$a > 0, 1 < p < 2, \alpha > 0, \beta \geq 0$$

Then

$$A_{\alpha,\beta}\big(e^{at^p}\big)(s)$$

converges for every real s.

Proof

Consider the exponent

$-st - \alpha t^2 + at^2$ Since 1<p<2, we have

$\frac{t^p}{t^2} = t^{p-2}$ approaches to zero as t tends to infinity

Thus, for sufficiently large t, $at^p < \frac{\alpha}{2}t^2$.

Hence $-\alpha t^2 + at^p \leq -\frac{\alpha}{2}t^2$

Therefore, $e^{-st-\alpha t^2+at^p}(1+\beta t)$ is eventually bounded by a Gaussian-decaying function of the form

$C(1+t)e^{\frac{-t^2}{2}+|s|t}$ which is integrable on [T,∞)
Therefore,

$$A_{\alpha,\beta}\left(e^{at^p}\right)(s)$$

converges for all s∈R.

9. RECOVERY OF THE CLASSICAL LAPLACE TRANSFORM

Set

α=0,β=0.
Then

$$A_{0,0}\big(f(t)\big)(s) = L(f(t))(s)$$

Therefore,

$$A_{0,0}\big(f(t)\big)(s) = L$$

This establishes consistency with classical transform theory.

10. LINEARITY PROPERTY

Theorem 10.1

Let f(t) and g(t) be functions for which the proposed transforms exist. For arbitrary constants c,d,

$$A_{\alpha,\beta}(cf(t)+dg(t))(s) = cA_{\alpha,\beta}(f(t))(s) + dA_{\alpha,\beta}(g(t))(s)$$

Proof

Using the definition,

$$A_{\alpha,\beta}\big(cf(t)+dg(t)\big)(s) = \int_0^\infty e^{-st-\alpha t^2}(1+\beta t)[cf(t)+dg(t)]dt$$

Using the linearity of integration,

$$= \int_0^\infty e^{-st-\alpha t^2}(1+\beta t)cf(t)dt + \int_0^\infty e^{-st-\alpha t^2}(1+\beta t)dg(t)dt$$

Therefore,

$A_{\alpha,\beta}(cf(t)+dg(t))(s) = cA_{\alpha,\beta}(f(t))(s) + dA_{\alpha,\beta}(g(t))(s).$

Thus, AKIT is a linear integral operator.

11. TRANSFORMS OF SELECTED STANDARD FUNCTIONS

Define

$$I_n(s,\alpha) = \int_0^\infty t^n e^{-st-\alpha t^2} dt$$

Then

$$A_{\alpha,\beta}(t^n)(s) = I_n(s,\alpha) + \beta I_{n+1}(s,\alpha)$$

11.1 Transform of 1

For f(t)=1

$$A_{\alpha,\beta}(1)(s) = I_0(s,\alpha) + \beta I_1(s,\alpha)$$

For α>0

$$I_0(s,\alpha) = \frac{\sqrt{\pi}}{2\sqrt{\alpha}} e^{\frac{s^2}{4\alpha}} erfc(\frac{s}{2\sqrt{\alpha}})$$

Furthermore,

$$I_1(s,\alpha) = \frac{1}{2\alpha} - \frac{s}{2\alpha} I_0(s,\alpha)$$

Hence,

$$A_{\alpha,\beta}(1)(s) = I_0(s,\alpha) + \beta(\frac{1}{2\alpha} - \frac{s}{2\alpha} I_0(s,\alpha))$$

11.2 Transform of t

We have

$A_{\alpha,\beta}(t)(s) = I_1(s,\alpha) + \beta(I_2(s,\alpha))$ Using $I_2(s,\alpha) = \frac{I_0 - sI_1}{2\alpha}$ we obtain

$A_{\alpha,\beta}(t)(s) = I_1(s,\alpha) + \beta(\frac{I_0 - sI_1}{2\alpha})$

11.3 General polynomial

For

$$f(t) = t^n,$$

we have $A_{\alpha,\beta}(t^n)(s) = I_n(s,\alpha) + \beta I_{n+1}(s,\alpha)$

12. A FUNCTION CLASS BEYOND THE CLASSICAL LAPLACE RANGE

, consider

$$f(t) = e^{at^2}, a > 0.$$

The classical Laplace transform is

$$L\left(e^{at^2}\right)(s)=\int_0^{\infty} e^{-st+at^2}\,dt$$

As t→∞, the positive quadratic term at2 dominates the linear term −st. Therefore,

$-st+at^2 \to +\infty$,

and hence

$$L(e^{at^2})(s)$$

does not exist for finite s.

