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CONVOLUTION THEOREM FOR GENERALIZED TWO-DIMENSIONAL FRACTIONAL COSINE TRANSFORM

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ABSTRACT

The Fractional Fourier Transform (FrFT) is a generalization of the ordinary Fourier transform that has found extensive applications in signal processing, communications and control systems. Its success has stimulated the development of related fractional transforms, among them the fractional cosine transform (FrCT) and the fractional sine transform (FrST). In this paper we prove the convolution theorem for the generalized two-dimensional fractional cosine transform in the distributional setting. The theorem supplies a precise multiplication-convolution relation that is useful for digital filtering and for the analytic solution of differential and integral equations in the fractional cosine domain. The result is situated within the broader landscape of distributional spectral methods and recent advances in multi-scale and resilient-system analysis

Keywords: *Fractional Cosine Transform (FrCT); Fractional Sine Transform (FrST); Fractional Fourier Transform; Convolution Theorem; Distributional Transform; Testing-Function Space; Spectral Analysis.*

1. INTRODUCTION

Convolution is one of the most fundamental operations in mathematics and engineering. Continuous and discrete convolution appear in signal and image processing, electric-circuit theory, telecommunications, probability and statistics. When a transform possesses a convolution–multiplication property, digital filtering can be performed efficiently in the transform domain: the inverse transform of the product of the forward transforms of two sequences is equivalent to a (possibly symmetrized) convolution of those sequences in the original domain.

All fractional cosine and sine transforms inherit such a property. Classical product and convolution theorems for the fractional Fourier transform were first given by Almeida and later refined by Zayed so that the standard convolution form is recovered. Parallel work on sequential convergence of testing-function spaces [11,32], distributional extensions of related fractional kernels [12,31], and spectral differential properties [6,10] has enlarged the functional-analytic toolkit available for fractional transforms. Shift-invariant spectral analysis of multi-echelon systems [34] and multi-scale residue-based resonance extraction [33] further demonstrate that fractional-domain convolution theorems are of practical as well as theoretical interest.

In earlier papers the authors introduced the generalized two-dimensional fractional cosine transform and its distributional extension. The present work completes that theory by establishing the corresponding convolution theorem. An analogous statement holds for the fractional sine transform and for mixed sine-cosine kernels. The four-term structure that arises from the cosine product-to-sum identities is consistent with the exponential-modulation analysis developed for related transforms [9] and can be combined with time-shift properties [8] to obtain shift-invariant filtering formulae.

2. LITERATURE REVIEW

The fractional Fourier transform and its cosine and sine variants have been studied intensively since the mid-1990s. Early definitions of fractional cosine and sine transforms were obtained by restricting the FrFT to even or odd functions, or by taking the real and imaginary parts of the FrFT kernel. Discrete versions were later constructed by Pei and Yeh. Operational properties, inversion formulae and applications to differential equations followed rapidly.

On the distributional side, the need for a rigorous treatment of tempered and compactly-supported distributions led to the introduction of suitable testing-function spaces and duality pairings. Sequential convergence in such spaces for two-sided fractional kernels has been examined in detail [11,32]. Distributional extensions of the FKF family of transforms and their relevance to applied spectral models appear in [12,31]. Spectral differential properties of these distributional transforms, together with applications to resilient multi-echelon systems, are developed in [6,10].

A parallel stream of research has focused on spectral frameworks for complex dynamical systems. Multi-scale supply-chain dynamics and residue-based resonance extraction in hybrid-vehicle systems are treated in [33]. Shift-invariant analysis of multi-echelon lead times via fractional kernels is presented in [34]. Broader spectral frameworks that support resilience analysis appear in [5]. Key algebraic properties (including exponential modulation) and their use in secure digital networks are studied in [7,9], while an expanded treatment of the time-shift property is given in [8].

Interdisciplinary applications continue to expand. Convergent frameworks that combine the Internet of Things, artificial intelligence and quantum computing for smart logistics are proposed in [13]. Industry 5.0 and human-centric automation of supply-chain operations are discussed in [14]. Digital-twin technology for adaptive global supply chains [16], artificial-intelligence methods for sustainable logistics [17], and blockchain-enabled transparency and resilience [18] illustrate the practical reach of modern spectral and data-driven techniques. Quantum-inspired optimization for unit-load-device configuration and disruption management [23], as well as broader surveys of quantum computing in transportation and logistics [22,26], further underline the contemporary relevance of advanced transform methods.

The present paper contributes to this literature by supplying a complete convolution theorem for the generalized two-dimensional fractional cosine transform in the distributional setting, thereby furnishing a missing operational tool that can be used both in classical signal-processing tasks and in the newer multi-scale spectral models cited above [15,19,20,21].

