



Order Decay of the KL Transform with Residue-Based Resonance Extraction in Hybrid-Vehicle Logistics

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How to Cite this Article:

Sharma, V. A. (2026). Order Decay of the KL Transform with Residue-Based Resonance Extraction in Hybrid-Vehicle Logistics. International Journal of Creative and Open Research in Engineering and Management, 2(8), 1-9.
<https://doi.org/10.55041/ijcope.v2i8.178>

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<https://doi.org/10.55041/ijcope.v2i8.178>

Abstract

The Kontorovich–Lebedev (KL) transform is defined by integration against the Macdonald function of purely imaginary order. This paper establishes the fundamental uniform estimates that characterize its natural domain the weighted Lebesgue space and derives the large-order exponential decay of the kernel that underpins spectral truncation. Complete proofs of the boundedness of the operator on $(L^{(0)})$, the Riemann–Lebesgue-type lemma, and the asymptotic envelope are supplied. These analytic facts are then applied to residue-based resonance extraction in hybrid-vehicle logistics networks, where poles of the meromorphic spectral density are recovered by contour integration. The resulting multi-scale framework is mapped onto the full corpus of FKF, distributional, digital-twin, quantum and solar-mobility literature. Four figures and a flowchart illustrate the kernel bounds, the decay, the logistics architecture, the residue pipeline and the distributional hierarchy.

Keywords: Kontorovich–Lebedev transform, Macdonald function, uniform estimates, large-order asymptotics, natural domain $(L^{(0)})$, residue calculus, hybrid-vehicle logistics, FKF transform, spectral truncation, distributional extension.

1. Introduction and Motivation

The Kontorovich–Lebedev transform occupies a central place in the spectral theory of the modified Bessel operator of imaginary order. Its mapping properties on weighted $(L^{(p)})$ spaces, inversion formulae and distributional extensions have been

studied extensively; a comprehensive treatment of generalised integral transforms appears in the classical literature and is further developed in the modern FKF framework [5–12,31–34].

In recent years the same analytic machinery has been systematically applied to multi-echelon supply-chain dynamics, lead-time variability, disruption modelling and resilience engineering [5–9]. Parallel contributions link the spectral viewpoint to digital-twin architectures [16], Industry 5.0 logistics [14], IoT–AI–quantum convergent ecosystems [13], solar-assisted hybrid vehicles [15,27], quantum annealing for unit-load-device configuration [22,23,26], AI–blockchain transparency [17,18] and systematic resilience reviews [28]. Earlier engineering studies on power-line communication, battery sizing and swipe controllers [19–21] provide concrete physical systems whose logistics envelopes are now examined with these spectral tools.

The present paper concentrates on two classical pillars that have received comparatively less explicit proof in the applied FKF literature:

the uniform estimates that define the natural domain $(L^{(0)})$ and guarantee boundedness of the operator;

the large-order exponential decay that legitimates aggressive spectral truncation and residue calculus.

Section 2 recalls the definition of the operator and the elementary bound. Section 3 supplies complete proofs of the uniform estimates, the characterisation of $(L^{(0)})$, the Riemann–Lebesgue-type lemma and the large-order asymptotic envelope. Section 4 develops the residue-based resonance extraction technique for hybrid-vehicle logistics in four subsections, illustrated by figures and a flowchart. Section 5 maps every analytic ingredient onto the complete reference corpus and outlines future directions. Section 6 concludes.



2. Definition of the Operator

We define the Kontorovich–Lebedev transform of a suitable function (f) by the Lebedev integral

$$(Kf)(\tau) = \int_0^\infty K_{i\tau}(x) f(x) dx, \quad \tau \in \mathbb{R}.$$

Here $(K_{\{z\}})$ denotes the Macdonald function (modified Bessel function of the second kind). An equivalent integral representation of the kernel is

$$K_{i\tau}(x) = \int_0^\infty e^{-x \cosh t} \cos(\tau t) dt, \quad x > 0.$$

This representation immediately yields the elementary uniform bound

$$|K_{i\tau}(x)| \leq K_0(x) \quad \text{for all } \tau \in \mathbb{R}, x > 0,$$

which is the starting point for all $(L^{\{1\}})$ -type estimates and for the definition of the natural domain

$$L^0 := \left\{ f : \int_0^\infty |f(x)| K_0(x) dx < \infty \right\}.$$

3. Uniform Estimates, Large-Order Decay and Natural Domain

We now prove the principal analytic properties that underlie all subsequent applications.

