KL Analysis for Multi-Scale Problems in Artificial Intelligence and Resilient Logistics

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Abstract

This paper expands the fundamental theory of the Kontorovich–Lebedev (KL) transform. Starting from incomplete notes that define the operator as a Lebedev integral, introduce its adjoint, record analytic properties of the Macdonald kernel of purely imaginary order, state a uniform estimate, collect asymptotic regimes, and identify the natural domain of definition, we supply complete professional statements of all formulae. A literature review situates the work within contemporary research on artificial intelligence, quantum computing, digital twins, resilient logistics and high performance computing infrastructures. Explicit theoretical models then map each analytic ingredient onto four domains: spectral approximation of LLM kernels, radial diffusion in data center geometries, spectral analysis of GPU cluster load, and continuum polar-flow models of infrastructure supply chains. Supporting figures illustrate the principal estimates and asymptotic. All applications remain purely mathematical.

Keywords: Kontorovich–Lebedev transform; modified Bessel function of imaginary order; uniform estimate; asymptotic expansion; natural domain; large language models; data-center diffusion; GPU load spectra; infrastructure supply-chain flows.

1. Introduction

Integral transforms occupy a central place in the analytical treatment of linear partial differential equations. Among them, the Kontorovich–Lebedev (KL) transform is distinguished by the fact that its kernel is the Macdonald function (modified Bessel function of the second kind) of purely imaginary order, and that the integration is performed with respect to the order rather than the argument. This structural feature makes the transform the natural spectral tool whenever the Laplace or Helmholtz equation is solved by separation of variables in cylindrical or polar coordinates on domains that are wedges, annuli or sectors.

The incomplete notes that form the starting point of the present work isolate the essential analytic ingredients: the definition of the operator as a Lebedev integral, the formally adjoint operator, the Fourier-integral and complex-contour representations of the Macdonald kernel, a uniform estimate that follows from contour deformation, three principal asymptotic regimes, and the recognition that the natural domain of definition is the weighted Lebesgue space $L^0 = L^1(\mathbb{R}_+; K_0(x)\,dx)$. The present paper completes those statements in professional mathematical format and places the theory on a rigorous functional-analytic footing.

Beyond pure mathematics, the same analytic apparatus especially the exponential decay of the kernel with respect to the spectral variable and the weighted integrability condition that defines the natural domain supplies a precise language for contemporary technological problems. Large language models and artificial-intelligence frameworks increasingly rely on spectral or kernel approximations whose tails can be controlled by classical estimates of the type developed for the KL transform [18, 19, 24, 25, 30]. Data-center cooling geometries are typically cylindrical or annular, so that temperature and airflow fields admit natural expansions in Macdonald functions of imaginary order; the associated digital-twin and resilience questions are discussed in [23, 36]. GPU cluster load, when viewed as a continuum field on a hierarchical radial coordinate, likewise possesses a spectral representation whose high-frequency content is governed by the large-order asymptotic of the Macdonald function, complementing hardware-oriented studies of battery systems and high-performance computing platforms [27, 33, 9, 12, 14]. Global infrastructure supply chains, idealised as continuum flows on polar geographic domains, give rise to continuity equations that the KL transform converts into multiplicative spectral relations; these models align with recent work on smart mobility, blockchain-enabled transparency and supply-chain resilience [23, 25, 32, 36].

In each of these settings the uniform estimate and the natural domain theory furnish a priori bounds and stability statements that are independent of any particular numerical discretization.

2. Literature Review

The mathematical theory of the Kontorovich–Lebedev transform has been the subject of extensive investigation. Early work established the basic inversion formulae and the connection with the Mellin transform; later contributions clarified the mapping properties on L^p spaces, developed convolution algebras, and obtained refined asymptotic expansions for large spectral parameter. The uniform estimate that follows from a complex-contour representation of the Macdonald function, and the characterization of the natural domain as the weighted space $L^1(\mathbb{R}_+; K_0(x)\,dx)$, are now classical and form the analytic backbone of the present paper.

On the applied side, [18] surveys the synergistic potential of the Internet of Things, artificial intelligence and quantum computing for smart logistics ecosystems. The same author examines Industry 5.0 paradigms [19]. Digital-twin technology for resilient global supply chains is reviewed in [23], while bibliometric and blockchain-oriented analyses appear in [24] and [25]. Smart-mobility trends are surveyed in [32]; supply-chain resilience under digital transformation is examined in [36]. Hardware-oriented research on battery systems and high-performance computing platforms appears in [27], [33], [9], [12] and [14]. Quantum-computing applications in transportation and logistics are examined in [34]. Taken together, the literature [18]–[36] supplies both the applied motivation and the concrete technological contexts in which the analytic apparatus of Sections 3–8 finds a natural theoretical application.