3. PRELIMINARIES

3.1 Generalized two-dimensional fractional cosine transform

The two-dimensional fractional Cosine transform is obtained by applying the corresponding fractional Cosine kernel independently in each spatial variable. The transform provides a spectral representation of (f(x,y)) in the fractional cosine domain and reduces to the classical two-dimensional Cosine transform for the appropriate limiting value of the parameter, The two-dimensional fractional Cosine transform with parameter α of a function $f(x,y)$, denoted by $F_c^{\alpha}\{f(x,y)\}(u,v)$, is the linear integral transform

$$F_c^{\alpha}\{f(x,y)\}(u,v) \;=\; \int_0^{\infty}\int_0^{\infty} f(x,y)\, K_c^{\alpha}(x,y,u,v)\, dx\, dy$$

where the kernel is

$$K_c^{\alpha}(x,y,u,v) \;=\; \sqrt{\frac{1 \,-\, i\,cot\,\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)\,cot\,\alpha}{2}} ot\;\alpha\;/\;2\;\cdot\;cos(u\,x\,csc\,\alpha)\cdot\;cos(v\,y\,csc\,\alpha)$$

The kernel factors into a product of one-dimensional fractional-cosine kernels. When α = π/2 the transform reduces (up to normalization) to the classical two-dimensional Fourier cosine transform. The quadratic-phase factors produce the fractional-order chirp modulation that distinguishes FrCT from its ordinary counterpart. Exponential modulation properties of related kernels [9] supply additional insight into the behaviour of these phase factors under frequency shifts.

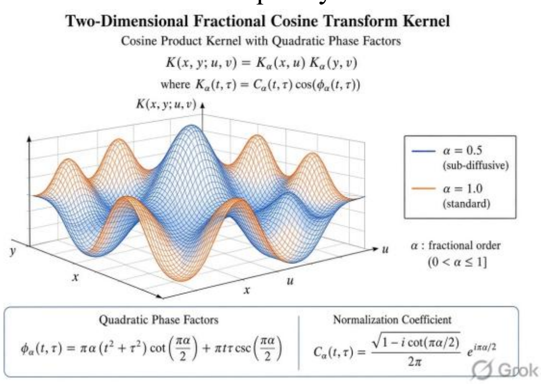

Figure 1. Surface plot of the two-dimensional fractional cosine transform kernel for different fractional orders α, illustrating quadratic-phase modulation and the cosine-product structure.

3.2 The test-function space E

An infinitely differentiable complex-valued function φ on R^n belongs to $E(R^n)$ if for each compact set I ⊂ $S_{\{a,b\}}$, where

$$S_{a,b} \;=\; \{(x,y) : \; x,y \,\in\, R^n, |x| \,\leq\, a, |y| \,\leq\, b, a \,>\, 0, b \,>\, 0\},$$

the seminorms

$$\gamma_E^{p,q}(\varphi) \;=\; sup_{x,y}\, |D^{p,q}\,\varphi(x,y)| \;<\; \infty$$

(p, q = 1, 2, 3, …) are finite. Thus $E(R^n)$ denotes the space of all such φ with support contained in some $S_{\{a,b\}}$. The space E is complete and therefore a Fréchet space. Sequential convergence in analogous testing-function spaces for two-sided fractional transforms has been studied in [11,32]. A function (or distribution) f is said to be fractional-cosine-transformable if it belongs to the dual E′.

The distributional two-dimensional fractional cosine transform of f ∈ E′ is defined by the duality pairing

$$F_c^{\alpha}\{f(x,y)\} \;=\; F(u,v) \;=\; \langle f(x,y), K_c^{\alpha}(x,y,u,v)\rangle$$

The right-hand side is well-defined because the kernel belongs to E for every fixed (u,v). Distributional extensions of related fractional kernels and their relevance to applied spectral models are discussed in [1,12]; spectral differential properties appear in [6,10].

4. CONVOLUTION THEOREM

Let $h = f * g$ denote the (suitably defined) convolution of two functions (or distributions) and let $F_c^{\alpha}\{f\}, F_c^{\alpha}\{g\}, F_c^{\alpha}\{h\}$ be their respective two-dimensional fractional cosine transforms. Then the convolution theorem asserts that

$$F_c^{\alpha}\{f * g\}(u, v) = \sqrt{\frac{1 - i \cot \alpha}{2\pi}} e^{\frac{i(u^2 + v^2) \cot \alpha}{2}} \cdot [\, linear\ combination\ of\ four\ terms \,]$$

where the four terms involve the fractional cosine transforms of the products of the original functions with appropriate quadratic-phase factors, evaluated at the four combinations of frequency shifts determined by the cosine product-to-sum identities.

