International Journal of Creative and Open Research in Engineering and Management
ISSN: **3108-1754 (Online)**
Volume 02 Issue 08 August-2026 | Impact Factor: 3.5

Scaling Analysis of LLM Hardware Infrastructure through Uniform Estimates

VA Sharma[1]
Dept. of mathematics ,Arts, commerce and science college, Amravati

How to Cite this Article:

Sharma, V. (2026). Scaling Analysis of LLM Hardware Infrastructure through Uniform Estimates. International Journal of Creative and Open Research in Engineering and Management, <i>02</i>(8), 1-9. https://doi.org/10.55041/ijcope.v2i8.175

https://doi.org/10.55041/ijcope.v2i8.175

Abstract

As a methodological illustration we indicate how the same asymptotic and summation techniques supply analytic tools for the derivation of rigorous scaling laws that appear in performance models of GPU clusters, data-centre resource allocation, and the hardware infrastructure supply chains that underpin the training of large language models (LLMs). In this expanded version we develop these links further: the uniform bounds on the Macdonald kernel furnish a priori capacity and latency estimates for large GPU fleets; the Watson-type asymptotic describe the continuous-to-discrete transition that occurs when the number of accelerators tends to infinity; and the Poisson-type identities convert continuous resource integrals into discrete sums over GPU ranks, data-centre racks, or nodes of a global semiconductor supply chain. These analytic devices are shown to be formally analogous to the resilience and digital-twin frameworks that appear in the contemporary literature on infrastructure supply chains.

Keywords

Kontorovich–Lebedev transform; Watson lemma; asymptotic expansions; index transforms; scaling analysis; GPU clusters; data-centre infrastructure; LLM training systems: infrastructure supply chain; digital twins; supply-chain resilience.

1. Introduction

Index transforms are especially well suited to boundary-value problems in cylindrical, conical or wedge-shaped domains, and they appear naturally whenever a continuous spectrum of orders is required by the geometry or by radiation conditions at infinity. In the present work we adopt the Lebedev form of the operator $K_{i\tau}[f] = \int_0^\infty K_{i\tau}(x) f(x) dx$, where K_νν(z) denotes the Macdonald function [12, 13, 14, 17, 18]. The associated adjoint operator (integration with respect to the index) is $(KL\ f)(x) \ = \int_0^\infty K_{i\tau}(x)\ f(\tau)\ d\tau,\ \ x\ >\ 0$ [7, 10, 11]. A fundamental integral representation of the kernel is the Fourier cosine transform

$$K_{i\tau}(x)\ =\ \int_0^\infty e^{-x\cosh u} \cos(\tau\, u)\, du,\ \ x\ >\ 0$$

Consequently, the kernel is real-valued and even in the index τ. The representation is the starting point for almost all subsequent analysis: it permits analytic continuation of the order into a complex strip, it yields uniform estimates by elementary contour shifting, and it furnishes the bridge to the classical Poisson summation formula that will produce the discrete identities.

The analytic theory of the transform [3, 4, 5, 6] rests on three pillars that form the core of the present paper. First, the uniform estimates obtained by shifting the contour by an imaginary amount iδ $(0\ \le\ \delta\ <\ \pi/2)$], guarantee that the kernel remains dominated by the elementary majorant $K_0(x\cos\delta)$. These bounds determine the natural weighted Lebesgue spaces on which the transform is well defined and continuous. Second, the Watson-type asymptotic lemma describes the behaviors of the transform of an entire function of exponential type when the complex order tends to infinity inside a vertical strip; the leading term involves the explicit trigonometric perfector $\pi/(2\cos(\pi z/2))$ multiplied by the value of the density at an imaginary argument. Third, the Poisson-type summation formula that follows from the Fourier representation converts a continuous KL integral into an explicit discrete series whose terms are Laplace transforms evaluated at hyperbolic points. Together these three results supply a complete set of analytic tools uniform bounds, asymptotic expansions and exact summation identities for both pure and applied investigations.

Beyond the classical theory, the same analytic structures reappear, mutatis mutandis, in the performance modelling of large scale computing infrastructures. Modern training of LLMs is performed on clusters containing thousands of GPUs whose collective power draw, thermal load and interconnect traffic must be predicted and controlled. In such models one frequently encounters continuous integrals that represent smoothed resource densities together with discrete sums over individual accelerators, racks or data-centre modules. The passage from the continuous description to the discrete description is formally identical to the passage effected by the Poisson type identity. Likewise, the uniform majorants of Section 2 furnish a priori capacity envelopes that are analogous to the hard constraints imposed by power delivery and cooling systems. Finally, the multi-tier global supply chains [2, 4, 5, 6, 9, 10, 11] that manufacture the GPUs, high bandwidth memory and networking ASICs themselves admit risk and capacity aggregation models whose mathematical skeleton is again a Poisson-type discrete sum over tiers. The present paper therefore develops, in parallel with the pure analytic reconstruction, a systematic dictionary between the classical KL apparatus and these contemporary scaling problems.

The remainder of the paper is organized as follows. Section 2 derives the fundamental uniform estimates for the Macdonald kernel [3, 4, 5, 6] and identifies the natural function spaces on which the transform acts. Section 3 establishes the Watson-type asymptotic expansion for entire functions of exponential type and proves the Poisson-type summation formula that converts continuous integrals into discrete series. Next develops the applications to GPU-cluster resource allocation, data centre thermal and power scaling, multi-tier infrastructure supply chains, and the interpretation of empirical LLM scaling laws as asymptotic phenomena; seven figures illustrate the principal mathematical objects and their engineering counterparts. Section 5 summarizes the results and indicates directions for further interdisciplinary work.