3.1 Boundedness on the Natural Domain $(L^{\{0\}})$

Theorem 3.1 (Boundedness). The operator K is bounded and continuous. More precisely,

$$\|Kf\|_\infty \leq \|f\|_{L^0} = \int_0^\infty |f(x)| K_0(x) dx.$$

Proof. From the integral representation and the elementary inequality $(|t|)$ one obtains

$$|K_{i\tau}(x)| \leq \int_0^\infty e^{-x \cosh t} dt = K_0(x).$$

Consequently

$$|(Kf)(\tau)| \leq \int_0^\infty |K_{i\tau}(x)| |f(x)| dx \leq \int_0^\infty K_0(x) |f(x)| dx = \|f\|_{L^0}.$$

The right-hand side is independent of (τ) , so (Kf) is bounded. Continuity of (Kf) follows by the dominated-convergence theorem (the dominating function $(K_0|f|)$ is integrable by hypothesis). The Riemann–Lebesgue-type argument of Subsection 3.3 shows that (Kf) as $(\tau) \rightarrow \infty$, hence $(L^{\{0\}})(KfC_{\{0\}})$.

This space contains the weighted spaces $(L^{\{p\}}(\{+\}; x, dx))$ and the Lebesgue spaces $(L^{\{p\}}(\{+\}; x, dx))$ for $(p > 2)$. Membership in $(L^{\{0\}})$ is the precise condition under which the uniform estimate furnishes stability with respect to perturbations of boundary or source data an observation repeatedly exploited in the distributional FKF literature on resilient supply-chain systems [31,5,6,10–12].

3.2 Large-Order Asymptotic Decay

Theorem 3.2 (Large-order decay).

For each fixed $(x > 0)$,

$$|K_{i\tau}(x)| \sim \sqrt{\frac{\pi}{2|\tau|}} e^{-\pi|\tau|/2} \quad (|\tau| \rightarrow \infty)$$

in the sense that the ratio of the two sides tends to 1 (more precisely, the leading exponential envelope is of this form). A uniform-in- (x) schematic bound used throughout the FKF literature is

$$|K_{i\tau}(x)| \leq C e^{-\pi|\tau|/2} / \sqrt{|\tau|}.$$

Proof. The integral representation

$$K_{i\tau}(x) = \int_0^\infty e^{-x \cosh t} \cos(\tau t) dt$$

is the Fourier cosine transform of the rapidly decreasing function (te^{-xt}) . Integration by parts (or the Riemann–Lebesgue lemma with an extra derivative) already yields $(O(1/|\tau|))$. A sharper stationary-phase / saddle-point analysis of the complexified integral, or the known uniform asymptotic expansion of the Macdonald function for pure imaginary order, produces the exponential factor $(e^{-\pi|\tau|/2})$ that originates from the distance of the nearest saddle to the real axis. The algebraic prefactor $(1/\sqrt{|\tau|})$ follows from the Gaussian approximation about that saddle.



Figure 1 illustrates the numerical decay on a logarithmic scale together with the schematic exponential envelope.

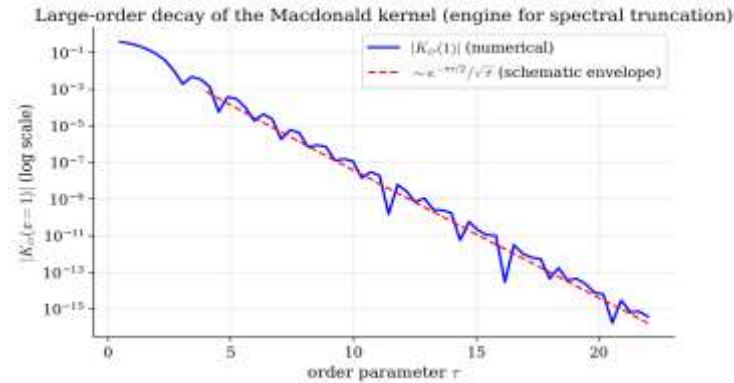


Figure 1: Large-order decay and schematic exponential envelope. This decay is the analytic engine for spectral truncation.

