Formulations for Quantum Annealed Project Scheduling

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Abstract—
Resource constrained project scheduling and related allocation tasks can be expressed as Quadratic Unconstrained Binary Optimization (QUBO) models and submitted to quantum annealers. This paper develops the QUBO formulation in detail: choice of binary variables, linear and quadratic objective terms, conversion of precedence and resource constraints into penalty functions, and calibration of penalty weights. The resulting Hamiltonian is embedded onto quantum hardware, annealed, and post-processed. Hybrid classical–quantum workflows that wrap the QUBO solver inside Monte Carlo risk analysis or a digital twin are examined. Supporting process flowcharts and energy-landscape figures illustrate every stage. The treatment remains grounded in construction and logistics applications while remaining transferable to other combinatorial domains such as ULD configuration.

Keywords: QUBO formulation; quantum annealing; Ising model; penalty methods; resource-constrained project scheduling; hybrid quantum-classical; minor embedding; Monte Carlo; digital twin.

I. INTRODUCTION

Construction projects exemplify these challenges. Modern infrastructure projects involve hundreds or thousands of interdependent activities competing for limited labour, equipment, materials, and workspace under uncertain operating conditions. Conventional scheduling methods, including the Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT), remain indispensable for planning and monitoring projects but are fundamentally deterministic and do not directly solve large scale resource constrained optimization problems. Although advances in mixed integer programming, genetic algorithms, simulated annealing, tabu search, and other metaheuristics have significantly improved scheduling performance, increasing project complexity continues to motivate exploration of alternative computational paradigms.

Quantum computing [3], [4] has emerged as a promising approach for solving difficult combinatorial optimization problems [5], [6]. Among current quantum technologies, quantum annealing [7], [9], [17] is particularly attractive because it is specifically designed to search for low energy solutions of optimization problems [8] represented as Ising Hamiltonians or their mathematically equivalent Quadratic Unconstrained Binary Optimization (QUBO) formulations. Instead of exhaustively enumerating feasible schedules, quantum annealing exploits quantum fluctuations and tunnelling to explore complex energy landscapes and identify near-optimal solutions efficiently. While present-day quantum hardware [18], [19] has practical limitations, hybrid quantum-classical [9], [20], algorithms have demonstrated increasing potential for addressing real world optimization tasks. An additional opportunity arises from integrating QUBO-based optimization with probabilistic and real-time decision-support frameworks. Monte Carlo simulation can repeatedly evaluate annealed schedules under uncertain durations, resource availability, or external disruptions to quantify schedule risk and confidence levels.

Similarly, digital twins [10], [11], [21] provide continuously updated representations of ongoing projects, allowing optimization to be repeated as site conditions evolve. These hybrid classical quantum [22] workflows enable optimization to become adaptive rather than static, improving resilience in dynamic construction and logistics environments.

This paper presents a comprehensive treatment of QUBO-based optimization for resource-constrained project scheduling and related allocation problems. It develops the mathematical formulation of QUBO models, including binary variable selection, objective-function construction, conversion of precedence and resource constraints into penalty functions, and systematic calibration of penalty weights. The paper further describes Hamiltonian generation, hardware embedding, quantum annealing execution, and post-processing procedures. Hybrid workflows combining quantum optimization with Monte Carlo simulation and digital-twin frameworks are examined to demonstrate practical implementation. Although the discussion is motivated primarily by construction project scheduling and logistics applications, the methodology is equally applicable to a wide range of combinatorial optimization problems, including unit load device (ULD) configuration and other complex resource allocation tasks.

II. THE COMBINATORIAL SPACE

Project schedules are networks of activities linked by precedence relations and competing for limited resources.

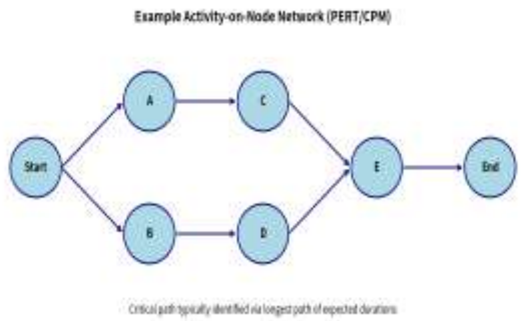

Figure 1. Precedence network whose feasible start-time assignments define the combinatorial space encoded by the QUBO.

many discrete time points, the set of feasible start-time combinations grows exponentially. Classical exact methods become intractable; meta-heuristics supply usable but rarely optimal solutions.

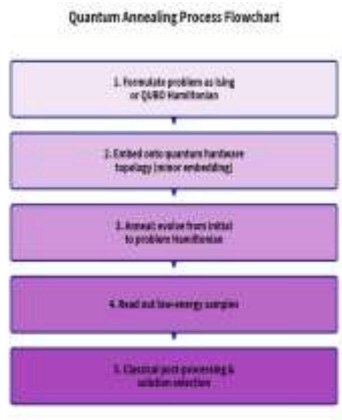

Figure 2. Quantum Annealing Process Flowchart emphasising the central role of QUBO formulation.

