Karhunen–Loève Expansion Theory in AI–Blockchain Supply Chains, and Sustainable Logistics

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Abstract—
The Karhunen–Loève (KL) expansion provides an optimal orthogonal series representation of second-order stochastic processes and random fields. This paper presents a complete mathematical formulation of the KL theory using native Office Math Markup Language (OMML) equations, covering the governing Fredholm integral eigenvalue problem, the expansion and its coefficients, truncation error bounds, and the normalized representation. Building upon this theoretical foundation, we systematically explore practical applications of the KL expansion in contemporary supply-chain research themes: quantum-inspired optimization and uncertainty quantification in logistics networks; construction of resilient and adaptive digital twins for global supply chains; stochastic modelling supporting artificial-intelligence and blockchain integration for transparency, traceability and resilience; and uncertainty-aware modelling in sustainable and green logistics. The KL expansion emerges as a rigorous, computationally tractable tool for dimensionality reduction, random-field generation and risk quantification across these domains, thereby bridging classical stochastic process theory with the emerging requirements of Industry 4.0 and quantum-era logistics systems.

Keywords— Karhunen–Loève expansion; stochastic processes; uncertainty quantification; digital twin; supply chain resilience; quantum logistics; green logistics; AI–blockchain integration.

I. INTRODUCTION

The Karhunen–Loève expansion occupies a central position in this landscape. Originating in the theory of stochastic processes [1,2], the KL theorem asserts that any second-order random process (or random field) admits an expansion into a series of deterministic orthonormal functions weighted by uncorrelated random coefficients. The expansion is optimal in the mean-square sense: for any fixed number of retained terms it minimizes the residual variance among all orthonormal bases [10]. Consequently, the KL expansion furnishes a natural, low-dimensional parametrization of uncertainty that can be injected into digital twins, AI-driven optimization models and quantum-inspired heuristic frameworks [31,32].

Spectral analysis provides a complementary perspective for understanding supply-chain uncertainty. The FKF framework has been extended to generalized functions and distributional signals [1], supported by a rigorous theory of sequential convergence in testing-function spaces [2]. Its applications include multi-scale spectral analysis [3], lead-time shift analysis [4], resilience modelling [5], and spectral differential operations [6]. Earlier studies of FKF properties also established key transformation relationships [7], time-shift behavior [8], and exponential modulation properties [9]. Together, these results provide a spectral foundation for analyzing temporal, scale-dependent, and dynamically changing supply-chain signals.

The engineering relevance of such mathematical modelling extends beyond logistics. Earlier work on power-line communication and anti-theft systems [19] and battery sizing for plug-in hybrid electric vehicles [20] demonstrates the application of analytical and computational methods to engineering systems. More recent research on smart mobility and intelligent transportation systems [24], electric-vehicle battery technologies [25], and solar-assisted electric mobility [27] highlights the increasing interaction between transportation, energy, sustainability, and intelligent computation. These applications provide additional contexts in which stochastic dimensionality reduction and uncertainty propagation can support system analysis and optimization.

The emergence of quantum computing introduces another important application area. Quantum computing has been investigated for logistics optimization and practical supply-chain applications [22,26], while quantum annealing has been explored for unit-load-device configuration and disruption-related optimization [23]. A KL-based representation can reduce the number of stochastic variables entering such optimization models, thereby facilitating computationally manageable hybrid classical–quantum formulations. This is particularly relevant when digital twins and AI systems continuously generate large numbers of uncertain scenarios [33,36].

II. LITERATURE REVIEW

Recent literature on the quantum revolution in logistics [3,33], digital-twin technology for resilient global supply chains [5,6,14], AI–blockchain integration for transparency and traceability [7,35], and artificial-intelligence applications in sustainable green logistics [8,36] underscores the growing importance of stochastic modelling. The present work therefore pursues two complementary objectives. First, it provides a self-contained, equation-rich exposition of the KL theory suitable for direct use in engineering and operations-research documents. Second, it maps the theoretical constructs onto the concrete modelling needs that arise in the aforementioned application domains, thereby demonstrating the practical utility of the expansion.

Modern engineering and supply-chain systems require rigorous mathematical tools for analyzing dynamic, uncertain, and complex signals [32,35]. The distributional FKF transform extends conventional spectral analysis to generalized and potentially singular supply-chain signals [32]. Its combination of temporal and scale-domain representations provides a useful framework for modelling complex operational behavior [35]. Earlier computational control research, including the Swipe Controller, further demonstrates the importance of structured mathematical and computational approaches in engineering applications [21].. Accurate uncertainty representation is therefore essential for simulation, optimization, resilience assessment, and intelligent decision-making [5,36]. Recent FKF-based research has introduced spectral approaches for multi-scale supply-chain dynamics [3], shift-invariant lead-time analysis [4], resilience modelling [5], and differential and distributional representations of complex supply-chain signals [1,6]. This property makes the KL expansion particularly suitable for reducing the dimensionality of correlated demand, inventory, production, transportation, and disruption uncertainties before simulation and optimization [2]. The underlying ideas of functional representation and spectral decomposition also complement the testing-function and convergence framework developed for the two-sided FKF transform [2,31].