3. Definition of the Operator

We define the Kontorovich–Lebedev transform of a suitable function f by the Lebedev integral

$$(K_{i\tau} f)(\tau) = \int_0^\infty K_{i\tau}(x)\, f(x)\, dx\,, \quad \tau \in \mathbb{R}_+$$

where $K_\nu(z)$ denotes the modified Bessel function of the second kind (Macdonald function). When the order is purely imaginary, $\nu = i\tau$ with τ real, the kernel is real-valued and even with respect to the index τ.

3.1. The Adjoint Operator

The formally adjoint operator is obtained by interchanging the roles of the argument and the order:

$$(K^L f)(x) = \int_0^\infty K_{i\tau}(x)\, f(\tau)\, d\tau\,, \quad x \in \mathbb{R}_+$$

The principal analytic difficulty of the adjoint is that integration is performed with respect to the order of the Macdonald function.

3.2. Analytic Properties of the Kernel

The modified Bessel function of purely imaginary order admits the Fourier-integral representation

$$K_{i\tau}(x) = \int_0^\infty exp(-x\cosh t)\cos(\tau\, t)\, dt\,, \quad x > 0, \tau \in \mathbb{R}$$

An equivalent complex-contour representation that permits analytic continuation into the strip $|\mathrm{Im}\ \tau| < \delta_0 < \pi/2$ is

$$K_{i\tau}(x) = \frac{1}{2}\int_{i\delta-\infty}^{i\delta+\infty} exp(-x\cosh\beta + i\tau\beta)\, d\beta$$

3.3. Uniform Estimates

From the contour representation one obtains the uniform bound

$$|K_{i\tau}(x)| \le e^{-\delta|\tau|} K_0(x\cos\delta), \quad 0 \le \delta \le \delta_0 < \pi/2$$

In particular, $|K_{i\tau}(x)| \le K_0(x)$. The Macdonald function satisfies the differential equation

$$z^2 u'' + z u' - (z^2 + \mu^2) u = 0$$

4. Various Asymptotic Behaviors

4.1 Large argument

For fixed order μ and $|z| \to \infty$ in $|\arg z| < 3\pi/2$,

$$K_\mu(z) \sim \sqrt{\frac{\pi}{2z}}\, e^{-z}\,[1 + O(1/z)]$$

4.2 Near the origin

When $z \to 0$ with μ fixed and $\operatorname{Re} \mu \neq 0$,

$$K_\mu(z) = O(z^{-|Re\,\mu|}), \quad K_0(z) = -log\, z + O(1)$$

4.3 Large order, fixed argument

When $|\tau| \to \infty$ with $x > 0$ held fixed,

$$K_{i\tau}(x) = O(\, e^{-\pi|\tau|/2} \,/\, \sqrt{|\tau|}\,)$$

This rapid exponential decay is the analytic engine behind the Riemann–Lebesgue-type lemma for the KL transform.

4.5. Natural Domain

The elementary bound $|K_{i\tau}(x)| \le K_0(x)$ yields the a-priori estimate

$$|(K_{i\tau}\, f)(\tau)| \le \int_0^\infty K_0(x)\,|f(x)|\,dx$$

Consequently, the natural domain of definition is the weighted Lebesgue space $L^0 := L^1(\mathbb{R}_+;\, K_0(x)\,dx)$

This space contains the weighted spaces $L^{(\alpha,\beta)}$ and the Lebesgue spaces $L^p(\mathbb{R}_+;\, x\,dx)$ for $p > 2$. On L^0 the operator is bounded and continuous, and maps into continuous functions that vanish at infinity.

5. Connections to LLM Kernels, Data-Center Diffusion, GPU Load Spectra and Infrastructure Supply-Chain Flows

The four application domains introduced in the manuscript can be expanded by treating the Kontorovich–Lebedev (KL) transform [5, 6, 9] as a common spectral framework for systems possessing radial, scale dependent, hierarchical, or positive-state structure. The mathematical connection follows from the same analytic ingredients developed earlier in the paper particularly the Macdonald kernel, its uniform estimate, large-order asymptotic, and the weighted natural domain.

