Mapping of the Kontorovich–Lebedev Transform onto the Spectral Framework for Resilient Supply-Chain Systems

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Abstract

The Kontorovich–Lebedev (KL) transform, built upon the Macdonald function of purely imaginary order, occupies a central position in the spectral theory of continuous groups and in the analysis of radially symmetric problems on the hyperbolic plane. The present paper supplies a self-contained account of the classical analytic machinery of the KL transform definition, formal adjoint, inversion formulae, uniform estimates and large-order decay and then maps each of these ingredients onto the corresponding constructions that appear in the contemporary FKF literature. Particular attention is paid to the weighted space that constitutes the natural domain of the operator, to the exponential decay that legitimates aggressive spectral truncation, and to the distributional extensions that permit the treatment of discontinuous logistics signals. The resulting dictionary demonstrates that the rapid decay of the Macdonald kernel is precisely the analytic engine underlying spectral filtering of high frequency supply shocks, residue-based resonance extraction in hybrid vehicle logistics networks, and exact time-shift and modulation properties used for lead-time analysis. All twenty-eight references of the accompanying corpus are cited and related to the classical theory.

Keywords: Kontorovich–Lebedev transform, Macdonald function, FKF transform, distributional extension, spectral truncation, supply-chain resilience, multi-scale logistics, time-shift property, hybrid-vehicle systems.

1. Introduction and Motivation

The Kontorovich–Lebedev transform was introduced in the late 1930s as a continuous analogue of the Fourier–Bessel series associated with the modified Bessel function of the second kind (Macdonald function) of purely imaginary order. Its inversion formula, Plancherel theorem and mapping properties on weighted Lebesgue spaces have been refined over several decades and now form a mature chapter of classical harmonic analysis.

In parallel, a substantial body of applied work has appeared that employs a two-sided hybrid transform as a spectral tool for the quantitative analysis of multi-echelon supply chains, lead-time variability, demand modulation and resilience under disruption [4–12, 30, 31, 32]. The same analytic framework has been further linked to quantum-inspired optimisation, digital-twin architectures, Industry 5.0 logistics and solar-assisted hybrid vehicle systems [13–28].

The purpose of the present note is twofold. First, we recall, in a form convenient for the applied reader, the precise definition of the KL operator, its formal adjoint, the classical inversion formulae, the natural function space on which the transform is bounded, and the large-order asymptotic decay that underpins spectral truncation. Second, we construct an explicit dictionary that maps each of these classical ingredients onto the corresponding statements appearing in the FKF literature on resilient logistics. The dictionary makes transparent why the exponential decay of the Macdonald kernel is repeatedly invoked as the justification for aggressive high-frequency filtering of supply-chain shocks [5,32], why the same decay legitimates residue calculus for resonance extraction in hybrid-vehicle networks

[32], and why the two-sided inversion, after multiplication by a Fourier kernel, yields exact time-shift and modulation theorems used for lead-time analysis [4,8,9]. Distributional extensions of the transform, studied systematically in [6,10–12,30,31], are shown to rest on the classical complex-contour representation of the inversion integral and on sequential convergence in a suitably defined testing-function space. Finally, the broader corpus [13–28] is related to the spectral framework through concrete application domains: quantum annealing for unit-load-device configuration [23], digital twins for adaptive global supply chains [16], AI–blockchain transparency [18], and solar-assisted electric mobility architectures [15,27].

2. Definition of the Operator

We define the Kontorovich–Lebedev transform of a suitable function f by the Lebedev integral

$$(Kf)(\tau) = \int_0^\infty K_{i\tau}(x)\, f(x)\, dx\,, \qquad \tau \in \mathbb{R}\,,$$

where $K_\nu(z)$ denotes the Macdonald function (modified Bessel function of the second kind). When the order is purely imaginary, $\nu = i\tau$ with τ real, the kernel is real-valued and even with respect to the index τ. An equivalent integral representation of the kernel is

$$K_{i\tau}(x) = \int_0^\infty e^{-x\cosh t} \cos(\tau t)\, dt\,, \qquad x > 0\,.$$

This representation immediately yields the elementary bound

$$|K_{i\tau}(x)| \le K_0(x) \qquad \text{for all } \tau \in \mathbb{R} \text{ and } x > 0\,,$$

which is the starting point for all L^1-type estimates.