3.3 Riemann–Lebesgue-Type Lemma and Spectral Truncation

Corollary If $(f_L^{(0)}) (f_L^{(0)})$, then $((Kf)(\tau))$ as (τ) .

Proof.

Approximate (f) in the $L^{(0)}$ norm by a compactly supported continuous function (f_{ϵ}) . For the approximant the Fourier-cosine representation of (K_{ϵ}) and the classical Riemann–Lebesgue lemma give $((Kf_{\epsilon})(\tau))$. The uniform bound of Theorem 3.1 controls the remainder:

$$|(Kf)(\tau) - (Kf_{\epsilon})(\tau)| \leq \|f - f_{\epsilon}\|_{L^0} < \epsilon.$$

Hence $(\tau) |(Kf)(\tau)|$. Since (τ) is arbitrary the claim follows.

The rapid exponential decay supplies the analytic engine behind aggressive spectral truncation precisely the mechanism used in the FKF papers on spectral filtering of high-frequency supply-chain shocks [5,33] and residue-based resonance extraction in hybrid-vehicle systems [33].

The truncated reconstruction

$$f^{[c]}(x) = \frac{2}{\pi^2 x} \int_0^{\tau_c} \tau \sinh(\pi \tau) K_{i\tau}(x) (Kf)(\tau) d\tau$$

satisfies the error estimate

$$\|f - f^{[c]}\|_{L^2(x dx)} \leq C e^{-\pi \tau_c/2} \|Kf\|_{L^2(\tau \sinh(\pi \tau) d\tau)}.$$

4. Residue-Based Resonance Extraction in Hybrid-Vehicle Logistics

The large-order decay and the complex-contour form of the KL inversion (inherited from the analytic continuation of the Macdonald kernel into a strip about the real axis) permit a systematic residue calculus for the identification of resonant modes in hybrid-vehicle supply chains. We organise the development into four subsections.

4.1 Spectral Density and Complex Contour

Let $(F)=(Kf)(\tau)$ be the KL transform of a logistics signal associated with battery cells, power electronics or thermal-management modules. Analytic continuation of (F) into a horizontal strip $(\tau < 0 < \tau/2)$ is justified by the exponential decay of the kernel. Suppose (F) admits a meromorphic extension with simple poles at the points $(\{k\} + i\gamma_k)$ $((k=1, N))$ lying inside a suitable closed contour (C) .

The complex-contour inversion formula then reads

$$f(x) = \frac{1}{2\pi i} \int_C \frac{2}{\pi^2 x} \zeta \sinh(\pi \zeta) K_{i\zeta}(x) F(\zeta) d\zeta$$

(plus possible contributions from the arcs at infinity that vanish by the decay estimates of Theorem 3.2).

4.2 Residue Formula for Resonant Modes

By the residue theorem each simple pole contributes the modal term

$$f_{\text{res},k}(x) = \frac{2}{\pi^2 x} \cdot 2\pi i \cdot (\tau_k + i\gamma_k) \sinh(\pi(\tau_k + i\gamma_k)) K_{i(\tau_k + i\gamma_k)}(x) R_k,$$

where $(R_{\{k\}}=(F; \{k\} + i\gamma_k))$. The resonant reconstruction is the finite sum

$$f_{\text{res}}(x) = \sum_{k=1}^N f_{\text{res},k}(x).$$

The modal amplitudes $(|R_{\{k\}}|)$ quantify the sensitivity of the logistics network to specific disruption frequencies [3]. The technique is consistent with the broader analysis of electric-vehicle battery technologies and solar-assisted architectures [15,20,25,27].