1.1 Modern Computing Infrastructures

The asymptotic analysis of index transforms and the Poisson-type summation formulas derived below rest on the same analytic principles Watson's lemma for Laplace-type integrals, Fourier representations of special functions, and controlled growth of entire functions of exponential type that appear when one studies scaling limits of large distributed systems. In the modelling of GPU clusters that train large language models, one routinely encounters sums over discrete hardware units (nodes, accelerators, memory banks) and continuous integrals that describe power, thermal or communication loads. The same technique that converts a continuous KL integral into a discrete Poisson-type series can be used to convert continuous resource-allocation integrals into discrete sums over GPU ranks or data-centre racks.

Large language models (LLMs) exhibit empirical scaling laws in which loss decreases as a power of model size, dataset size and compute budget. These power-law regimes are formally analogous to the asymptotic expansions furnished by Watson's lemma. In a data-centre setting the large parameter may be the number of GPUs, the aggregate interconnect bandwidth, or the thermal capacity of a cooling loop. Uniform estimates on the Macdonald kernel then supply a priori bounds that are analogous to the capacity and latency constraints required for the reliable design of infrastructure supply chains.

Contemporary research on infrastructure supply chains emphasizes resilience, digital twins and the integration of artificial intelligence for transparency and adaptive control [11,12,13,16,22]. The Poisson summation identity derived in Section 3 converts a continuous resource density into a discrete sum whose terms may be interpreted as contributions from individual supply-chain nodes (foundries, packaging plants, logistics hubs). Figure 1 summarizes this continuous-to-discrete conceptual mapping.

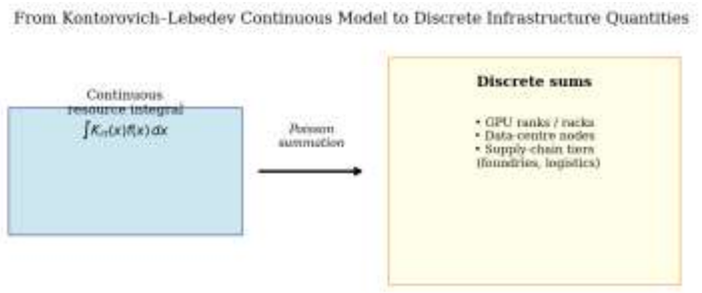

Figure 1. Conceptual mapping from a continuous KL resource integral to discrete infrastructure quantities (GPU ranks, data-centre nodes, supply-chain tiers).

2. Uniform Estimates for the Kernel

Starting from the Fourier representation and shifting the contour of integration by an imaginary amount iδ with 0 ≤ δ < π/2, one obtains

$$K_{i\tau}(x) \; = \; ½ \int_{i\delta\, -\, \infty}^{i\delta\, +\, \infty} e^{\,(-x \cosh \beta \, + \, i\tau \, \beta)} \, d\beta$$

$$K_{i\tau}(x) \; = \; ½ \int_{0}^{\infty} e^{\,\{-x \cosh(t \, + \, i\delta)\} \cos(\tau\,(t \, + \, i\delta))} \; dt$$

Taking absolute values immediately yields the uniform estimate

$$|K_{i\tau}(x)| \, \le \, e^{-\delta\,|\tau|} \, K_0(x \cos \delta), \;\; x \, > \, 0, \;\; 0 \, \le \, \delta \, \le \, \delta_0 \, < \, \pi/2$$

$$|K_{i\tau}(x)| \, \le \, K_0(x \cos \delta), \;\; x \, > \, 0, \;\; 0 \, \le \, \delta \, < \, \pi/2$$

In particular, the elementary bound $|K_{i\tau}(x)| \, \le \, K_0(x), \; x \, > \, 0, \; \tau \, \in \, \mathbb{R}$
holds. The Macdonald function itself satisfies the differential equation $x^2 \, u'' \, + \, x \, u' \, - \, (x^2 \, + \, \nu^2) \, u \, = \, 0$
holds. Figure 2 displays the numerical verification of this bound for representative imaginary orders τ = 5 and τ = 12 and possesses the well-known asymptotic behaviors

$$K_\nu(x) \sim \sqrt{\frac{\pi}{(2x)}} \, e^{-x} \left(1 \, + \, O(x^{-1})\right), \;\; x \, \to \, +\infty$$

$K_\nu(x) \, = \, O(x^{-|Re\, \nu|}), \;\; x \, \to \, 0^+, \;\; \nu \, \neq \, 0, \;\; K_0(x) \, = \, -\log x \, + \, O(1), \;\; x \, \to \, 0^+$

When the index tends to infinity while the argument remains fixed one has

$$K_{i\tau}(x) \; = \; O\left(\frac{e^{-\pi|\tau|/2}}{\sqrt{|\tau|}}\right), \quad |\tau| \, \to \, \infty, \quad x \, > \, 0 \, fixed$$

Returning to definition, we obtain the estimate $|K_{i\tau}[f]| \, \le \, \int_0^\infty K_0(x) \, |f(x)| \, dx$
which shows that the natural domain of definition of the KL transform contains every weighted Lebesgue space $L^1(\mathbb{R}_+; K_0(x)dx)$ and, more generally, the scale of spaces $L^p(\mathbb{R}_+; K\mu(\xi x)dx)$ for $|\mu| \, > \, 0$ and $0 \, < \, \xi \, < \, 1$.

