Hybrid Quantum Classical Integration

Aayushee G[1]

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Abstract—Quantum annealing has emerged as an effective meta-heuristic for solving combinatorial optimization problems by leveraging quantum tunnelling to navigate complex energy landscapes. This paper examines both the theoretical foundations and the practical implementation of quantum annealing for project scheduling and resource-allocation problems formulated as Ising or Quadratic Unconstrained Binary Optimization (QUBO) models. A comprehensive explanation of the quantum annealing process is provided through a stage-by-stage process flowchart. In addition, the study discusses hybrid quantum classical frameworks in which quantum annealing is integrated with Monte Carlo simulations or digital twin environments, supported by illustrations of the optimization energy landscape and the hybrid system architecture. The adaptability of the proposed approach is further demonstrated through analogies with unit load device (ULD) configuration and other related combinatorial optimization problems across multiple application domains. Finally, the paper outlines key implementation considerations and evaluates the current level of technological maturity of quantum annealing for applications in construction planning and logistics optimization.

Keywords: quantum annealing implementation; Ising model; QUBO; combinatorial optimization; project scheduling; hybrid quantum-classical; energy landscape; digital twin; Monte Carlo; supply-chain resilience; ULD configuration

I. INTRODUCTION

Quantum annealing [11], [12] is most powerful when embedded inside a larger classical loop. Classical Monte Carlo methods or a digital twin [13], [14] generates scenarios and identify hard combinatorial sub-problems; the Quantum Annealing [15] Process Flowchart solves those sub problems; the refined configurations are re inserted and the residual schedule risk is reevaluated. This hybrid pattern combines the strengths of probabilistic risk analysis with the combinatorial power of quantum annealing.

The effectiveness of quantum annealing [16] depends fundamentally on constructing an appropriate QUBO formulation. In a QUBO model, decision variables are represented by binary variables, optimization [17] objectives are expressed through linear and quadratic coefficients, and problem constraints are transformed into penalty terms that preserve feasibility while maintaining compatibility with unconstrained optimization [18] . The quality of the resulting solution is therefore strongly influenced by variable encoding strategies, formulation of precedence and resource constraints, selection of objective functions, and calibration of penalty weights. Careful Hamiltonian design is essential to ensure that feasible schedules correspond to low energy states accessible to the annealing process.

Practical implementation extends beyond mathematical formulation [19], [20] alone. Once a QUBO model is constructed, it must be embedded onto the connectivity graph of quantum annealing hardware, executed through an annealing schedule, and subsequently post-processed to repair infeasible solutions and improve objective quality. Increasingly, these quantum optimization [4] stages are incorporated into hybrid computational [24] workflows in which classical algorithms perform preprocessing, decomposition, validation, and refinement. Such hybrid architectures enable quantum annealers [5], [6] to address the computationally demanding optimization core while leveraging classical computing for data management and large scale problem coordination.

II. RESOURCE CONSTRAINED PROJECT SCHEDULING AS A QUBO

Present day quantum annealers [7], [8], such as those based on superconducting flux qubits, can already address modest sized scheduling sub problems. Larger instances still require decomposition strategies. Noise, embedding overhead, and parameter tuning remain practical challenges; yet improving hardware and hybrid software stacks are steadily expanding the set of viable applications.

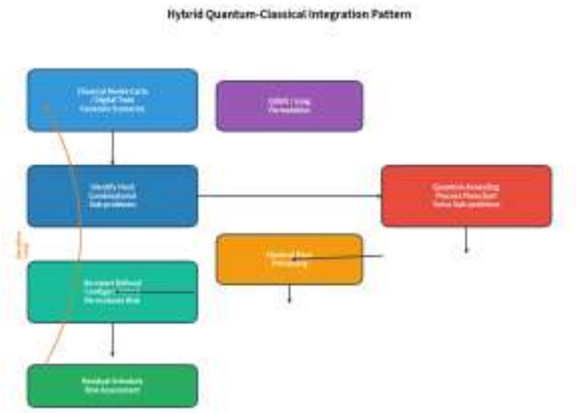

Figure 1. Hybrid Quantum-Classical Integration Pattern showing the iterative loop between classical scenario generation and quantum annealing of combinatorial sub-problems.

Resource constrained project scheduling (RCPS) can be formulated with binary variables indicating activity start times. Let the set of activities be J and the discrete time horizon be T. Define the binary decision variable: $x_{j,t} =$ $1 \;\; if\ activity\ j\ starts\ at\ time\ t, \quad x_{j,t} \in \{0,1\}$

Precedence, resource capacity, and time window constraints are encoded as quadratic penalty terms that augment the objective function. The resulting model is a Quadratic Unconstrained Binary Optimization (QUBO) problem of the form: $min \;\; x^T\, Q\, x \quad subject\ to \;\; x \in \{0,1\}$^$n$

where Q is an $n \times n$ real-valued matrix whose diagonal entries encode linear costs and self penalties, and whose off diagonal entries encode pairwise interactions arising from constraints.

Mapping to the Ising Model

Quantum annealers natively implement the Ising spin glass Hamiltonian. The QUBO is therefore transformed into Ising form via the substitution $x_i = (1 - \sigma_i)/2$, yielding:

$$H(\sigma) = \sum_i h_i\, \sigma_i + \sum_{i<j} J_{ij}\, \sigma_i\, \sigma_j$$

Here $\sigma_i \in \{-1, +1\}$ are the spin variables, h_i are local fields, and J_{ij} are coupling strengths. The ground state of H corresponds to the optimal (or near-optimal) binary solution of the original scheduling problem.