The growing integration of digital technologies further increases the importance of efficient uncertainty quantification. IoT, artificial intelligence, and quantum computing are converging to form intelligent logistics ecosystems [33], while Industry 5.0 promotes intelligent, adaptive, and human-centric supply-chain operations [34]. Digital-twin technology enables continuous representation and monitoring of physical supply-chain systems for resilience and adaptive planning [36]. AI applications in sustainable and green logistics provide additional opportunities for predictive and prescriptive decision-making under uncertainty [17], while AI–blockchain integration supports transparency, traceability, and resilient information management [18].

The KL expansion extracts statistically dominant modes from covariance structure, whereas the FKF framework provides temporal and positive-scale spectral information concerning shifts, modulation, differential behavior, and multi-scale dynamics [1,4,6,8,9]. These methods can consequently be incorporated into digital-twin architectures [36], AI-based sustainable logistics [17], blockchain-enabled traceability systems [18], and supply-chain resilience frameworks [28]. The mathematical development is connected with contemporary application s in intelligent logistics [33], Industry 5.0 [14], digital twins [36], sustainable logistics [17], blockchain-enabled supply chains [18], quantum logistics [22,23], and smart transportation [24]. The resulting framework positions the KL expansion as a practical uncertainty-quantification and dimensionality-reduction technique that can complement emerging FKF spectral methods and support resilient, intelligent, sustainable, and computationally efficient supply-chain systems [3,5,26,28].

III. STATEMENT OF THE THEOREM

Let X_t be a zero-mean, square-integrable stochastic process defined on a probability space $(\Omega, \mathcal{F}, P)$ and indexed by a closed bounded interval $[a,b]$. Assume the covariance function

$$K_X(s,t) = E[X_s X_t]$$

is continuous. Then the covariance operator

$$(T_K f)(t) = \int_a^b K_X(t,s)\, f(s)\, ds$$

admits a countable set of non-negative eigenvalues λ_k (ordered non-increasingly) with corresponding orthonormal eigenfunctions $e_k(t)$ that form a complete basis of $L^2[a,b]$ [1,2,10].

3.1. The Karhunen–Loève Expansion

The process admits the mean-square convergent expansion

$$X_t = \sum_{k=1}^{\infty} Z_k\, e_k(t)$$

where the random coefficients are the projections

$$Z_k = \int_a^b X_t\, e_k(t)\, dt$$

3.2. The Governing Integral Equation

The eigenfunctions and eigenvalues satisfy the homogeneous Fredholm integral equation of the second kind:

$$\int_a^b K_X(t,s)\, e_k(s)\, ds = \lambda_k\, e_k(t)$$

together with the orthonormality condition

$$\int_a^b e_i(t)\, e_j(t)\, dt = \delta_{ij}$$

IV. PROPERTIES OF THE EXPANSION COEFFICIENTS

The random variables Z_k satisfy the following fundamental properties [1,2,10]:

(i) Zero mean: $E[Z_k] = 0$ for all k.

(ii) Uncorrelatedness: $E[Z_i\, Z_j] = 0$ whenever i ≠ j.

(iii) Variance equal to the eigenvalue: $Var(Z_k) = E[{Z_k}^2] = \lambda_k$.

When the underlying process is Gaussian, the coefficients Z_k are mutually independent and normally distributed. This property greatly simplifies Monte-Carlo sampling and analytic risk calculations.

4.1. Finite Truncation and Optimality

In computational practice only the first N eigenpairs corresponding to the largest eigenvalues are retained:

$$X_t^{(N)} = \sum_{k=1}^{N} Z_k\, e_k(t)$$

The mean-square truncation error equals the sum of the neglected eigenvalues:

$$E[\int_a^b |X_t - X_t^{(N)}|^2\, dt] = \sum_{k=N+1}^{\infty} \lambda_k$$

Among all N-term expansions that employ orthonormal deterministic functions, the KL expansion is optimal: it attains the smallest possible mean-square residual [10]. This optimality underpins its widespread use for dimensionality reduction in high-dimensional stochastic models of logistics networks [31,32].

4.2. Normalised Representation

An equivalent and frequently preferred representation employs unit-variance coefficients:

$$X_t = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\, \xi_k\, e_k(t)$$

where the ξ_k are uncorrelated random variables of zero mean and unit variance (standard normal when the process is Gaussian). This form is particularly convenient for sampling-based UQ pipelines and for interfacing with machine-learning models that expect standardized inputs [31].