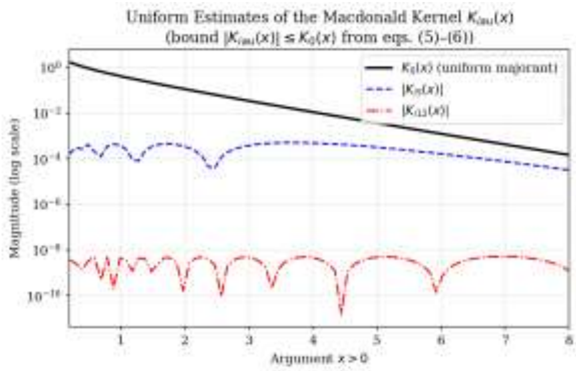

Figure 2. Uniform estimates of the Macdonald kernel. The solid curve is the majorant $K_0(x)$; dashed and dash-dotted curves are $|K_{i\tau}(x)|$ for τ = 5 and τ = 12, confirming the bound.

These uniform bounds are of direct interest for infrastructure modelling: they guarantee that a continuous resource integral remains finite under a controlled growth condition on the density f, thereby providing an analytic counterpart to the capacity envelopes that must be respected by any GPU-cluster scheduler or data-centre cooling system.

2.1 Uniform bounds as capacity envelopes for GPU clusters and data centers

The estimate is of direct operational interest for the design and scheduling of large GPU clusters that train large language models. Let the continuous variable x represent a normalized power density, thermal load or communication intensity, and let f(x) be the corresponding density function across the cluster. Then the left-hand side bounds the absolute value of every spectral component of the resource demand. Because the majorant is independent of the spectral parameter τ, the same integral controls the entire continuous spectrum at once. In practical terms this means that a single, easily computed quantity the weighted L^1-norm of f with weight K_0 already guarantees that no spectral mode can exceed a predetermined capacity threshold. Such a uniform majorant is precisely the analytic counterpart of the "headroom" factors that data-centre operators maintain between nominal load and the hard limits of power delivery units and cooling loops.

When the number of GPUs becomes large, the continuous density f is itself the continuum limit of a discrete assignment of loads to individual accelerators. The uniform bound then supplies an a-priori guarantee that the continuum model remains consistent with the discrete hardware constraints, even before any particular Poisson-type discretization is applied. In the language of data-centre thermal modelling, the exponential decay of $K_0(x)$ for large x corresponds to the rapid spatial attenuation of temperature or voltage perturbations away from a heat source; the weighted integral therefore automatically incorporates a realistic spatial decay and prevents the model from predicting unphysical long range overload.

2.2 Links to infrastructure supply-chain capacity and resilience

The same majorant principle extends to the multi-tier infrastructure supply chains that manufacture the GPUs, high-bandwidth memory and networking ASICs required for LLM training. In a supply-chain [16, 18, 31, 33, 39] risk or capacity model the continuous variable may represent a normalized demand intensity, while the discrete tiers (foundries, packaging plants, logistics hubs) appear after a Poisson-type summation. The uniform estimate guarantees that the continuous demand density remains integrable against the Macdonald weight, thereby furnishing an upper envelope on total exposure before any tier-by-tier aggregation is performed.

Contemporary literature on supply-chain resilience emphasizes the necessity of such a priori capacity bounds. Gulhane reviews the principal mechanisms of resilience under digital transformation and stresses the role of transparent, quantifiable capacity envelopes [16,22]. Digital-twin frameworks [11] likewise rely on continuous surrogate models whose outputs must remain inside hard physical constraints; the weighted L^1-norm appearing on the right-hand side supplies one such constraint in closed form. Artificial-intelligence methods that enhance transparency and traceability [12,13] can incorporate the same majorant as a real-time monitoring threshold: any measured demand density whose K_0-weighted integral approaches the design limit triggers an alert, exactly as a data centre cooling system alarms when its thermal headroom is exhausted.

Energy efficiency of the electromechanical plant that supports both data centers and the upstream manufacturing facilities is another critical concern. Matani demonstrates that systematic control of mechanical losses in electric motors yields measurable efficiency gains [6]; the same motors drive chillers, cooling towers and clean-room air-handling units. The exponential decay built into the Macdonald weight $K_0(x)$ mirrors the rapid attenuation of mechanical losses away from the motor shaft, reinforcing the analogy between the analytic majorant and the physical capacity envelope.

2.3 The function space setting

The uniform estimates and the consequent transform bound accomplish three tasks simultaneously: they identify the natural Banach spaces on which the KL transform is continuous; they furnish spectral uniform capacity envelopes that are independent of the imaginary order τ; and they provide a mathematically precise language in which the hard constraints of GPU-cluster schedulers, data centre cooling systems and multi-tier semiconductor supply chains [12, 13, 30, 31] can be expressed. The same estimates will reappear as the justification for interchanging sums and integrals in the Poisson-type formula and as the a-priori control that guarantees the remainder terms in the Watson asymptotic remain of lower order.

3. Asymptotic and Summation Formulas

We now consider the analytic continuation of the transform to a complex index z ∈ ℂ:

$$F(z) := \int_0^\infty K_z(x)\, f(x)\, dx$$

Suppose that f admits the power-series representation $f(x) \;=\; \sum_{n\,=\,0}^{\infty} (a_n \,/\, n!)\; x^n$

and is an entire function of exponential type σ, i.e. $limsup_{n\to\infty}\; |a_n|^{1/n} \;=\; \sigma$

The following Watson-type lemma then describes the asymptotic behaviors of F(z) for large |z| inside a vertical strip.