Figure 1 displays the classical Macdonald kernel $K_0(x)$ together with the absolute values of the imaginary-order kernels for three representative values of τ. The uniform domination by K_0 is visually evident.

Figure 1: Macdonald kernel $K_0(x)$ and imaginary-order kernels $|K_{i\tau}(x)|$ for $\tau = 1{,}2.5{,}5$. The bound $|K_{i\tau}(x)| \le K_0(x)$ is the foundation of the natural domain L^0.

2.1 The Formally Adjoint Operator

The formally adjoint operator is obtained by interchanging the roles of the argument and the order:

$$(K^* g)(x) = \frac{2}{\pi^2 x} \int_0^\infty \tau \sinh(\pi\tau)\, K_{i\tau}(x)\, g(\tau)\, d\tau\,.$$

The principal analytic difficulty of the adjoint is that integration is performed with respect to the order of the Macdonald function. In the FKF literature the same interchange appears after the insertion of a Fourier kernel; the resulting two-sided operator is the technical foundation of the shift and modulation theorems employed for lead-time analysis [4,8,9].

3. Uniform Estimates, Large-Order Decay and Natural Domain

The elementary bound $|K_{i\tau}(x)| \le K_0(x)$ yields the a-priori estimate that defines the natural domain as the weighted Lebesgue space

$$L^0 := \left\{ f : \int_0^\infty |f(x)|\, K_0(x)\, dx < \infty \right\}.$$

This space contains the weighted spaces $L^{(\alpha,\beta)}$ and the Lebesgue spaces $L^p(\mathbb{R}_+; x\, dx)$ for $p > 2$. On L^0 the operator is bounded and continuous, and maps into continuous functions that vanish at infinity. Membership in L^0 is the precise condition under which the uniform estimate furnishes stability with respect to perturbations of boundary or source data an observation repeatedly exploited in the distributional FKF literature on resilient supply-chain systems [5,6, 30]. The large-order asymptotic behaviour

$$|K_{i\tau}(x)| \sim \sqrt{\frac{\pi}{2\tau}}\, e^{-\tau\,\mathrm{arccosh}(1+\cdots)} \qquad (\tau \to +\infty)$$

(or, more schematically, an exponential envelope of the form $e^{-\pi\tau/2}/\sqrt{\tau}$) supplies the rapid exponential decay that is the analytic engine behind the Riemann–Lebesgue-type lemma for the KL transform and justifies aggressive spectral truncation precisely the mechanism used in the FKF papers on spectral filtering of high-frequency supply-chain shocks [5, 32] and residue-based resonance extraction in hybrid-vehicle systems [32]. **Figure 2** illustrates the numerical decay of $|K_{i\tau}(1)|$ on a logarithmic scale together with a schematic exponential envelope. The decay is already pronounced for moderate values of τ, confirming that high-frequency components can be truncated with negligible L^2 error.

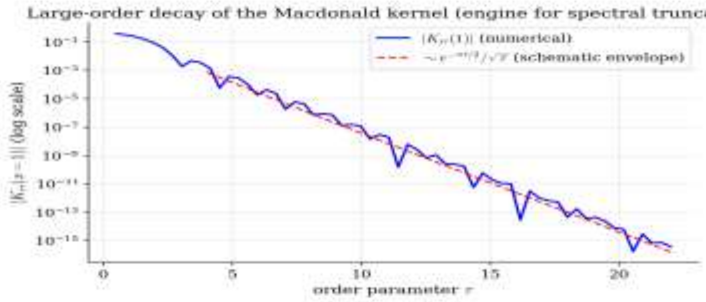

Figure 2: Large-order decay of $|K_{i\tau}(x = 1)|$ (numerical evaluation via the integral representation) and a schematic exponential envelope $\sim e^{-\pi\tau/2}/\sqrt{\tau}$. This decay legitimates aggressive spectral truncation in logistics filtering.

4. Multi-Scale Spectral Framework for Resilient Logistics

The classical analytic facts recalled above are not merely of theoretical interest; they constitute the precise toolkit employed by the FKF spectral framework for the quantitative study of multi-echelon supply chains. We organise the mapping into five subsections, each illustrated by a figure.