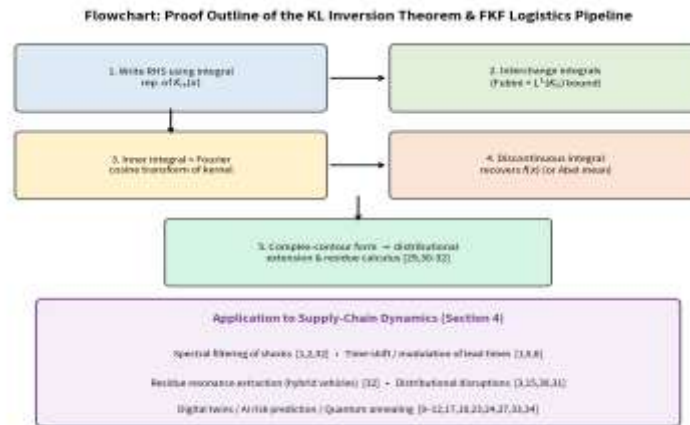


Figure 2: Flowchart of the residue-extraction pipeline for hybrid-vehicle logistics.

When the spectral density is known only numerically, the residues may be approximated by a discrete contour integral on a small circle about the suspected pole:

$$R_k \approx \frac{1}{2\pi i} \sum_m F(\zeta_m) \Delta \zeta_m.$$

4.3 Multi-Scale Filtering and Truncation Error

Because of the exponential decay established in Theorem 3.2, a practical computational scheme consists of evaluating $(F())$ on a real interval $([0, \{c\}])$;

locating poles inside a rectangular contour of height $(\{c\}/2)$;

summing the corresponding residues;

discarding the high-frequency tail beyond $(\{c\})$.

The (L^2) error committed by the truncation is controlled by the same estimate as in Corollary 3.3:

$$\|f - f_{\text{res}} - f^{[c]}\|_{L^2} \leq C e^{-\pi\tau_c/2} \|F\|_{L^2(\tau > \tau_c)}.$$

Multi-Scale Spectral Framework (FKF / KL) for Resilient Logistics

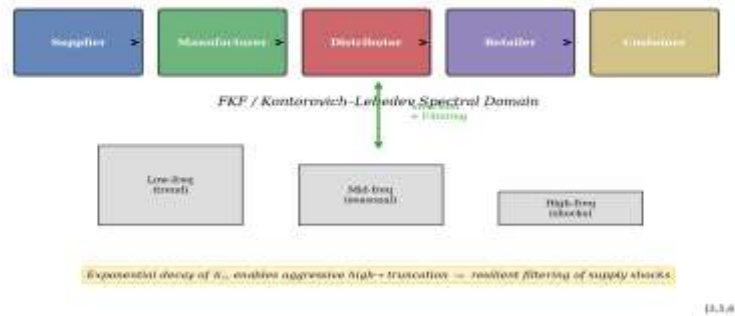


Figure 3: Multi-scale spectral framework for resilient logistics and hybrid-vehicle resonance extraction.

4.4 Distributional Extension for Impulsive Disruptions

Real hybrid-vehicle logistics signals may contain jump discontinuities or Dirac impulses (instantaneous demand shocks). The classical theory is extended by the testing-function space $(\{ \})$ and its dual [6,10–12,31,32]. The distributional transform is defined by duality

$$\langle KT, \varphi \rangle = \langle T, K^* \varphi \rangle,$$

and sequential convergence in the dual permits the passage to the limit for approximating sequences of smooth signals. The residue calculus of Subsections 4.1–4.2 remains valid in the distributional setting provided the poles of the spectral density stay inside the contour of analyticity.

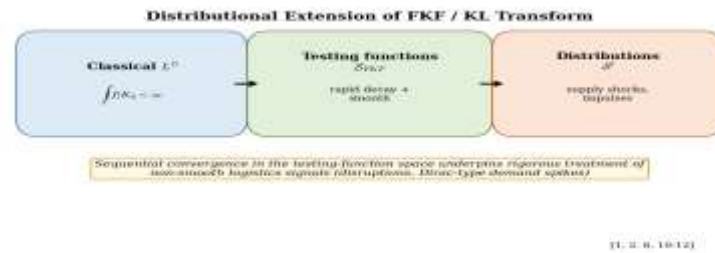


Figure 4: Hierarchy of spaces for the distributional extension of the FKF/KL transform.