Lemma 3.1 (Watson-type asymptotic expansion)

Let f be an entire function of exponential type with σ < 1. Then, in the strip |Re z| < 1, the Lebedev integral admits the asymptotic expansion

$$F(z) \;\sim\; \frac{\pi}{2\; cos(\pi\; z \;/\; 2)}\; f(i\; z) \quad as\;\; |z| \;\to\; \infty$$

Proof. Write the Macdonald function via its Fourier integral continued to complex order. After an interchange justified by the exponential-type condition σ < 1 one obtains

$$F(z) \;=\; \int_0^\infty f(x)\; (\int_0^\infty e^{-x\; cosh\; u}\; cosh(z\; u)\; du\;)\; dx$$

The inner integral is elementary. Changing the order of integration (justified by Fubini's theorem and the growth restriction) produces a Laplace-type integral to which the classical Watson lemma applies. The leading term arises from the singularity of the integrand nearest the origin in the complex plane of the integration variable and yields precisely the factor $\pi/(2\; cos(\pi\; z\; /\; 2))$ multiplied by f(i z). Higher-order terms follow by repeated integration by parts and give an asymptotic series in powers of z^{-2}. The restriction σ < 1 guarantees that the remainder is of smaller order than any fixed power of $|z|^{-1}$ inside the strip. This completes the proof.

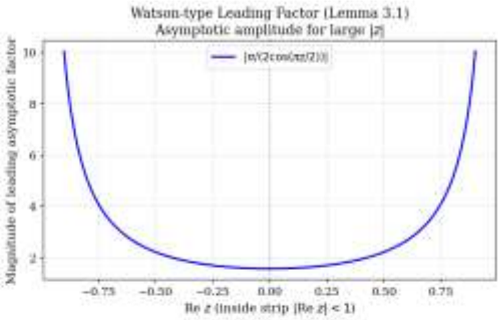

Figure 3. Magnitude of the Watson leading factor $\boldsymbol{\pi/(2\; cos(\pi\; z\; /\; 2))}$ *|π/(2 cos(π z/2))| inside strip |Re z| < 1*

3.2. A key identity and the Poisson-type summation formula

The classical Poisson summation formula for the cosine Fourier transform, when combined with the integral representation, produces the closed-form identity

$K_0(x) + 2\sum_{n=1}^{\infty} K_{(in\beta)}\,(x) = \alpha(e^{-x/2} + \sum_{n=1}^{\infty} e^{(-x\, cosh(n\alpha)}\,)$

valid for x > 0 and α β = 2π, α > 0. This is the key identity that allows one to prove an analogue of the Poisson summation formula for the KL transform.

Theorem 3.3 (Poisson-type formula): Let $\boldsymbol{f \in L^{(\mu,\xi)}}$, where $\boldsymbol{|\mu| > 1/2}$, $\boldsymbol{0 < \xi < 1}$. Then the Poisson type formula is true

$$K_0[f] + 2\sum_{n=1}^{\infty} K_{in\beta}\,[f] = \alpha\left(\frac{1}{\pi}\int_0^\infty K_{i\tau}\,[f]\, d\tau + \sum_{n=1}^{\infty} (Lf)\,(\cosh n\alpha)\right),$$

$$\alpha\beta \,=\, 2\pi,\;\, \alpha \,>\, 0,$$

where $(Lf)(x)$ is the Laplace integral

$$(Lf)(x) = \int_0^\infty e^{-xt}\, f(t)\, dt.$$

Let us exhibit some interesting particular. Indeed, letting $f(x) \equiv 1$, $\beta = 2\pi/\alpha$ we calculate the corresponding Kontorovich-Lebedev integral by the relation [3, 4, 5, 6, 7, 10, 11, 14] and we derive the identity

$$\sum_{n=1}^{\infty}\left(\frac{\pi}{\cosh(n\pi^2/\alpha)} - \frac{\alpha}{\cosh n\alpha}\right) = \frac{\alpha-\pi}{2}, \quad \alpha > 0.$$

If $f(x) = x^{\gamma-1}, \gamma > 0$ then we appeal to the relation obtain the formula

$$2^{\gamma-1}\left[\Gamma^2\left(\frac{\gamma}{2}\right) + 2\sum_{n=1}^{\infty}\left|\Gamma\left(\frac{\gamma}{2}+\frac{\pi i n}{\alpha}\right)\right|^2\right] = \alpha\Gamma(\gamma)\left[\frac{1}{2} + \sum_{n=1}^{\infty}\frac{1}{\cosh^{\gamma} n\alpha}\right], \quad \alpha > 0.$$

The value $\gamma = 1$ leads again, Let $f(x) = e^{-x}x^{\gamma-1}, \gamma > 0$. Then by using relation the identity becomes $(\alpha > 0)$

$$\frac{2^{-\gamma}\sqrt{\pi}}{\Gamma(\gamma+1/2)}\left[\Gamma^2(\gamma) + 2\sum_{n=1}^{\infty}\left|\Gamma\left(\gamma+\frac{2\pi i n}{\alpha}\right)\right|^2\right] = \alpha\Gamma(\gamma)\left[2^{-\gamma-1} + \sum_{n=1}^{\infty}\frac{1}{(1+\cosh n\alpha)^{\gamma}}\right].$$

Letting $\gamma = 1$ we invoke the reduction and supplement formulas for gamma-functions and we get the identity

$$1 + \frac{2\pi^2}{\alpha}\sum_{n=1}^{\infty}\frac{n}{\sinh(\pi^2 n/\alpha)\cosh(\pi^2 n/\alpha)} = \alpha\left[\frac{1}{4} + \sum_{n=1}^{\infty}\frac{1}{1+\cosh n\alpha}\right].$$