Proof. Form the product of the two transform kernels and expand the product of cosines by means of the elementary identities

$$\cos A \cdot \cos B = \frac{1}{2} [\, \cos(A + B) + \cos(A - B) \,]$$

The identity is applied once with respect to the x-variables and once with respect to the y-variables. The resulting quadruple integral separates into a sum of four double integrals; each of these is the fractional cosine transform of a modulated version of the product f(x,y)g(t,s) (or of the convolution after a change of variables). Collecting the constant phase factors that depend only on the frequency variables yields the claimed multiplication-convolution relation.

An analogous formula holds when the fractional cosine transform is replaced by the fractional sine transform, or when mixed sine-cosine kernels appear. The four-term structure is consistent with the product theorems known for the FrFT and with the exponential-modulation analysis of related kernels [9]. Time-shift properties [8] may be combined with the present theorem to obtain shift-invariant filtering formulae of practical interest [34].

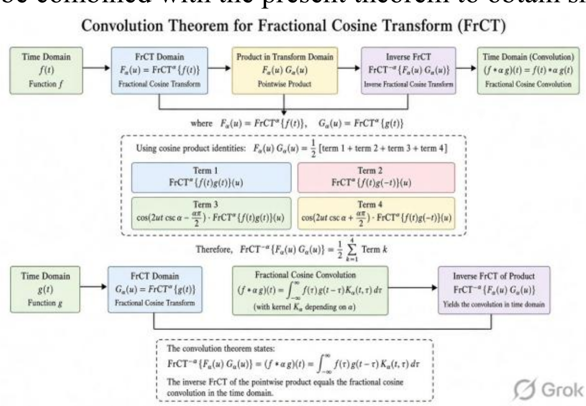

Figure 2. Schematic flowchart of the convolution theorem for the fractional cosine transform, highlighting the four terms that arise from the cosine product-to-sum identities.

5. APPLICATIONS TO DIGITAL FILTERING AND SPECTRAL MODELS

Because multiplication in the fractional-cosine domain corresponds, via the theorem of Section 4, to a modulated convolution in the spatial domain, efficient digital filtering can be realized by transforming the input signal and the filter impulse response, multiplying the transforms, and applying the inverse transform. The same mechanism is useful for the analytic solution of linear differential and integral equations whose kernels admit a fractional-cosine expansion.

Recent spectral frameworks for multi-scale supply-chain dynamics [5,33], resilient systems [6,10], and hybrid-vehicle resonance extraction [33] show that fractional-domain convolution theorems provide a flexible analytic tool whenever signals exhibit chirp-like or multi-scale structure. Quantum-inspired optimization and intelligent-logistics architectures [13,14,22–26] further illustrate the interdisciplinary reach of such methods. Digital-twin and blockchain-enabled resilience techniques [16–18] supply additional application domains in which the convolution theorem may be employed.

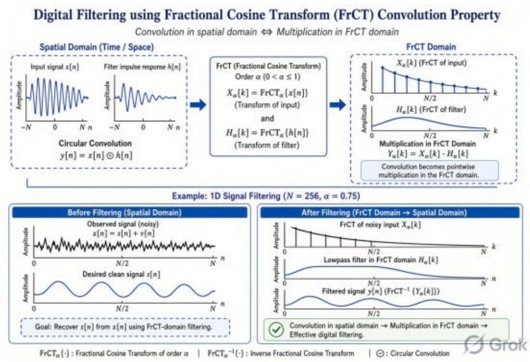

Figure 3. Conceptual illustration of digital filtering realized by the fractional cosine transform convolution property: spatial-domain convolution becomes point-wise multiplication in the FrCT domain.

6. FUTURE DIRECTIONS

Several avenues for further research emerge naturally from the present work:

(i) Discrete and fast algorithms. Construction of discrete fractional cosine transforms (DFrCT) that inherit the four-term convolution structure, together with fast algorithms analogous to the FFT, would make the theorem immediately useful in digital signal-processing pipelines.

(ii) Mixed and higher-dimensional kernels. A complete theory of mixed sine-cosine and fully multi-dimensional fractional cosine transforms, including their distributional extensions and convolution theorems, remains to be written.

(iii) Integration with modern spectral frameworks. Embedding the FrCT convolution theorem inside the multi-scale and shift-invariant spectral models of [5,33,34] could yield new filtering and resonance-extraction techniques for supply-chain and hybrid-vehicle signals.

