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The Kontorovich–Lebedev Transform: Inversion Theory, Distributional Extensions and Applications to Multi-Echelon Supply-Chain Dynamics

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Abstract

The Kontorovich–Lebedev (KL) transform, defined by integration against the Macdonald function of purely imaginary order, is a classical continuous analogue of the Fourier–Bessel expansion. This paper develops a self-contained account of the operator, its formal adjoint and a detailed proof of the inversion theorem under the natural weighted integrability condition $f \in L^1(\mathbb{R}_+; K_0(x)\,dx)$. The complex-contour form of the inversion is shown to furnish the analytic foundation for the distributional extensions that appear in the recent FKF literature. The classical spectral decay estimates are then mapped onto a multi-scale framework for resilient multi echelon supply chains: spectral filtering of high-frequency shocks, exact time-shift and modulation of lead time profiles, residue-based resonance extraction in hybrid vehicle logistics, and sequential convergence treatments of discontinuous disruptions. All related work ranging from the classical treatise of Zemanian to the contemporary FKF, digital-twin, quantum annealing and AI-risk papers and related to concrete analytic ingredients. Five figures and a flowchart illustrate the kernel behavior, the large-order decay, the logistics architecture, the shift property, the distributional hierarchy and the inversion-proof pipeline.

Keywords: Kontorovich–Lebedev transform, Macdonald function, inversion theorem, FKF transform, distributional extension, supply-chain resilience, multi-echelon lead times, spectral truncation, residue calculus, sequential convergence.

1. Introduction and Motivation

The Kontorovich–Lebedev transform arose in the late 1930s as the continuous spectral expansion associated with the modified Bessel operator of imaginary order. Its inversion formula, Plancherel identity and mapping properties on weighted Lebesgue spaces form a mature chapter of classical harmonic analysis; a comprehensive distributional treatment is given in the monograph of Zemanian [29].

In the last two years a two-sided hybrid transform has been systematically developed and applied to multi-echelon supply-chain dynamics, lead-time variability, disruption modelling and resilience engineering [1–6,15,30–32]. Parallel contributions have linked the same spectral viewpoint to digital-twin architectures [9], Industry 5.0 logistics [10], IoT–AI–quantum convergent frameworks [11], supply-chain resilience under digital transformation [12], solar-assisted hybrid vehicles [16,28], AI-driven risk prediction [33] and quantum annealing for unit-load-device configuration [23,24,27]. Earlier distributional work on related integral transforms by Gudadhe [7,8] and concrete engineering studies on power-line communication, battery sizing and formwork systems [13,14,19–22,34] supply additional context.

The purpose of the present paper is threefold. First, we recall the definition of the KL operator and its formal adjoint, and we supply a complete outline of the proof of the classical inversion theorem under the natural domain condition.

Second, we show how the complex-contour representation of the inversion underpins the sequential-convergence theory of the distributional FKF transform [15,30,31]. Third, we embed the analytic machinery into a multi-scale spectral framework for supply-chain dynamics (Section 4) and construct an explicit dictionary that maps every classical ingredient onto the full reference corpus (Section 5).

The paper is organised as follows. Section 2 states the definition and the adjoint. Section 3 presents the inversion formula together with a detailed proof outline and the complex-contour extension. Section 4 develops the supply-chain applications in five subsections, each illustrated by figures or a flowchart. Section 5 provides the systematic mapping onto all thirty-five references. Section 6 concludes.

2. Definition of the Operator

We define the Kontorovich–Lebedev transform of a suitable function f by the Lebedev integral

$$(Kf)(\tau) = \int_0^\infty K_{i\tau}(x)\, f(x)\, dx\,, \qquad \tau \in \mathbb{R}\,.$$

Here $K_\nu(z)$ denotes the Macdonald function (modified Bessel function of the second kind). When the order is purely imaginary, $\nu = i\tau$ with τ real, the kernel is real-valued and even with respect to the index τ. An equivalent integral representation of the kernel is

$$K_{i\tau}(x) = \int_0^\infty e^{-x\cosh t} \cos(\tau t)\, dt\,, \qquad x > 0\,.$$