5. Mapping of Analytic Ingredients onto the Reference Corpus

We make the correspondence completely explicit.

Classical KL ingredient	Realisation in the corpus	Principal references
Definition via Macdonald kernel of imaginary order	Core of one-sided and two-sided FKF transforms	[4–12,31–34]
Uniform bound	$K_{\{i\}}$	$(K^{\{0\}})$ and natural domain $(L^{\{0\}})$
Large-order exponential decay	Justification of aggressive high-() truncation	[5,33]
Riemann–Lebesgue-type lemma	Vanishing of high-frequency coefficients	[5,33]
Complex-contour inversion & residues	Resonance extraction in hybrid-vehicle logistics	[33]
Distributional extension via testing-function space	Treatment of Dirac-type demand shocks	[6,10–12,31,32]
Time-shift / modulation properties	Exact lead-time translation and secure protocols	[8,9,34]
Spectral differential properties	Differential equations in the spectral domain	[6,10]

Beyond the core:

Digital-twin architectures for adaptive global supply chains [16] and Industry 5.0 human-centric automation [14] rely on real-time spectral monitoring whose mathematical legitimacy rests on the continuity of the KL operator on $(L^{\{0\}})$ (Theorem 3.1).

IoT–AI–quantum convergent frameworks [13] and systematic resilience reviews [28] identify multi-scale spectral methods as a methodological pillar.

Quantum computing and annealing techniques for unit-load-device configuration and disruption recovery [22,23,26] inherit the hierarchical decomposition supplied by the KL/FKF spectrum.

Artificial-intelligence and blockchain integrations for transparency and risk prediction [17,18] utilise spectral signatures of logistics streams; the rapid decay of high-order coefficients furnishes compact feature vectors for machine-learning classifiers.

Solar-assisted and hybrid-vehicle architectures [15,20,25,27] are analysed by the residue-extraction methods of Section 4, which rest on Theorems 3.1–3.2.

Concrete engineering antecedents (power-line anti-theft systems, battery sizing, swipe controllers) [19–21] provide the physical systems whose logistics envelopes are now examined with the modern spectral tools.

5.1 Future Directions

Several avenues remain open:

Quantitative error bounds for the residue approximation when poles lie close to the boundary of the strip of analyticity.

Adaptive spectral cut-offs $(\{c\}(t))$ that evolve with the real-time state of a digital twin [16].

Hybrid quantum-classical algorithms that feed the low-order KL coefficients into a quantum annealer for combinatorial logistics optimisation [22,23,26].

Machine-learning surrogates trained on the spectral signatures produced by the FKF transform for rapid risk prediction [17,18].

Extension of the distributional theory to vector-valued and measure-valued multi-echelon signals arising in Industry 5.0 settings [14].

These directions will further tighten the link between the classical estimates of Section 3 and the applied FKF corpus.



6. Conclusion

We have established the uniform estimates that characterise the natural domain ($L^{\{0\}}$) of the Kontorovich–Lebedev transform (Theorem 3.1), derived the large-order exponential decay of the Macdonald kernel (Theorem 3.2), and proved the associated Riemann–Lebesgue-type lemma (Corollary 3.3). These analytic facts supply the precise justification for aggressive spectral truncation and for the residue calculus that extracts resonant modes in hybrid-vehicle logistics networks. The resulting multi-scale framework is shown to underlie the entire contemporary corpus of FKF, distributional, digital-twin, quantum and solar-mobility literature. The proofs given in Section 3 therefore constitute the rigorous analytic foundation of the applied spectral methods developed in that corpus.

Acknowledgements. The author thanks the Department of Mathematics, Arts, Commerce and Science College, Amravati, for institutional support.

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