For the proposed transform,

$$A_{\alpha,\beta}\left(e^{at^2}\right)(s)=\int_0^{\infty} e^{-st-\alpha t^2}(1+\beta t)e^{at^2}dt$$

Combining the quadratic terms gives

$$A_{\alpha,\beta}\left(e^{at^2}\right)(s)=\int_0^{\infty} e^{-st-(\alpha-a)t^2}(1+\beta t)dt$$

Therefore, if

$\alpha > a,$ then $\alpha - a > 0$, and the integrand contains the damping factor $e^{-(\alpha-a)t^2}$. Consequently, the integral converges for every s>0,β≥0.

This gives the important observation $L(e^{at^2})(s)$ does not exist,$A_{\alpha,\beta}(e^{at^2})\ exists\ if\ \alpha > a.$

It gives a specific class of functions for which the additional Gaussian damping can change the convergence behavior.

13. COMPARISON WITH THE LAPLACE TRANSFORM

The convergence behaviour can be summarized as follows.

Function	**Laplace Transform**	**AKIT**
e^{at^p} $0 < p < 1,$	Converges for suitable s	Converges
e^{at}	Converges for s>as>a	Converges for α>0
e^{at^p} $1 < p < 2$	Diverges	Converges for α>0
e^{at^2}	Diverges	$Converges\ when\ \alpha > a$
e^{at^p} $p > 2$	Diverges	Generally diverges

The important region is

1<p<2 In this range, $t^p = o(t^2)\ as\ t \to \infty$
Therefore, the Gaussian damping can dominate the growth.

For example,

$$f(t) = e^{t^{3/2}}$$

The Laplace integrand is $e^{-st+t^{3/2}}$which eventually grows.

The AKIT integrand with α=1 and β=0 is $e^{-st-t^2+t^{3/2}}$, which eventually decays.

Therefore, AKIT provides a different convergence mechanism for this class.

It should nevertheless be emphasized that this does not establish universal superiority over the Laplace transform. The advantage is restricted to specific growth classes.