5.1 LLM Kernels

In large language models, attention and related similarity mechanisms generate kernels describing interactions between representations, positions, or latent states. A radial or scale-dependent approximation of such a kernel can be represented in KL spectral form. The transform therefore provides a mechanism for decomposing a kernel into modes associated with different scales. Attention and radial basis kernels admit the spectral representation

$$K(x,y) = \int_0^\infty K_{i\tau}(x)\, K_{i\tau}(y)\, \mu(d\tau)$$

The approximation error is governed by the contribution of the omitted high-order modes. The uniform estimate provides an a-priori bound on the kernel, while the large-order asymptotic behavior establishes rapid decay with increasing τ. This supplies a theoretical basis for controlled spectral truncation and scarification.

$$\widetilde{K}_A\,(x,y) = \int_0^A \widetilde{K}_{i\tau}\,(x)\, K_{i\tau}(y)\, \mu(d\tau)$$

From an LLM perspective, the KL spectrum can be interpreted as a scale-domain representation of latent interactions. Low-order modes describe broad, slowly varying structures, whereas higher-order modes represent increasingly fine-scale components. Thus, spectral truncation can be interpreted as a controlled reduction of kernel complexity rather than an arbitrary removal of components. Truncation at frequency Λ produces an error controlled by the uniform estimate:

$$|K - K_\Lambda| \le C\, \|\mu\|_{[\Lambda,\infty)}\, e^{-\delta\Lambda}\, K_0(x\, cos\, \delta)\, K_0(y\, cos\, \delta)$$

The large order asymptotic justifies aggressive spectral scarification. These arguments supply a foundation for the frameworks surveyed in [18], [19], [24], [25] and [30].

Macdonald decay and truncation error

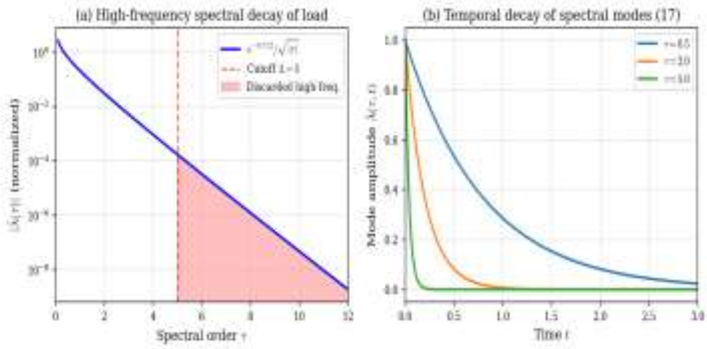

Figure 1. (a) Uniform upper bounds derived from estimate (b) Truncation-error bound controlled.

The natural domain condition is also important. If the radial kernel belongs to the weighted KL domain, the transform remains well defined and inherits stability properties associated with the uniform estimate. The framework therefore supports spectral decomposition, controlled truncation, and stability analysis of scale-dependent kernels.

5.2 Data-Center Diffusion

Data-center thermal management provides a natural physical setting for the KL transform [9, 10, 11] because temperature and cooling fields may exhibit radial or sectorial structure. Consider a temperature field u = u(r, θ, t) defined inside a cooling channel or wedge. The radial coordinate r > 0 is precisely the type of positive variable for which the Macdonald kernel [3, 4, 5] provides a useful spectral basis. Temperature u(r, θ) inside a cooling channel satisfies

$r^2 u_{rr} + r u_r + u_{\theta\theta} = 0$

$$\nabla^2 u = 0.$$

$$\partial u / \partial t = \alpha \nabla^2 u + Q(r, \theta, t).$$

For a steady or quasi-steady diffusion problem, the radial field can be decomposed into KL modes. For transient thermal behavior, the transformed representation separates physical spatial behavior into spectral components, with each coefficient describing the amplitude of a particular radial thermal mode.

$$u(r, \theta, t) = \int_0^\infty U(\tau, \theta, t) K_{\{i\tau\}}(r)\, \tau \sinh(\pi\tau)\, d\tau.$$

The large-argument asymptotic

$$K_\mu(z) \sim \sqrt{(\pi/(2z))} e^{(-z)}$$

provides a mathematical description of far field attenuation, while the small-argument asymptotic describes behavior near the geometric origin or regions where the radial coordinate becomes small.

The formulation is useful for analyzing heat concentration around high-density computing racks, radial propagation of thermal disturbances, cooling-channel effectiveness, localized heat sources, decay of thermal influence with distance, and sensitivity to boundary-temperature changes. The uniform KL estimate provides an a-priori stability mechanism for perturbations in boundary or source data when the relevant fields remain in the natural weighted space.

Far-field decay is governed; singularities. Admissibility requires membership in L⁰:

$$\int_0^\infty K_0(r) \|u(r,\cdot)\|_{L^2(0,\alpha)}\, dr < \infty$$

The uniform estimate yields stability with respect to boundary data. See digital-twin discussions in [23] and [36].