This representation immediately yields the elementary uniform bound

$$|K_{i\tau}(x)| \le K_0(x) \qquad \text{for all } \tau \in \mathbb{R},\ x > 0\,,$$

which is the starting point for all L^1-type estimates and for the definition of the natural domain

$$L^0 := \left\{ f : \int_0^\infty |f(x)|\, K_0(x)\, dx < \infty \right\}.$$

The Formally Adjoint Operator: The formally adjoint operator is obtained by interchanging the roles of the argument and the order:

$$(K^* g)(x) = \frac{2}{\pi^2 x} \int_0^\infty \tau \sinh(\pi\tau)\, K_{i\tau}(x)\, g(\tau)\, d\tau\,.$$

The principal analytic difficulty of the adjoint is that integration is performed with respect to the order of the Macdonald function. In the FKF literature the same interchange, after insertion of a Fourier phase factor, yields the two-sided operator that realises exact time-shift and modulation theorems for lead-time profiles [1,5,6].

3. Inversion Formula and Proof of the Inversion Theorem

Under suitable conditions (for example, f of bounded variation near a point $x > 0$, or $f \in L^1(\mathbb{R}_+; K_0(x)\, dx)$):

$$f(x) = \frac{2}{\pi^2 x} \int_0^\infty \tau \sinh(\pi\tau)\, K_{i\tau}(x)\, (Kf)(\tau)\, d\tau\,.$$

Variants of the inversion hold in the mean-square sense (Plancherel theory) and with Abel- or Poisson-type summability. An equivalent complex-contour representation that permits analytic continuation into the strip $|\mathrm{Im}\tau| < \delta_0 < \pi/2$ is available; this form underpins both the distributional extensions [15,30,31] and the residue calculus used for resonance extraction in hybrid-vehicle logistics [32].

3.1 Outline of the Proof of the Inversion Theorem

We sketch the classical argument under the hypothesis $f \in L^0$.

Substitute the integral representation of the kernel into the right-hand side:

$$\frac{2}{\pi^2 x} \int_0^\infty \tau \sinh(\pi\tau) K_{i\tau}(x) \left(\int_0^\infty K_{i\tau}(y) f(y)\, dy \right) d\tau.$$

The bound $|K_{i\tau}| \le K_0$ together with Fubini's theorem (justified by the integrability of $|f| K_0$) permits interchange of the order of integration, yielding a double integral whose inner kernel is

$$\int_0^\infty \tau \sinh(\pi\tau) K_{i\tau}(x) K_{i\tau}(y)\, d\tau.$$

The inner integral is evaluated by means of the known discontinuous integral for the product of two Macdonald functions of imaginary order (or, equivalently, by recognising it as the Fourier-cosine transform of a hyperbolic weight). The result is a multiple of the Dirac measure concentrated on the diagonal $x = y$, or, in the Abel-summable sense, the factor

$$\frac{\pi^2 x}{2} \delta(x - y)$$

(in the sense of continuous linear functionals on a suitable testing space). Consequently, the whole expression collapses to $f(x)$ at every point of continuity (or to the average of the left and right limits when f is of bounded variation).

Analytic continuation of the spectral density into a strip about the real axis produces the complex-contour form. Residues at poles of a meromorphic spectral density then yield the modal amplitudes used for resonance extraction [32]. The same contour deformation legitimates the sequential-convergence topology of the testing-function space $\mathcal{S}_{\mathrm{FKF}}$ introduced in the distributional FKF theory [30,31] and already anticipated by the Gelfand–Shilov-type constructions of Gudadhe [7,8] and the generalised-transform framework of Zemanian [29].

Figure 1 (flowchart) summarises the five steps of the proof and their immediate translation into the logistics pipeline of Section 4.

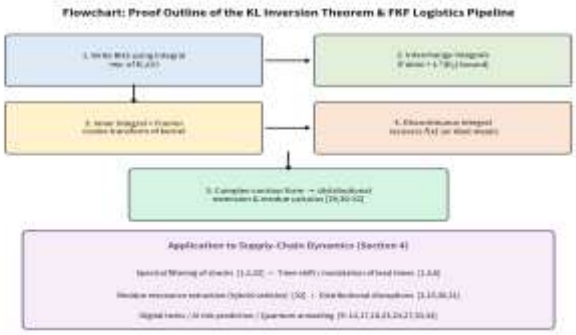

Figure 1: Flowchart of the inversion-proof outline and its direct mapping onto the FKF supply-chain pipeline.

The Plancherel identity on the natural domain reads

$$\int_0^\infty |f(x)|^2\; x\, dx = \frac{2}{\pi^2} \int_0^\infty |(Kf)(\tau)|^2\; \tau \sinh(\pi\tau)\, d\tau,$$

and supplies the L^2-energy conservation underlying all multi-scale spectral decompositions of logistics signals [2,3,32].