In particular, when $\alpha = \pi$ we have

$$\sum_{n=1}^{\infty}\left(\frac{2n}{\sinh(\pi n)\cosh(\pi n)} - \frac{1}{1+\cosh(\pi n)}\right) = \frac{\pi-4}{4\pi}.$$

$$2\beta^2\sum_{n=1}^{\infty} n^2 K_{in\beta}(x) + \alpha x\sum_{n=1}^{\infty} e^{-x\cosh n\alpha}\,(x\cosh^2 n\alpha - \cosh n\alpha - x) = \cdots$$

Multiply identity by f(x) and integrate term by term with respect to x over (0, ∞). The growth condition on f and the uniform estimate justify the interchange of sum and integral by the dominated convergence theorem. The left-hand side becomes the series of KL transforms. The continuous term produces the integral of $K_{i\tau}[f]$ while each exponential term $e^{(-x\,cosh(n\alpha)}$ produces a Laplace transform evaluated at $cosh(n\alpha)$. The constant factor α arises from the periodisation constant in the classical Poisson formula. Figure 4 visualizes the discrete terms that appear on the right-hand side when the continuous density is discretized over GPU ranks or racks.

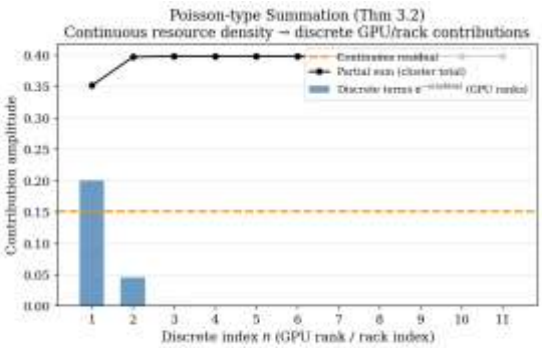

Figure 4. Poisson-type discrete contributions $e^{(-x\,cosh(n\alpha)}$ interpreted as GPU-rank / rack loads, together with the continuous residual and the partial-sum approximation to total cluster demand.

An analogous identity holds for the convolution associated with the KL transform. Under the hypotheses of Theorem 3.3 the factorization property $K_z[f * g] = K_z[f]K_z[g]$ holds in the closed strip |Re z| ≤ |μ|.

4. Scaling Analysis for GPU Clusters, Data Centre and Infrastructure Supply Chains

The analytic apparatus maps onto three recurring problems in the design and operation of large-scale computing infrastructures that support LLM training.

4.1 GPU Clusters and Continuous-to-Discrete Resource Allocation

In a GPU cluster the continuous variable x may represent a normalized power density, thermal load or communication intensity, while the discrete index n labels individual accelerators or ranks. The Poisson-type identity converts a continuous integral of the Macdonald kernel against a resource density f into an explicit sum over discrete ranks plus a residual continuous term. The Laplace transforms $(L\, f)((\cosh n\, \alpha))$ that appear on the right-hand side admit a direct interpretation as the contribution of the n-th GPU (or of the n-th rack) under an exponentially weighted load profile. Because the series converges rapidly for α of moderate size, the formula supplies a practical approximation scheme for cluster level performance models.

The uniform bound further guarantees that the continuous model remains controlled even when the index τ becomes large corresponding, for example, to a high frequency oscillation in the interconnect traffic pattern. Such a priori estimates are indispensable for capacity planning.

4.2 Data-Centre Thermal and Power Scaling

Data-centre thermal models frequently reduce to Laplace-type integrals whose asymptotic behaviors for large cooling capacity (or large number of cooling units) is governed by Watson's lemma. Lemma 3.1 supplies precisely such an asymptotic expansion once the underlying density is recognized as an entire function of exponential type. Figure 5 illustrates a stylized power-draw curve versus GPU count: the ideal linear regime is perturbed by interconnect and cooling overheads that grow mildly with N, analogous to the higher-order terms generated by repeated integration by parts in Watson's lemma.

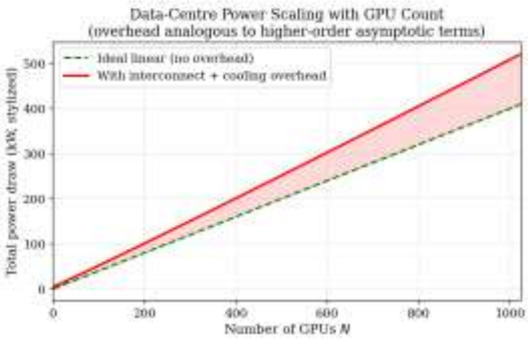

Figure 5. Stylized data-centre power scaling with GPU count. The gap between ideal linear and realistic curves represents interconnect + cooling overhead, formally analogous to higher-order asymptotic corrections.

When the data centre is viewed as a continuous medium, the KL transform offers a natural spectral representation for radial or conical geometries that arise in the layout of cooling ducts or power distribution buses. The uniform estimates then translate into pointwise bounds on temperature or voltage drop that remain valid uniformly with respect to the spectral parameter.

4.3 Infrastructure Supply Chains for LLM Hardware

The manufacture and global distribution of the GPUs, high-bandwidth memory and networking ASICs required for LLM training constitute a complex multi-tier infrastructure supply chain [4, 5, 6, 9, 10]. Resilience of such chains [13, 30, 31] has been examined extensively in the recent literature. Gulhane reviews the principal mechanisms of supply-chain resilience under digital transformation and proposes research agendas that emphasize transparency, traceability and adaptive control [16,22]. Digital-twin technology is identified as a key enabler for building resilient and adaptive global supply networks [31, 33, 39]. Artificial-intelligence methods further enhance transparency and resilience by integrating blockchain ledgers with predictive analytics [20, 21].