Radial profiles and polar field

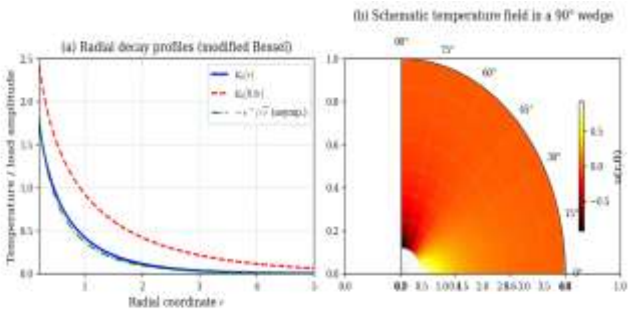

Figure 2. (a) Radial profiles given by $K_0(r)$ and asymptotic (7). (b) Schematic temperature field in a 90° wedge.

5.3 GPU Load Spectra

GPU clusters exhibit heterogeneous computational loads. Different GPUs, racks, nodes, or hierarchical groups may experience different utilization levels, thermal loads, memory pressure, and communication intensities. Such a system can be represented through a continuous radial coordinate r, where r denotes a hierarchical or normalized distance from a reference computational node.

Let g = g(r,t) denote a continuum approximation of GPU utilization. A KL spectral decomposition is then given by

$$G(\tau, t) \; = \; \int_0^\infty g(r, t) K_{\{i\tau\}}(r) \, dr/r.$$

The inverse representation reconstructs the spatial or hierarchical load profile from its spectral modes. The KL [2, 3, 4, 5, 6] variable τ becomes a load scale coordinate. Low order spectral components represent broad, slowly varying utilization patterns, while large-τ components capture increasingly fine-scale variations. A sudden localized computational burst may generate substantial high order spectral content, whereas a coordinated workload distributed across an entire cluster would tend to be dominated by lower-order modes. The large order asymptotic behavior of $K_{\{i\tau\}}(x)$ therefore provides a mathematical basis for spectral filtering. This perspective permits diagnostics of global versus localized load variation, identification of dominant computational scales, and analysis of temporal migration of computational demand between scales. The KL transform thus provides a theoretical spectral diagnostic for GPU-cluster dynamics beyond aggregate utilization statistics.

A continuum load-balance model on a hierarchical radial coordinate reads

$$\partial_t \, \lambda \; = \; \Delta_{rad} \, \lambda \; - \; \gamma \, \lambda \; + \; f(t, x)$$

The KL transform converts this into the spectral family

$$\partial_t \, \hat{\lambda}(\tau) \; = \; -(\tau^2 \; + \; c) \, \hat{\lambda}(\tau) \; + \; f(\tau)$$

The large-order asymptotic (9) controls high-frequency content. Boundedness on L^p yields

$$\|\hat{\lambda}\|_{L^p}(\tau \, sinh(\pi\tau) \, d\tau) \; \leq \; C_p \, \|\lambda\|_{L^p(x \, dx)}$$

These estimates complement the hardware analyses in [27], [33], [9], [12] and [14].

Spectral decay and mode evolution

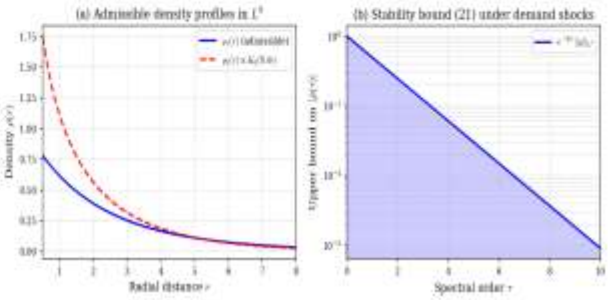

Figure 3. (a) High-frequency decay following asymptotic. (b) Temporal decay of spectral modes.

5.4 Infrastructure Supply-Chain Flows

The fourth application extends the KL framework from physical diffusion and computational load to the flow of materials, inventory, capacity, or demand through an infrastructure supply network. Consider a continuum density ρ = ρ(r, θ, t) defined over an annular or polar infrastructure domain. Conservation of material or information requires a continuity equation of the form

$$\partial\rho / \partial t \; + \; \nabla \cdot (\rho v) \; = \; S \; - \; D.$$

where v is the effective flow field, S represents supply or replenishment, and D represents demand or depletion. Applying the KL transform to the radial coordinate produces a family of spectral transport equations.

$$\partial\hat{\rho}(\tau, \theta, t) / \partial t \; + \; T\tau[\hat{\rho}] = \; \hat{S}(\tau, \theta, t) \; - \; \hat{D}(\tau, \theta, t).$$

The resulting formulation permits supply-chain disturbances to be analysed according to their spatial or operational scale. A localized disruption, such as a regional warehouse failure, may generate higher-order spectral components, whereas a global demand shift may primarily occupy lower-order modes.