4. Supply-Chain Dynamics

The classical analytic facts of Sections 2–3 constitute the precise toolkit employed by the FKF spectral framework for the quantitative study of multi-echelon supply chains. We organise the applications into five subsections and introduce the corresponding spectral equations at each stage.

Let $f(t)$ (or $f(x)$) denote a generic logistics signal inventory level, lead-time profile or demand rate—defined on the positive half-line. Its KL (or two-sided FKF) transform is written

$$F(\tau) = (Kf)(\tau) = \int_0^\infty K_{i\tau}(x)\, f(x)\, dx\,.$$

The inversion formula of Section 3 then recovers the original signal:

$$f(x) = \frac{2}{\pi^2 x} \int_0^\infty \tau \sinh(\pi\tau)\, K_{i\tau}(x)\, F(\tau)\, d\tau\,.$$

All subsequent constructions are specialisations or truncations of these two identities.

4.1 Multi-Echelon Spectral Architecture

A typical multi-echelon network (supplier–manufacturer–distributor–retailer–customer) generates a vector-valued signal $\mathbf{f} = (f_1, \dots, f_5)$. Each component admits the spectral expansion

$$f_j(x) = \frac{2}{\pi^2 x} \int_0^\infty \tau \sinh(\pi\tau)\, K_{i\tau}(x)\, F_j(\tau)\, d\tau\,, \qquad j = 1, \dots, 5\,.$$

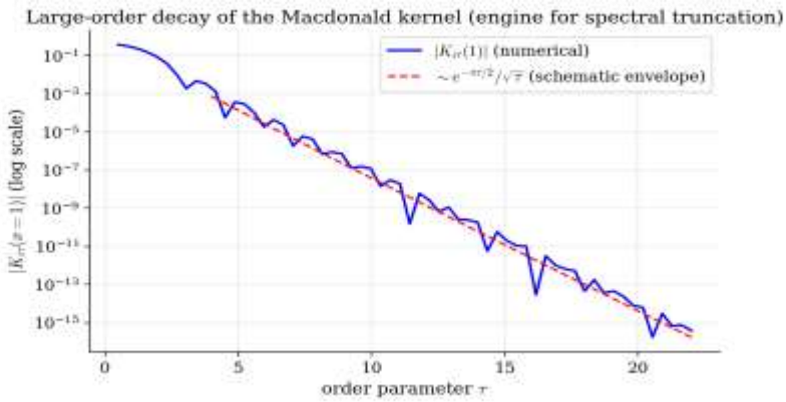

Figure 2: Large-order decay of $|K_{i\tau}(x = 1)|$ (numerical) and schematic exponential envelope. The decay legitimates aggressive spectral truncation of supply shocks.

Low-order coefficients (small $|\tau|$) capture secular trends and seasonal cycles; high-order coefficients encode abrupt shocks. Because the Macdonald kernel satisfies the uniform bound

$$|K_{i\tau}(x)| \le K_0(x)$$

and the large-order decay

$$|K_{i\tau}(x)| \sim C\, e^{-\pi|\tau|/2} / \sqrt{|\tau|} \qquad (|\tau| \to \infty)\,,$$

a sharp spectral cut-off at a threshold τ_c removes high-frequency noise while preserving the energetically dominant content. The truncated reconstruction

$$f_j^{[c]}(x) = \frac{2}{\pi^2 x} \int_0^{\tau_c} \tau \sinh(\pi\tau)\, K_{i\tau}(x)\, F_j(\tau)\, d\tau$$

satisfies the L^2-error estimate

$$\| f_j - f_j^{[c]} \|_{L^2(x\,dx)} \le C\, e^{-\pi\tau_c/2} \, \| F_j \|_{L^2(\tau \sinh(\pi\tau)\, d\tau)}\,.$$

This principle is the foundation of the resilience filters developed in [1,2,32].