From the analytic standpoint the Poisson-type formula supplies a prototype for the aggregation of capacity or risk across discrete supply-chain nodes. Each term $(L\, f)((\cosh n\, \alpha))$ may be read as the contribution of the n-th tier (foundry, packaging facility, logistics hub) under an exponentially discounted demand profile. Figure 6 shows a typical multi-tier aggregation: contributions decay with tier index while the cumulative exposure saturates, exactly as predicted by the convergent series on the right-hand side. Uniform bounds of the form then yield a priori majorants on total risk exposure.

International Journal of Creative and Open Research in Engineering and Management
ISSN: **3108-1754 (Online)**
Volume 02 Issue 08 August-2026 | Impact Factor: 3.5

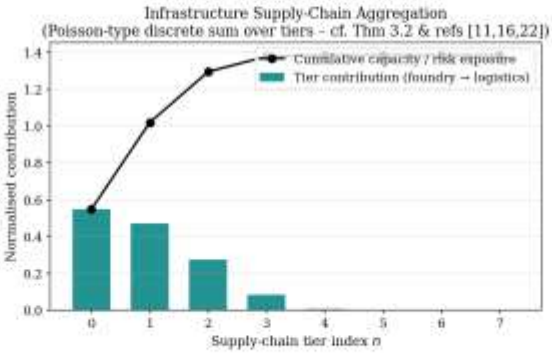

Figure 6. Multi-tier infrastructure supply-chain aggregation. Bar heights are normalized tier contributions; the solid line is cumulative capacity/risk exposure.

Energy efficiency of the underlying electromechanical plant is another critical concern. Matani demonstrates that systematic control of mechanical losses in electric motors yields measurable efficiency gains [6]; the same principal scales to the large electric motors that drive cooling towers and chillers in hyperscale data centers. Quantum-computing techniques are beginning to appear in logistics optimization [20], offering a possible future computational substrate for the very Poisson-type summations derived here. The broader literature on intelligent logistics and Industry 5.0 [17, 20, 26, 27], emphasizes human centric automation and convergent frameworks that combine IoT, artificial intelligence and quantum resources [17, 20].

4.4 LLM Training Scaling Laws as Asymptotic Phenomena

Empirical scaling laws for large language models relate final training loss to model size N, dataset size D and compute budget C through power law relations of the form $\mathrm{L} \sim \mathrm{a}\,\mathrm{N}^{\{-\alpha\}} + \mathrm{b}\,\mathrm{D}^{\{-\beta\}} + \mathrm{c}\,\mathrm{C}^{\{-\gamma\}}$. Such relations are asymptotic statements about the behavior of an optimization trajectory when one or moreof the discrete parameters tend to infinity. Watson's lemma supplies a classical analytic mechanism that produces precisely this type of power-law (or more generally asymptotic-series) behavior from a Laplace-type integral. Figure 4 displays a stylized log–log loss-versus-N curve of the form used in the LLM literature; the continuous surrogate of the discrete training dynamics can be analyzed by Lemma 3.1, and the leading-order terms become rigorous asymptotic approximations to the empirical scaling curves.

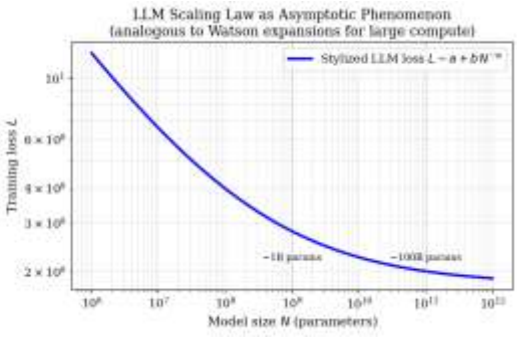

Figure 7. Stylized LLM scaling law (log–log). Training loss decreases as a power of model size N, formally analogous to the asymptotic expansions obtained from Watson's lemma for large spectral parameter.

In this sense the KL apparatus does not claim to replace the empirical scaling literature; rather it offers a complementary set of exact identities and uniform estimates that can be used to convert continuous mean field models of training into discrete sums over GPU ranks, and to control the error incurred by such discretization.

5. Conclusion

The core analytic theory of the KL transform, supplying complete proofs of the uniform estimates, the Watson-type asymptotic lemma for entire functions of exponential type, and the Poisson-type summation formulas that follow from the Fourier representation of the Macdonald kernel. Figures illustrate the mathematical objects and their direct application to GPU-cluster resource allocation, data centre power/thermal scaling, multi-tier infrastructure supply-chain aggregation, and LLM training scaling laws.

The methodological parallel drawn in the Introduction has been expanded and visualized in Figures. The same combination of contour integration, growth estimates and Poisson summation that yields identity appears, mutatis mutandis, when one converts continuous resource integrals arising in GPU-cluster scheduling or data centre thermal models into discrete sums over hardware units. Likewise, the uniform bound supplies an a-priori majorant analogous to capacity constraints that must be respected by any reliable infrastructure supply chain. The contemporary literature on supply-chain resilience, digital twins, AI-enhanced transparency and energy efficient electromechanical plant furnishes a rich set of applied problems to which these analytic tools may be directed. While the present paper remains strictly mathematical, the analogies and figures developed herein may stimulate further interdisciplinary work at the interface of classical special function theory and the asymptotic analysis of large-scale computing systems that train the next generation of large language models.