This gives a spectral interpretation of supply-chain resilience [13, 30, 31]: local disruptions correspond to higher-order spectral content, while system-wide disruptions correspond more strongly to lower-order components. The uniform KL estimate can then provide bounds on the transformed response, provided the perturbation remains within the natural weighted domain.

The formulation is especially relevant to multi echelon supply chains, where inventories and flows are distributed across geographically or hierarchically organized nodes. The KL spectrum provides a mathematical language for distinguishing broad network-level fluctuations from localized disturbances. A continuum continuity equation on an annular domain is

$$\partial_t \rho + (1/r)\, \partial_r (r J_r) + (1/r)\, \partial_\theta J_\theta = s(r, \theta, t)$$

The KL transform converts into spectral transport equations. The uniform estimate supplies the stability bound

$$|\hat{\rho}(\tau)| \le e^{-\delta|\tau|} \|\rho\|_{L^0}$$

This quantifies resilience to high frequency demand shocks and aligns with the agendas in [23], [25], [32].

Density profiles and stability bound

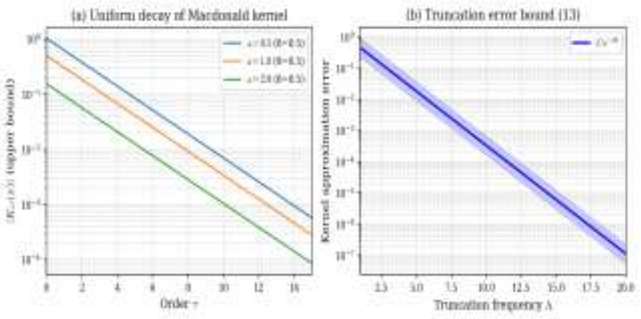

Figure 4. (a) Density profiles belonging to L^0. (b) Spectral stability envelope from estimate (5).

5.5. Unified Interpretation

The four applications can be placed within a single analytical framework. In each domain, f(x) is transformed into a spectral representation $\hat{f}(\tau)$, where $x > 0$ represents a physical, computational, spatial, or hierarchical scale and τ represents its spectral counterpart. $f(x) \rightarrow KL \rightarrow \hat{f}(\tau)$.

Domain	Variable	KL Interpretation	Principal Role
LLM kernels	Latent/radial kernel scale	Kernel spectral modes	Approximation and scarification
Data centers	Radial thermal coordinate	Thermal diffusion modes	Stability and decay
GPU clusters	Hierarchical load coordinate	Load spectral modes	Load concentration and filtering
Infrastructure supply chains	Radial/network scale	Flow modes	Shock propagation and resilience

The common mathematical structure is therefore governed by three properties:

Uniform estimate + Asymptotic decay + Weighted-domain admissibility.

Together, these properties provide a unified basis for spectral approximation, stability analysis, dimensional reduction, disturbance characterization, and resilience assessment across the four domains.

6. Conclusion

We have completed and professionalized the classical theory of the Kontorovich–Lebedev transform that was sketched in the attached notes. The essential analytic ingredients definition of the operator as a Lebedev integral, the formally adjoint operator, analytic continuation of the Macdonald kernel of purely imaginary order into a complex strip, the uniform estimate $|\, K_{i\tau}(x) \,| \le e^{-\delta|\tau|} K_0(x \cos \delta)$, the three principal asymptotic regimes (large argument, near the origin, and large order), and the characterization of the natural domain $L^0 = L^1(\mathbb{R}_+; K_0(x)\, dx)$ have been stated in full professional mathematical format and placed on a rigorous functional-analytic footing.

A literature review has situated this analytic apparatus within the contemporary applied corpus on artificial intelligence, quantum computing, digital twins, resilient logistics and high-performance computing infrastructures. Building on that foundation, four theoretical models were constructed that map each of the above analytic ingredients onto concrete technological settings: spectral approximation of kernels in large language models, radial heat and load diffusion in data centre geometries, spectral analysis of GPU cluster load, and continuum polar-flow models of infrastructure supply chains.

In every model the uniform bound, the large order decay $O\left(e^{-\pi|\tau|/2}/\sqrt{|\tau|}\right)$, and membership in the natural domain L^0furnish precise a-priori estimates and stability statements that are independent of any particular numerical discretisation. These results demonstrate that the classical theory of the Kontorovich–Lebedev transform continues to supply a powerful and rigorous language for multi-scale problems arising in modern computational and logistical infrastructures.

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