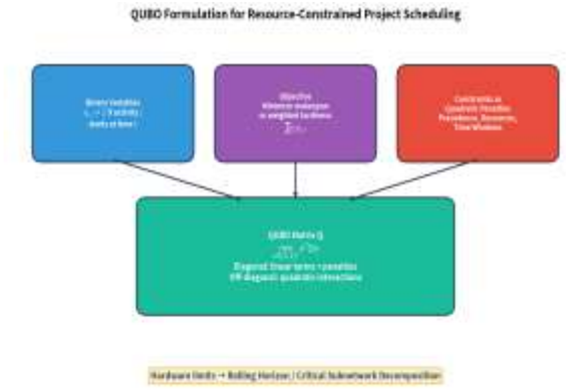

Figure 2. Conceptual structure of the QUBO formulation for resource-constrained project scheduling.

Constraint Encoding

Typical quadratic penalty terms include:

Precedence constraints. If activity i must finish before activity j can start, with processing time p_i:

$$P_{prec} = \sum_{i<j} \sum_{t,t'} \lambda_{prec}\; x_{i,t}\, x_{j,t'} \quad \forall\, t' < t + p_i$$

Resource capacity constraints. For each renewable resource k with capacity R_k and requirement r_{jk} the squared penalty is applied over all time periods:

$$P_{res} = \sum_{k,t} \lambda_{res}\; (\textstyle\sum j \quad r_{jk}\; \sum \tau\; x_{j,\tau} \; - \; R_k\;)\text{^}2$$

The penalty weights λ must be chosen large enough to enforce feasibility yet not so large that they overwhelm the original objective, a classic challenge in penalty-method formulations.

III. DECOMPOSITION STRATEGIES

Because current quantum annealing hardware limits problem size (typically a few thousand logical variables after embedding overhead), practical implementations decompose the full schedule into manageable sub-problems. Two widely used strategies are:

Rolling-horizon decomposition: The planning horizon is partitioned into overlapping time windows. At each step only the activities that can start inside the current window are optimized quantum mechanically [9], [10]; decisions for earlier windows are fixed.

Critical-subnetwork extraction: Classical critical path or resource conflict analysis identifies a dense subset of tightly coupled activities. Only this subnetwork is mapped to the quantum annealer; the remaining activities are scheduled classically around the quantum solution.

The end-to-end quantum annealing workflow consists of several tightly coupled stages, each of which critically affects solution quality.

Figure 3. Quantum Annealing Process Flowchart from problem definition through solution validation.

Logical variables of the QUBO must be mapped onto the sparse hardware graph (e.g., Chimera, Pegasus, or Zephyr topologies). Chains of physical qubits represent a single logical variable; the chain strength must be set high enough to keep the chain consistent yet low enough not to dominate the problem Hamiltonian. Embedding overhead can increase the required number of physical qubits by a factor of 5–20 for dense interaction graphs. The annealing time t_a and the number of independent reads (samples) control the trade-off between exploration and exploitation. Typical production settings use annealing times of 20–200 µs and collect several thousand samples. Advanced schedules with pauses, reverse annealing [20], [21], or quantum classical [22],, [23] feedback loops further improve success probability on hard instances. The same Ising/QUBO modelling style and the same Quantum Annealing Process Flowchart transfer to other combinatorial domains. Notable examples include:

Figure 4. Domains in which the same Ising/QUBO modelling approach has been successfully applied.

Multi-project portfolio selection: Binary variables select projects under budget, resource, and strategic-fit constraints.

Crew rostering: Assignment of personnel to shifts while respecting labor regulations, skills, and fairness criteria.

Construction logistics routing: Vehicle routing with time windows, capacity, and site-access constraints.

ULD configuration in air cargo: Three-dimensional packing of unit load devices under weight, balance, and volume constraints maps naturally onto quadratic binary models and has been explored with both classical heuristics and quantum-inspired solvers.

Successful hybrid quantum-classical implementations depend on several interlocking factors:

Figure 5. Key success factors that determine the practical viability of a quantum-annealing solution.

High-quality QUBO formulation — accurate encoding of the objective and tight, well-scaled penalty terms.

Effective minor embedding — low-overhead mappings that preserve logical connectivity.

Sufficient annealing time and number of reads — adequate statistical sampling of the low-energy landscape.

Robust classical post-processing — unembedding, majority voting on chains, local search refinement, and feasibility repair.

Seamless integration with the surrounding Monte Carlo or digital-twin workflow so that quantum results are consumed and residual risk is correctly quantified.

Present day annealers can already address modest sized scheduling sub-problems (typically a few hundred logical variables after embedding). Larger instances still require decomposition. Noise, embedding overhead, and the need for careful parameter tuning remain practical challenges. Nevertheless, successive generations of hardware featuring denser connectivity, lower noise, and larger qubit counts, together with maturing hybrid software frameworks, are steadily expanding the set of industrially relevant applications that can benefit from quantum annealing.

The hybrid pattern described here classical scenario generation and risk evaluation wrapped around quantum solution of combinatorial kernels offers a pragmatic and scalable pathway for extracting value from near-term quantum devices while classical high-performance computing continues to handle the bulk of the probabilistic analysis.

IV. CONCLUSION

Hybrid quantum classical integration leverages quantum annealing precisely where it is strongest: solving hard combinatorial sub-problems that arise inside larger probabilistic or simulation-driven workflows. By formulating resource-constrained scheduling (and related logistics problems) as QUBOs, embedding them onto quantum hardware, and closing the loop with classical post-processing and risk re-evaluation, organizations can begin to realize practical advantages even with today's limited quantum resources. Continued advances in both hardware and hybrid algorithms promise to broaden the scope of these applications in the coming years.

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