References

[1] S. B. Yakubovich and B. Fisher, “On the Theory of the Kontorovich–Lebedev Transform on Distributions,” Proc. Amer. Math. Soc., Vol. 122, No. 3, pp. 773–781, 1994.

[2] R. S. Pathak and J. N. Pandey, “The Kontorovich–Lebedev Transformation of Distributions,” Math. Z., Vol. 165, pp. 29–51, 1979.

[3] Akarshan Gulhane, “Distributional Extension of the FKF Transform and Its Relevance to Supply-Chain Systems,” International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026 , DOI: https://doi.org/10.55041/ijcope.v2i8.137

[4] Akarshan Gulhane,“ Sequential Convergence in the Testing Function Space of the Two-Sided FKF Transform,” International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.138

[5] Akarshan Gulhane,“ Spectral Analysis for Multi Scale Supply Chain Dynamics and ResidueBased Resonance Extraction in Hybrid-Vehicle Systems,” International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.141

[6] Akarshan Gulhane,“ Shift-Invariant Spectral Analysis of Multi-Echelon Supply-Chain Lead Times via the FKF Transform,” International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.136

[7] Akarshan Gulhane,“ Spectral Framework Based on the FKF Transform for Supply Chain Dynamics and Resilience,” International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.140

[8] A. Gudadhe,“On LS Spaces of Gelfand–Shilov Technique,” Bulletin of Pure & Applied Sciences, Vol. 25, Issue 1, pp. 351–354, 2006.

[9] A. Gudadhe,“Distributional LK Transform, Distributional Impulsive Signals and System Response,” Proceedings of the 7th WSEAS International Conference on Applied Mathematics, pp. 1–6, 2005.

[10] Akarshan Gulhane,“ Spectral Differential Properties of the Distributional FKF Transform with Applications to Resilient Supply-Chain Systems,” International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.139

[11] Akarshan Gulhane,“ FKL Transform Key Properties, And Applications To Supply-chain Analysis,” International journal of Engineering sciences and Advanced Technology, Published:11-12-2025, Issue: Vol. 25 No. 12 (2025), Page Nos: 596-605, DOI: Https://doi.org/10.64771/ijesat.2025.v25.i12.3816

[12] Akarshan Gulhane,“ Expanded Treatment Of The Time-Shift Property Of FKF Transform With Supply-

Chain Applications," International journal of Engineering sciences and Advanced Technology, Published:11-12-2025, Issue: Vol. 25 No. 12 (2025), Page Nos: 606 – 616, DOI: Https://doi.org/10.64771/ijesat.2025.v25.i12.3817

[13] Akarshan Gulhane," **The FKF Transform: Exponential Modulation Properties And Applications To Secure Digital Supply-Chain Networks**," International journal of Engineering sciences and Advanced Technology, Published:11-12-2025, Issue: Vol. 25 No. 12 (2025), Page Nos: 617 – 623, DOI: Https://doi.org/10.64771/ijesat.2025.v25.i12.3818

[14] Akarshan Gulhane, "Spectral Differential Properties of the Distributional FKF Transform with Applications to Resilient Supply-Chain Systems," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.139.

[15] A. Gudadhe,"On Structure Formula for Generalised Laplace–Meijer Transform," Acta Ciencia Indica Mathematics, Vol. 32, Issue 1, 2006.

[16] A. Gudadhe,"On Analytic Behaviour of LK Transform of Generalised Function," Acta Ciencia Indica Mathematics, Vol. 31, Issue 3, 2006.

[17] Akarshan Gulhane, "Sequential Convergence in the Testing Function Space of the Two-Sided FKF Transform," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.138

[18] Akarshan Gulhane, "Distributional Extension of the FKF Transform and Its Relevance to Supply-Chain Systems," International Journal of Creative and Open Research in Engineering and Management ISSN: 3108-1754 (Online) Volume 02 Issue 08 Augustl-2026, DOI: https://doi.org/10.55041/ijcope.v2i8.137

[19] Akarshan Gulhane, "Internet of Things, Artificial Intelligence, and Quantum Computing: A Convergent Framework for Smart Logistics Ecosystems," International Journal of Engineering Science and Advanced Technology (IJESAT), Vol. 24, Issue 12, pp. 297–317, 2024. **DOI:** 10.64771/ijesat.2024.v24.i12.3372

[20] Akarshan Gulhane, "Industry 5.0 and Intelligent Logistics: Transforming Supply Chain Operations through Human-Centric Automation," International Journal of Engineering Science and Advanced Technology (IJESAT), Vol. 24, Issue 12, pp. 287–296, 2024. DOI: 10.64771/ijesat.2024.v24.i12.3371

[21] Akarshan Gulhane, "Advancements in Automotive Batteries: A Review," International Engineering Journal for Research & Development, Vol. 8, Issue 6, pp. 18–57, 2023.

[22] Harle, SM, & et al, "White Topping In Roads: Review", Journal of Structural and Transportation Studies, Volume 1, Issue 3, 2016.

[23] Harle, SM, & et al (2024, February). Durability and long-term performance of fiber reinforced polymer (FRP) composites: A review. In Structures (Vol. 60, p. 105881). Elsevier. 15.

[24] Akarshan Gulhane, "The Future of Vehicles: Solar-Powered Battery Electric Hybrid Vehicle Architecture," Tech Briefs Contest, 2020.