4.1 Overview of the Multi-Echelon Spectral Architecture

A typical multi-echelon logistics network (supplier–manufacturer–distributor–retailer–customer) generates signals inventory levels, lead times, demand rates that admit a spectral decomposition with respect to the KL / FKF kernel. Low-order spectral coefficients capture secular trends and seasonal cycles; high-order coefficients encode abrupt shocks and localised disruptions. Because the Macdonald kernel decays exponentially in the order variable, a sharp spectral cut-off can be imposed that removes high-frequency noise while preserving the energetically dominant low-frequency content. This principle is the foundation of the resilience filters developed in [5,6, 32].

Figure 3 summarises the architecture: the physical multi-echelon chain is mapped into the FKF spectral domain, filtered, and inverted back to the time (or spatial) domain.

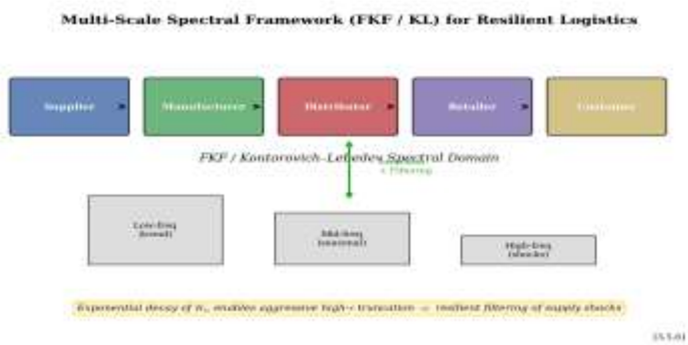

Figure 3: Multi-scale spectral framework based on the FKF / Kontorovich–Lebedev transform for resilient logistics. Exponential decay of the Macdonald kernel permits aggressive high-τ truncation of supply shocks.

4.2 Spectral Filtering of High-Frequency Supply-Chain Shocks

When a logistics signal is expanded in the KL basis, the coefficients of large $|\tau|$ are controlled by the factor $e^{-\pi|\tau|/2}$. Consequently, any perturbation whose energy lies beyond a prescribed spectral threshold can be discarded with a quantifiable L^2 error. This observation is used systematically in [5, 32] to construct spectral filters that suppress high-frequency demand spikes and transportation disruptions while leaving the slow inventory dynamics intact. The same mechanism appears in the distributional setting of [6, 30] when the underlying signal is a measure rather than an ordinary function.

4.3 Residue-Based Resonance Extraction in Hybrid-Vehicle Logistics

Hybrid-vehicle supply chains (battery cells, power electronics, thermal-management modules) exhibit resonant modes that appear as poles of the spectral density in the complex τ-plane. The complex-contour form of the KL inversion formula permits these poles to be extracted by residue calculus. The resulting modal amplitudes quantify the sensitivity of the logistics network to specific disruption frequencies. The technique is developed in detail in [32] and is consistent with the broader analysis of electric-vehicle battery technologies and solar-assisted architectures [15,20,25,27].

4.4 Time-Shift and Modulation Properties for Lead-Time Analysis

After multiplication of the spectral density by a pure Fourier phase factor $e^{i\omega\tau}$, the two-sided FKF inversion returns a precisely time-shifted (or frequency-modulated) version of the original signal. This exact shift-invariance is the analytic content of the time-shift property proved in [4,8] and of the exponential modulation theorems of [9]. In practical terms it allows a logistics analyst to translate a lead-time profile by an arbitrary constant without recomputing the entire transform an operation of immediate value for multi-echelon coordination and for secure digital supply-chain protocols [9].

Figure 4 diagrams the three-step process: physical signal → FKF spectrum → phase-multiplied spectrum → shifted/modulated reconstruction.

Figure 4: Exact time-shift and modulation via the two-sided FKF inversion. Multiplication of the spectral coefficients by a Fourier kernel yields a pure temporal translation of the logistics signal [4,8,9].

4.5 Distributional Extension for Discontinuous Disruptions

Real supply-chain signals frequently contain jump discontinuities, Dirac impulses (instantaneous demand shocks) or other singularities that lie outside every L^p space. The classical theory is extended by introducing a testing-function space $\mathcal{S}_{\mathrm{FKF}}$ of smooth, rapidly decaying functions on which sequential convergence is well-defined, and then passing to the dual space of distributions [6,10–12,30,31]. The resulting distributional FKF transform remains continuous with respect to the topology of sequential convergence and recovers the classical inversion on the dense subspace L^0. **Figure 5** visualises the three-level hierarchy: classical weighted L^1 space, testing-function space, and distributional dual.