Quantum annealing [23], [24] attacks the same search space by encoding decisions as binary variables and letting a physical system relax toward low-energy configurations of the corresponding QUBO Hamiltonian.

The practical pathway from a scheduling statement to a quantum [17] solution consists of five modelling and execution steps: (1) select binary decision variables, (2) write the linear and quadratic objective, (3) transform every constraint into a non-negative penalty, (4) embed the logical QUBO [13], [14] onto the hardware graph, and (5) anneal, read out and classically repair the samples. The remainder of the paper examines each of these steps in depth and situates the QUBO solver inside larger Monte Carlo and digital twin [1], [9], [30] loops.

III. QUBO FORMULATION – THEORY AND CONSTRUCTION

Binary Decision Variables

Let the project contain n activities and a discrete planning horizon of T time slots. Define the binary indicator

$x_{j,t} \in \{0,1\}, \quad j = 1 \ldots n, \quad t = 0 \ldots T$

where x_{j,t} = 1 if and only if activity j starts at time t. Exactly-one-start constraints will later force each activity to begin at precisely one slot. Alternative encodings (end-time variables, binary expansion of continuous start times, assignment variables for renewable resources) are possible; the start-time encoding above is the most common for medium sized networks.

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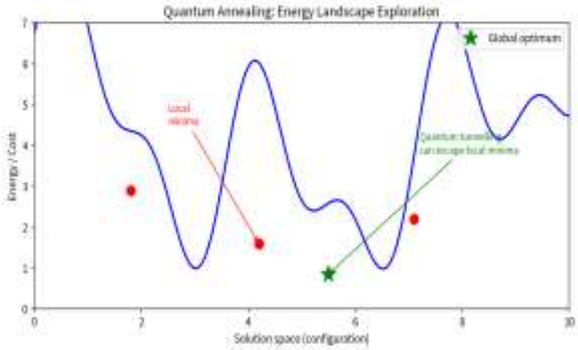

Figure 3. Energy landscape after QUBO-to-Ising conversion; the global minimum corresponds to an optimal (or near-optimal) feasible schedule.

Objective Function

A typical objective minimises a weighted combination of project makespan, total resource cost and optional earliness/tardiness penalties. In pure binary form the makespan contribution can be written

$$\min \; \Sigma_t \; t \cdot y_t$$

where y_t is an auxiliary binary variable that equals 1 if the project is still unfinished at time t. Resource-cost terms appear as linear sums over the $x_{j,t}$. Soft preferences (preferred start windows, balanced workloads) add further linear or quadratic contributions. The complete objective is therefore a quadratic polynomial in the binary variables and can be stored as a real-valued upper-triangular matrix Q such that

$$H_{obj} = x^T Q x$$

is the quantity to be minimised.

Constraint-to-Penalty Transformation

Quantum annealers accept only unconstrained quadratic objectives. Every hard constraint must therefore be converted into a non-negative penalty term that vanishes on feasible solutions and is strictly positive on infeasible ones. Three families of constraints dominate project scheduling.

Exactly-one-start (assignment) constraints

Each activity must start at exactly one time:

$$\Sigma_t \, x_{j,t} = 1 \quad \forall j$$

The corresponding penalty is

$$P_{assign} = \Sigma_j \, (\Sigma_t \, x_{j,t} - 1)^2$$

which expands into linear and quadratic terms readily absorbed by the Q matrix.

Precedence constraints

If activity i must finish before activity j may start, and d_i is the (constant) duration of i, then

$$\Sigma_t \, (t + d_i) \, x_{i,t} \;\le\; \Sigma_t \, t \, x_{j,t}$$

A quadratic penalty that enforces the inequality can be constructed by introducing slack variables or by pairwise products that punish violating pairs of start times. One compact form uses

$$P_{prec} = \Sigma_{(i,j)\in A} \; \Sigma_{t,t'} \; \max(0,\; t + d_i - t')^2 \; x_{i,t} \, x_{j,t'}$$

Only pairs that violate the precedence contribute a positive value.

Renewable-resource capacity constraints

Let R_k be the capacity of renewable resource k and $r_{j,k}$ the requirement of activity j. At every time τ the total demand must not exceed capacity:

$$\Sigma_j \, \Sigma_{t:\, t \le \tau < t+d_j} \, r_{j,k} \, x_{j,t} \;\le\; R_k$$

Again the inequality is turned into a squared residual (with optional slack binaries) and added to the QUBO. The resulting quadratic terms couple all activities that can overlap in time.

IV PENALTY WEIGHT CALIBRATION

Each penalty is multiplied by a positive weight λ. If λ is too small, the annealer may return infeasible solutions that look attractive on the soft objective. If λ is excessively large, the energy landscape becomes dominated by the penalty ridges and the annealer loses resolution on the true objective.