14. NUMERICAL AND GRAPHICAL ILLUSTRATION

Consider

$$f(t) = e^{t^{3/2}}$$

Take

s=1.s=1.
For the Laplace transform, the integrand is

$$F_L(t) = e^{-t+t^{3/2}}.$$

For AKIT, choose α=1,β=0 Then

$F_A(t) = e^{-t-t^2+t^{\frac{3}{2}}}$
For large tt,

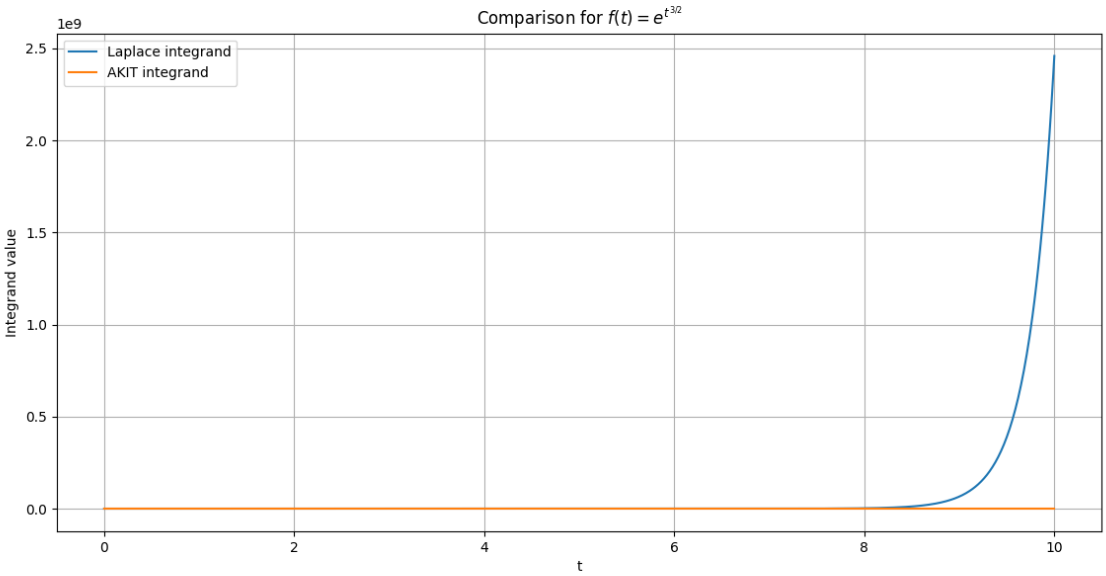
$-t + t^{3/2}$→+∞,
so

$F_L(t)$→∞
On the other hand, $-t - t^2 + t^{\frac{3}{2}}$→−∞ and therefore

$F_A(t) = e^{-t-t^2+t^{\frac{3}{2}}}$. Approaches to zero
Thus,

$$F_L(t)\ grows, F_A(t)\ decays$$

A graph of the two functions over a suitable finite interval provides a visual illustration of the theoretical convergence analysis



15. DISCUSSION

The proposed kernel introduces two independent parameters with different mathematical roles. The parameter α\alpha determines the strength of the additional quadratic damping, whereas β\beta modifies the weighting factor.

When α=0, the additional Gaussian damping disappears and the kernel becomes

$$K_{0,\beta}(t,s) = e^{-st}(1+\beta t)$$

When β=0, the kernel becomes

$$K_{\alpha,0} = e^{-st-\alpha t^2}$$

Finally, when

α = β = 0,

the classical Laplace kernel is obtained.

The convergence analysis shows that the most interesting new regime is associated with functions satisfying

$$f(t) = e^{at^p}, 1 < p < 2$$

For these functions, the classical Laplace damping e^{-st} is insufficient, while the additional factor

$$e^{-at^2}$$

can provide sufficient decay.

This demonstrates the mathematical motivation for introducing the Gaussian factor.

However, the presence of the Gaussian factor may also make closed-form expressions more complicated. For example, even the transform of the constant function involves the complementary error function. Therefore, the proposed transform should be viewed as a tool for specific problems rather than as a replacement for the Laplace transform in all applications.

16. CONCLUSION

In this paper, a parameterized Adaptive Kernel Integral Transform has been introduced through the kernel

$K_{\alpha,\beta}(t,s) = e^{-st-at^2}(1+\beta t)$

The proposed kernel combines the classical Laplace exponential factor with a Gaussian-type damping factor and a linear weighting term.

Several fundamental properties of the kernel have been established. In particular, the kernel is continuous, positive, smooth, and decays to zero as $t\to\infty$ for the prescribed parameter range.

A sufficient existence theorem was obtained for functions of exponential order. An extended convergence result was then established for functions of the form

$$e^{at^p}, 1 < p < 2$$

For this class, the classical Laplace transform diverges because the positive term at^v eventually dominates the linear damping term st. In contrast, the proposed transform contains the additional negative quadratic term $-\alpha t^2$ which dominates at^p $when\ 1 < p < 2\ and\ \alpha > 0$.The classical Laplace transform is recovered as the special case $\alpha=\beta=0$. The study therefore establishes a mathematically consistent connection between the proposed parameterized transform and the classical Laplace transform.

The results suggest that the proposed transform may be useful for selected function classes with growth characteristics lying between exponential and quadratic-exponential scales. Nevertheless, further analysis is required before claiming broader applicability or superiority over existing transforms.

17. FUTURE SCOPE

Several directions remain open for future investigation.

First, an explicit inversion theory for the proposed transform may be developed. Second, operational formulas involving derivatives and integrals may be investigated. Third, an appropriate convolution structure may be formulated.

Further work may also investigate kernels of the more general form

$$K_{\alpha,\beta,q}(t,s) = e^{-st-\alpha t^q}(1+\beta t)$$

Where q>1.Such a generalization would allow the damping order itself to be treated as a parameter.Possible applications include selected differential equations, integral equations, fractional differential equations, and numerical approximation problems.

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