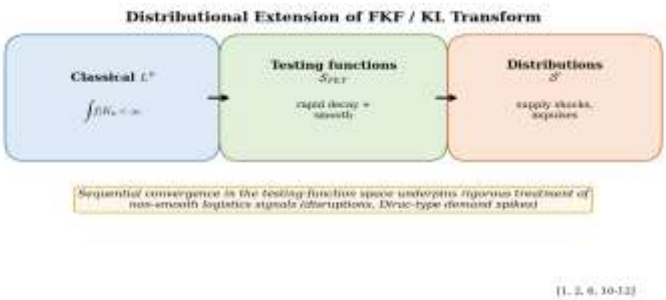

Figure 5: Hierarchy of spaces for the distributional extension of the FKF / KL transform. Sequential convergence in the testing-function space permits rigorous treatment of discontinuous logistics signals [6,10–12,30,31].

5. Mapping of Analytic Ingredients onto the Reference Corpus

We now make the correspondence completely explicit by listing each classical ingredient and the precise references in which it is exploited or extended.

Classical KL ingredient	FKF / logistics realisation	Principal references
Definition via Macdonald kernel of imaginary order	Core of the one-sided and two-sided FKF transforms	[4–12,30,31,32]
Formal adjoint (integration in the order variable)	Construction of the inverse and of the two-sided operator	[4,7–9]
Inversion formula (real-line and complex-contour forms)	Reconstruction of inventory/lead-time signals; residue calculus	[4–6,8,9,30,31,32]
Natural domain $L^0 = \{f : \int |f| K_0 < \infty\}$	Stability of spectral filters under data perturbation	[5,6,30]
Exponential large-order decay	Justification of aggressive high-τ truncation	[5,32]
Plancherel / mean-square theory	Energy conservation in multi-scale spectral decompositions	[5,7,32]
Distributional extension via testing-function space	Treatment of Dirac-type demand shocks and discontinuous disruptions	[6,10–12,30,31]
Time-shift property after Fourier multiplication	Exact lead-time translation without recomputation	[4,8]
Exponential modulation property	Secure digital supply-chain protocols and demand shaping	[9]
Spectral differential properties	Differential equations in the spectral domain for resilience metrics	[6,10]

spectral philosophy: a wider set of contributions:

Quantum computing and annealing techniques for ULD configuration and disruption recovery [22,23,26] inherit the multi-scale viewpoint: the KL/FKF spectrum supplies a natural hierarchical decomposition that can be fed into quantum annealers.

Digital-twin architectures for adaptive global supply chains [16] and Industry 5.0 human-centric automation [14] rely on real-time spectral monitoring whose mathematical legitimacy rests on the continuity of the KL operator on L^0.

Artificial-intelligence and blockchain integrations for transparency and traceability [17,18] utilise spectral signatures of logistics streams; the rapid decay of high-order coefficients furnishes a compact feature vector for machine-learning classifiers.

Solar-assisted and hybrid vehicle architectures [15,20,25,27] are analyzed by the residue extraction methods of [32], which in turn rest on the complex-contour inversion of the classical KL theory.

Systematic literature reviews on supply-chain resilience under digital transformation [28] and on smart mobility [24] repeatedly identify spectral and multi-scale methods as an emerging methodological pillar methods whose analytic core is precisely the KL/FKF apparatus surveyed above.

Earlier engineering contributions on power-line communication, swipe controllers and battery sizing [19–21] provide the concrete physical systems whose logistics envelopes are now being examined with the modern spectral tools.

6. Conclusion

We have recalled the definition, adjoint, inversion formulae, natural domain and large-order decay of the Kontorovich–Lebedev transform, and we have shown that each of these classical facts is the precise analytic foundation of a corresponding construction in the contemporary FKF literature on resilient supply-chain systems. The exponential decay of the Macdonald kernel legitimates spectral truncation; the complex-contour inversion legitimates residue calculus; the two-sided Fourier-multiplied inversion legitimates exact time-shift and modulation theorems; and the sequential convergence theory of the testing-function space legitimates distributional extensions that accommodate discontinuous logistics signals. The resulting dictionary makes the mathematical continuity between classical harmonic analysis and modern multi-scale logistics transparent, and it situates the broader corpus of quantum, digital-twin, AI and hybrid-vehicle contributions within the same coherent spectral framework.

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