V. APPLICATIONS IN CONTEMPORARY SUPPLY-CHAIN RESEARCH

We now map the KL apparatus onto four interrelated research themes that appear prominently in recent literature on quantum logistics, digital twins, AI–blockchain integration and sustainable green supply-chain management [3–8,33–36].

5.1 Quantum Revolution and Uncertainty Quantification in Logistics

Quantum-inspired algorithms and quantum annealing approaches for combinatorial logistics problems (vehicle routing, facility location, inventory allocation) typically operate on a classical description of uncertainty [3,4,33]. The KL expansion supplies a compact, orthogonal parametrization of demand, travel-time or cost random fields that can be fed directly into quantum-inspired solvers. Because only a modest number of dominant eigenmodes need be retained, the resulting stochastic programmes remain of manageable dimension, facilitating hybrid quantum classical optimization workflows [4]. Moreover, the explicit spectral representation permits analytic propagation of uncertainty through quantum inspired heuristics, yielding confidence intervals on solution quality that are essential for practical adoption in supply-chain decision support.

5.2 Digital Twins for Resilient and Adaptive Global Supply Chains

A digital twin of a global supply network must continuously assimilate streaming data while maintaining a statistically consistent representation of residual uncertainty [5,6,34]. The KL expansion realizes this requirement by decomposing spatially and temporally correlated residual fields (inventory levels, transit delays, capacity fluctuations) into a small set of independent random coefficients. Real-time Bayesian updating of these coefficients, combined with the deterministic eigenfunctions, yields a low-dimensional state that can be propagated inside the twin. Resilience metrics such as the probability of stock-out under disruption scenarios then become computable by inexpensive Monte-Carlo sampling of the retained KL modes, enabling adaptive reconfiguration of sourcing and routing policies [9], [34].

5.3 AI and Blockchain Integration for Transparency, Traceability and Resilience

Artificial-intelligence models that forecast demand or detect anomalies, and blockchain ledgers that record provenance events, both benefit from a rigorous treatment of stochastic inputs [7], [35]. KL-expanded demand or quality random fields can serve as generative models that produce realistic synthetic scenarios for training robust AI predictors. On the blockchain side, the same expansion can quantify the residual uncertainty attached to sensor readings or IoT measurements before they are committed to an immutable ledger, thereby supporting probabilistic smart contracts and risk-aware consensus mechanisms. The resulting fusion of AI predictive power, blockchain trust and KL-based UQ strengthens end-to-end transparency and operational resilience.

5.4 Artificial Intelligence Applications in Sustainable Logistics and Green Supply Chain Management

Sustainable logistics must account for the stochastic nature of renewable-energy availability, emission factors, weather-dependent transport modes and circular-economy reverse flows [8], [36]. The KL expansion provides an efficient representation of these environmental random fields, allowing multi-objective AI optimizers (cost versus carbon footprint versus social impact) to operate on a reduced stochastic basis. Because the expansion is optimal, the number of scenarios needed for robust green-route planning or green-inventory policies is minimized, rendering large-scale sustainable optimization computationally tractable. Bibliometric evidence of the rapid growth of AI-for-green-logistics research further underscores the value of a mathematically sound UQ foundation such as the KL framework [8].

5.5 Cross-Cutting Benefits

Across the four domains the KL expansion consistently delivers:

Optimal dimensionality reduction of high-dimensional uncertainty [10], [31], [32].

Uncorrelated (and, under Gaussianity, independent) random coefficients that simplify sampling and analytic calculations.

Explicit, controllable truncation-error bounds that support risk-aware decision making.

A modular interface that can be embedded inside digital twins, AI pipelines, blockchain oracles and quantum-inspired solvers alike [3–7,31].

research findings effectively involve a strategic V. CONCLUSION

This paper has presented a complete mathematical formulation of the Karhunen–Loève expansion, expressed throughout in native OMML so that every equation remains fully editable inside Microsoft Word's equation editor. The theoretical development encompasses the statement of the theorem, the series expansion, the governing Fredholm eigenvalue problem, the statistical properties of the coefficients, the finite-truncation error formula and the normalized unit-variance representation.

Building on this foundation, we have demonstrated the relevance of the KL expansion to four contemporary research frontiers that appear in the recent literature: the quantum revolution in logistics, digital-twin technology for resilient global supply chains, AI–blockchain integration for transparency and traceability, and AI-driven sustainable green logistics. In each setting the expansion supplies an optimal, low-dimensional parametrization of uncertainty that enhances computational tractability, supports rigorous risk quantification and facilitates hybrid classical quantum or AI–blockchain workflows.

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