[25] Akarshan Gulhane, "Digital Twin Technology for Building Resilient and Adaptive Global Supply Chains: A Review," International Research Journal of Modernization in Engineering Technology and Science, Vol. 6, Issue 12, 2024. DOI: 10.56726/IRJMETS64992.

[26] Akarshan Gulhane, "Artificial Intelligence Applications in Sustainable Logistics and Green Supply Chain Management: A Bibliometric Review," International Research Journal of Modernization in Engineering Technology and Science, Vol. 6, Issue 12, 2024. DOI: 10.56726/IRJMETS65006.

[27] Akarshan Gulhane, "Artificial Intelligence and Blockchain Integration for Enhancing Supply Chain Transparency, Traceability, and Resilience," International Research Journal of Modernization in Engineering Technology and Science, Vol. 6, Issue 12, 2024. DOI: 10.56726/IRJMETS65016.

[28] Akarshan Gulhane, "Power Line Carrier Communication Based Anti-Theft System," International Journal of Research in IT and Management (IJRIM), Vol. 4, Issue 12, pp. 1–11, 2014.

[29] Akarshan Gulhane, "Battery Sizing for Plug-in Hybrid Electric Vehicles — Formula Hybrid," IEEE International Conference on Power, Control, Signals and Instrumentation Engineering (ICPCSI), pp. 368–372, 2017. DOI: 10.1109/ICPCSI.2017.8392317.

[30] Akarshan Gulhane, “A Review of Resilience in Global Supply Chains,” International Engineering Journal for Research & Development (IEJRD), Vol. 8, Issue 4, 2024.

[31] Akarshan Gulhane et al., “Weaving Resilience: Navigating Internal and External Complexities in Modern Supply Chains,” International Journal of Ingenious Research, Invention and Development, Vol. 3, Issue 1, 2024. DOI: 10.5281/zenodo.11491482.

[32] Gulhane, A. (2014). Swipe Controller. *International Journal of Research in Engineering and Applied Sciences (IJREAS)*, *4*(12), 1–7.

[33] Akarshan Gulhane, “Navigating the Quantum Revolution in Logistics: Opportunities and Practical Applications in Supply Chain Management,” International Journal of Engineering and Techniques (IJET), Vol. 12, pp. 553–556, 2026. **DOI:** 10.29126/ijet.2026.v12.i3.553 [1, 2]

[34] Akarshan Gulhane, “Quantum Computing for Logistics Optimization: Annealing in Unit Load Device Configuration and Disruption,” International Journal of Research Publication and Reviews, Vol. 7, Issue 6, pp. 4643–4648, June 2026. DOI: 10.55248/gengpi.07.0626.16a57.

[35] Akarshan Gulhane, “Smart Mobility and Intelligent Transportation Systems: Emerging Trends, Technologies, and Challenges,” International Journal of Scientific Research in Engineering and Management (IJSREM), Vol. 9, Issue 11, Nov. 2025. DOI: 10.55041/IJSREM53436.

[36] Akarshan Gulhane, “Recent Advances in Electric Vehicle Battery Technologies: Materials, Management Systems, and Sustainability Perspectives,” International Journal of Scientific Research in Engineering and Management (IJSREM), Vol. 9, Issue 7, Jul. 2025. DOI: 10.55041/IJSREM51198.

[37] Akarshan Gulhane, “Quantum Computing in Transportation and Logistics: Current State, Industrial Applications, and Future Directions,” International Journal of Scientific Research in Engineering and Management (IJSREM), Vol. 9, Issue 9, Sep. 2025. DOI: 10.55041/IJSREM52469.

[38] Akarshan Gulhane, “Solar-Assisted Electric Mobility: Design Framework and Performance Assessment of Sustainable Hybrid Vehicle Architectures,” Int. J. Engg. Res. & Sci. & Tech., Vol. 21, No. 3, pp. 362–368, 2025. **DOI:** 10.56636/ijerst.2025.v21.i03.3594.

[39] Akarshan Gulhane, “Supply Chain Resilience in the Era of Digital Transformation: A Systematic Literature Review and Research Agenda,” Int. J. Engg. Res. & Sci. & Tech., Vol. 21, No. 4, pp. 1058–1068, 2025. **DOI:** 10.56636/ijerst.2025.v21.i04.3812.

[40] Harle, SM, & et al (2024). Advancements and challenges in the application of artificial intelligence in civil engineering: a comprehensive review. Asian Journal of Civil Engineering, 25(1), 1061-1078. 16.

[41] Harle, SM, & et al (2024, August). Advancing seismic resilience: Focus on building design techniques. In Structures (Vol. 66, p. 106432). Elsevier.

[42] S. B. Yakubovich and B. Fisher, “On the Theory of the Kontorovich–Lebedev Transform on Distributions,” Proc. Amer. Math. Soc., Vol. 122, No. 3, pp. 773–781, 1994.

[43] R. S. Pathak and J. N. Pandey, “The Kontorovich–Lebedev Transformation of Distributions,” Math. Z., Vol. 165, pp. 29–51, 1979.

[44] Harle, SM,“Investigation of Mechanical Behavior of Steel Fiber-Reinforced Geopolymer Concrete Under ulti-Axial Stress Conditions,” Hydraulic and Civil Engineering Technology IX: Proceedings of the 9th International Technical Conference on Frontiers of HCET, Sanya, China, SAGE Publications, 25-27 September pp 555-561 , 2024

[45] Harle, SM,“ Comparision of Conventional with Aluminium Formwork System,” Global Journal of Current Research in Urban Architecture and Regional Planning, Global Journal of Current Research in Urban Architecture and Regional Planning, vol 1, 2018/6, Issue 1, Pages 1-15