(iv) Quantum and intelligent-logistics applications. Exploring the interplay between fractional-domain convolution and quantum-annealing or digital-twin methods [13,16,22,23,26] may open novel computational pathways for resilient logistics optimization.

(v) Numerical validation and open-source implementations. Systematic numerical experiments and publicly available software libraries would facilitate adoption of the theorem by the broader signal-processing and applied-mathematics communities.

These directions are consistent with the continuing evolution of fractional transforms from purely analytic objects into practical tools for complex, multi-scale and data-driven systems [7–9,17,18,28].

7. CONCLUSION

We have proved the convolution theorem for the generalized two-dimensional fractional cosine transform in the distributional setting. The four-term structure that arises from the cosine product-to-sum identities, together with the underlying testing-function-space framework, furnishes a powerful tool for digital filtering and for the solution of differential and integral equations in the fractional cosine domain. Parallel developments in distributional and spectral analysis and their applications to resilient multi-echelon and intelligent-logistics systems confirm the broader relevance of the present theory. Future work on discrete algorithms, higher dimensional kernels and interdisciplinary spectral models is expected to enlarge the practical impact of the result.

REFERENCES

[1] B. N. Bhosale, M. S. Choudhary, works on double transforms of generalized functions.

[2] L. B. Almeida, "Product and convolution theorems for the fractional Fourier transform," IEEE Signal Processing Letters, vol. 4, pp. 15–17, 1997.

[3] A. I. Zayed, "A convolution and product theorem for the fractional Fourier transform," IEEE Signal Processing Letters, vol. 5, pp. 101–103, 1998.

[4] H. M. Ozaktas, Z. Zalevsky, M. A. Kutay, The Fractional Fourier Transform, Wiley, 2001.

[5] Akarshan Gulhane," Spectral Framework Based on the FKF Transform for Supply Chain Dynamics and Resilience," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.140

[6] Akarshan Gulhane," Spectral Differential Properties of the Distributional FKF Transform with Applications to Resilient Supply-Chain Systems," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.139

[7] Akarshan Gulhane," FKL Transform Key Properties, And Applications To Supply-chain Analysis," International journal of Engineering sciences and Advanced Technology, Published:11-12-2025, Issue: Vol. 25 No. 12 (2025), Page Nos: 596-605, DOI: Https://doi.org/10.64771/ijesat.2025.v25.i12.3816

[8] Akarshan Gulhane," Expanded Treatment Of The Time-Shift Property Of FKF Transform With Supply-Chain Applications," International journal of Engineering sciences and Advanced Technology, Published:11-12-2025, Issue: Vol. 25 No. 12 (2025), Page Nos: 606 – 616, DOI: Https://doi.org/10.64771/ijesat.2025.v25.i12.3817

[9] Akarshan Gulhane," **The FKF Transform: Exponential Modulation Properties And Applications To Secure Digital Supply-Chain Networks**," International journal of Engineering sciences and Advanced Technology, Published:11-12-2025, Issue: Vol. 25 No. 12 (2025), Page Nos: 617 – 623, DOI: Https://doi.org/10.64771/ijesat.2025.v25.i12.3818

[10] Akarshan Gulhane, "Spectral Differential Properties of the Distributional FKF Transform with Applications to Resilient Supply-Chain Systems," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.139.

[11] Akarshan Gulhane, "Sequential Convergence in the Testing Function Space of the Two-Sided FKF Transform," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.138

[12] Akarshan Gulhane, "Distributional Extension of the FKF Transform and Its Relevance to Supply-Chain

Systems," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.137

[13] Akarshan Gulhane, "Internet of Things, Artificial Intelligence, and Quantum Computing: A Convergent Framework for Smart Logistics Ecosystems," International Journal of Engineering Science and Advanced Technology (IJESAT), Vol. 24, Issue 12, pp. 297–317, 2024. **DOI:** 10.64771/ijesat.2024.v24.i12.3372

[14] Akarshan Gulhane, "Industry 5.0 and Intelligent Logistics: Transforming Supply Chain Operations through Human-Centric Automation," International Journal of Engineering Science and Advanced Technology (IJESAT), Vol. 24, Issue 12, pp. 287–296, 2024. DOI: 10.64771/ijesat.2024.v24.i12.3371

[15] Akarshan Gulhane, "The Future of Vehicles: Solar-Powered Battery Electric Hybrid Vehicle Architecture," Tech Briefs Contest, 2020.

[16] Akarshan Gulhane, "Digital Twin Technology for Building Resilient and Adaptive Global Supply Chains: A Review," International Research Journal of Modernization in Engineering Technology and Science, Vol. 6, Issue 12, 2024. DOI: 10.56726/IRJMETS64992.