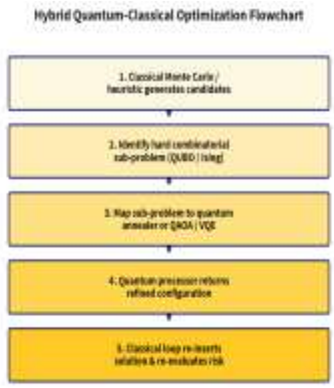

Figure 4. Hybrid Quantum-Classical Flowchart – the QUBO annealer appears as the inner combinatorial engine.

A practical rule is to set λ larger than any possible soft-objective difference between two feasible solutions, then refine by binary search on a small validation set of instances. Adaptive or iterative penalty schemes that increase λ only for persistently violated constraints are also reported in the literature.

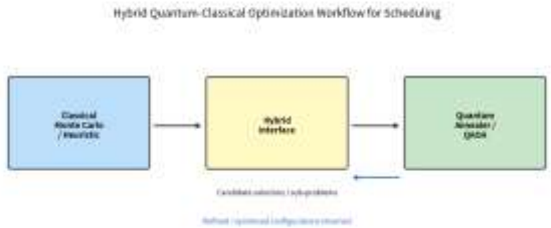

Figure 5. Hybrid architecture in which a classical risk layer repeatedly invokes the QUBO-based quantum annealer.

Mapping to the Ising Hamiltonian

Most quantum-annealing hardware natively minimises an Ising Hamiltonian. The linear change of variables

$x_i = (1 - \sigma_i^z)/2$

converts the QUBO matrix into Ising local fields h_i and couplings J_{ij}. The transformation is exact and is performed automatically by most software development kits before embedding.

From QUBO to Hardware Execution Pipeline

Once the QUBO matrix has been assembled, the remaining stages follow the Quantum Annealing [16], [15] Process Flowchart: minor embedding onto the physical topology, execution of the annealing schedule, readout of spin configurations, chain repair, and classical local-search polishing. The polished binary vectors are finally decoded back into start time assignments and evaluated against the original (non-penalised) objective and constraints.

Embedding the QUBO Solver inside Monte Carlo and Digital-Twin Loops

A single deterministic QUBO solution ignores duration uncertainty. In practice the QUBO annealer is called from within a Monte Carlo loop: each outer iteration samples activity durations from PERT or other distributions, rebuilds the time indexed QUBO for that realization, solves it, and records the resulting make span. The collection of make spans yields the same probabilistic risk metrics that classical Monte Carlo produces, now conditioned on near-optimal resource allocations.

When the same loop is placed inside a digital twin, duration distributions and resource availabilities are refreshed continuously from site data; the QUBO is therefore always solved for the most current conditions.

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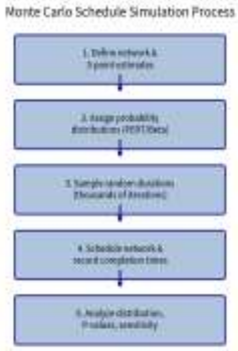

Figure 6. Monte Carlo Process Flowchart whose Stage 4 can invoke a QUBO quantum-annealing sub-solver.

IV. RESULT AND LIMITATIONS

The number of binary variables scales as O(nT). Current annealers support a few thousand logical qubits after embedding, limiting direct solution to modest networks or forcing rolling-horizon / critical subnetwork decomposition. Sparse precedence graphs and limited resource types keep the Q matrix manageable; dense overlapping resource demands rapidly increase fill-in. Penalty calibration remains partly empirical. Nonetheless, for the sub-problems that fit the hardware daily resource levelling, short-horizon sequencing, or selected multi project subsets QUBO-based quantum annealing already supplies competitive or superior solutions compared with classical meta heuristics of similar wall clock budget.

Transfer to Other Combinatorial Domains

Any problem whose decisions are binary and whose objective and constraints are at most quadratic admits the same QUBO treatment. Unit-load-device configuration under weight, volume and balance constraints is one logistics example; the identical penalty construction technique applies.

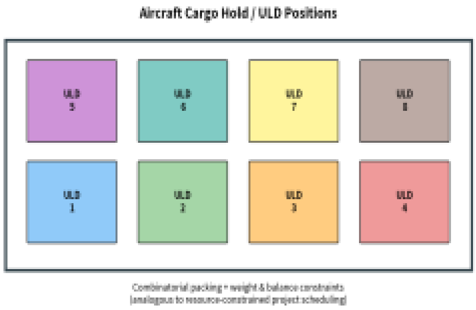

Figure 7. ULD configuration – another quadratic-binary problem solvable by the same QUBO and annealing pipeline.

V. CONCLUSION

Quadratic Unconstrained Binary Optimization supplies a precise mathematical bridge between project-scheduling statements and quantum-annealing hardware. By selecting start-time binaries, encoding the soft objective, and converting precedence and resource constraints into carefully weighted penalty terms, a practitioner obtains a QUBO matrix that can be embedded, annealed and post-processed. When this solver is wrapped inside Monte Carlo sampling or a digital twin, the result is a hybrid system that simultaneously respects combinatorial feasibility and quantifies schedule risk under uncertainty. As quantum hardware connectivity and hybrid software improve, the range of construction and logistics instances amenable to exact QUBO-based quantum annealing will continue to expand.

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