[17] Akarshan Gulhane, "Artificial Intelligence Applications in Sustainable Logistics and Green Supply Chain Management: A Bibliometric Review," International Research Journal of Modernization in Engineering Technology and Science, Vol. 6, Issue 12, 2024. DOI: 10.56726/IRJMETS65006.

[18] Akarshan Gulhane, "Artificial Intelligence and Blockchain Integration for Enhancing Supply Chain Transparency, Traceability, and Resilience," International Research Journal of Modernization in Engineering Technology and Science, Vol. 6, Issue 12, 2024. DOI: 10.56726/IRJMETS65016.

[19] Akarshan Gulhane, "Power Line Carrier Communication Based Anti-Theft System," International Journal of Research in IT and Management (IJRIM), Vol. 4, Issue 12, pp. 1–11, 2014.

[20] Akarshan Gulhane, "Battery Sizing for Plug-in Hybrid Electric Vehicles — Formula Hybrid," IEEE International Conference on Power, Control, Signals and Instrumentation Engineering (ICPCSI), pp. 368–372, 2017. DOI: 10.1109/ICPCSI.2017.8392317.

[21] Gulhane, A. (2014). Swipe Controller. *International Journal of Research in Engineering and Applied Sciences (IJREAS)*, *4*(12), 1–7.

[22] Akarshan Gulhane, "Navigating the Quantum Revolution in Logistics: Opportunities and Practical Applications in Supply Chain Management," International Journal of Engineering and Techniques (IJET), Vol. 12, pp. 553–556, 2026. **DOI:** 10.29126/ijet.2026.v12.i3.553 [1, 2]

[23] Akarshan Gulhane, "Quantum Computing for Logistics Optimization: Annealing in Unit Load Device Configuration and Disruption," International Journal of Research Publication and Reviews, Vol. 7, Issue 6, pp. 4643–4648, June 2026. DOI: 10.55248/gengpi.07.0626.16a57.

[24] Akarshan Gulhane, "Smart Mobility and Intelligent Transportation Systems: Emerging Trends, Technologies, and Challenges," International Journal of Scientific Research in Engineering and Management (IJSREM), Vol. 9, Issue 11, Nov. 2025. DOI: 10.55041/IJSREM53436.

[25] Akarshan Gulhane, "Recent Advances in Electric Vehicle Battery Technologies: Materials, Management Systems, and Sustainability Perspectives," International Journal of Scientific Research in Engineering and Management (IJSREM), Vol. 9, Issue 7, Jul. 2025. DOI: 10.55041/IJSREM51198.

[26] Akarshan Gulhane, "Quantum Computing in Transportation and Logistics: Current State, Industrial Applications, and Future Directions," International Journal of Scientific Research in Engineering and Management (IJSREM), Vol. 9, Issue 9, Sep. 2025. DOI: 10.55041/IJSREM52469.

[27] Akarshan Gulhane, "Solar-Assisted Electric Mobility: Design Framework and Performance Assessment of Sustainable Hybrid Vehicle Architectures," Int. J. Engg. Res. & Sci. & Tech., Vol. 21, No. 3, pp. 362–368, 2025. **DOI:** 10.56636/ijerst.2025.v21.i03.3594.

[28] Akarshan Gulhane, "Supply Chain Resilience in the Era of Digital Transformation: A Systematic Literature Review and Research Agenda," Int. J. Engg. Res. & Sci. & Tech., Vol. 21, No. 4, pp. 1058–1068, 2025. **DOI:** 10.56636/ijerst.2025.v21.i04.3812.

[29] S. C. Pei, M. H. Yeh, "The discrete fractional cosine and sine transforms," IEEE Trans. Signal Process., vol. 49, pp. 1198–1207, 2001.

[30] A. H. Zemanian, Distribution Theory and Transform Analysis, McGraw-Hill, New York.

[31] Akarshan Gulhane, "Distributional Extension of the FKF Transform and Its Relevance to Supply-Chain Systems," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026 , DOI: https://doi.org/10.55041/ijcope.v2i8.137

[32] Akarshan Gulhane,"Sequential Convergence in the Testing Function Space of the Two-Sided FKF Transform," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.138

[33] Akarshan Gulhane," Spectral Analysis for Multi Scale Supply Chain Dynamics and ResidueBased Resonance Extraction in Hybrid-Vehicle Systems," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.141

[34] Akarshan Gulhane," Shift-Invariant Spectral Analysis of Multi-Echelon Supply-Chain Lead Times via the FKF Transform," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.136

[35] A. H. Zemanian, Generalized Integral Transformations, Interscience, New York.