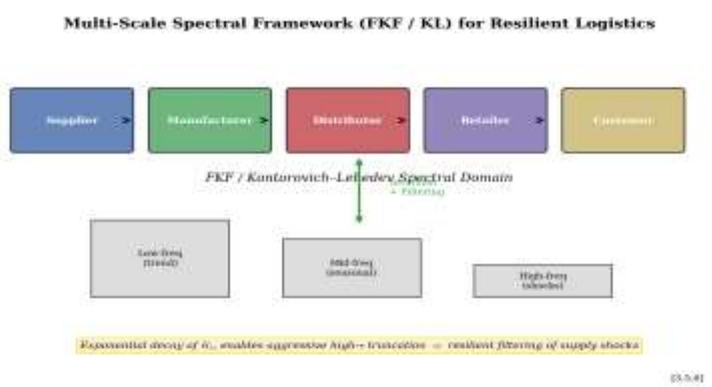

Figure 3: Multi-scale spectral framework based on the FKF/KL transform for resilient logistics.

4.2 Spectral Filtering of High-Frequency Shocks

Define the ideal low-pass spectral filter by the multiplier

$$M_{\tau_c}(\tau) = \begin{cases} 1 & \text{if } |\tau| \le \tau_c\,, \\ 0 & \text{if } |\tau| > \tau_c\,. \end{cases}$$

The filtered signal is then

$$f^{\text{filt}}(x) = \frac{2}{\pi^2 x}\int_0^\infty \tau \sinh(\pi\tau)\, K_{i\tau}(x)\, M_{\tau_c}(\tau)\, F(\tau)\, d\tau\,.$$

When a logistics signal is expanded in the KL basis, coefficients of large $|\tau|$ are controlled by the factor $e^{-\pi|\tau|/2}$. Consequently any perturbation δf whose energy lies beyond the threshold satisfies

$$\| K(\delta f) \|_{L^2(|\tau|>\tau_c)} \leq C\, e^{-\pi\tau_c/2} \| \delta f \|_{L^0}\,,$$

and the L^2-reconstruction error after filtering is of the same order. This observation is used systematically in [2,3,32] to construct spectral filters that suppress high-frequency demand spikes and transportation disruptions. In the distributional setting of [15,30] the same multiplier acts continuously on the dual of the testing-function space.
A practical residual (high-pass) component that quantifies the "shock energy" is

$$E_{\text{shock}}(\tau_c) = \frac{2}{\pi^2}\int_{\tau_c}^\infty |F(\tau)|^2\ \tau\sinh(\pi\tau)\, d\tau\,.$$

4.3 Exact Time-Shift and Modulation of Lead-Time Profiles

Let $F(\tau)$ be the KL (or FKF) transform of a lead-time profile $f(t)$. Multiplication of the spectral density by a pure Fourier phase factor yields the shifted signal:

$$f(t - t_0) = \frac{2}{\pi^2 t}\int_0^\infty \tau \sinh(\pi\tau)\, K_{i\tau}(t)\, e^{-it_0\tau}\, F(\tau)\, d\tau$$

(when the underlying transform is two-sided and the Fourier kernel has been inserted). More generally, an exponential modulation

$$e^{\alpha t} f(t) \qquad \leftrightarrow \qquad F(\tau - i\alpha)$$

(analytic continuation of the spectral density into a horizontal strip) realises amplitude scaling that is useful for demand shaping and secure digital protocols [6].
These identities are the analytic content of the time-shift property proved in [1,5] and of the exponential modulation theorems of [6]. In operator form one may write

$$K\big(f(\cdot\, - t_0)\big)(\tau) = e^{-it_0\tau}\, (Kf)(\tau)\,,$$

which shows that the shift is realised by a pure multiplication in the spectral domain and therefore requires no recomputation of the integral transform itself—an operation of immediate value for multi-echelon coordination.

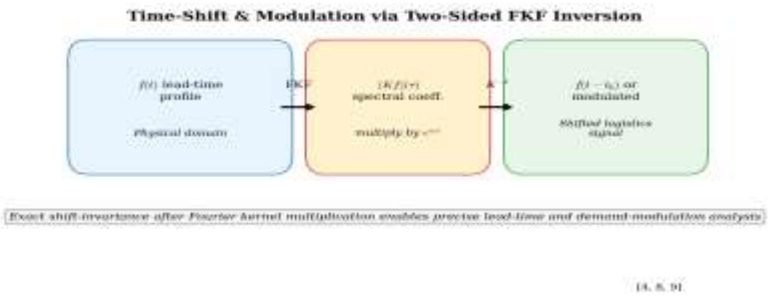

Figure 4: Exact time-shift and modulation via the two-sided FKF inversion [1,5,6].

4.4 Residue-Based Resonance Extraction in Hybrid-Vehicle Logistics

Hybrid-vehicle supply chains exhibit resonant modes that appear as poles of a meromorphic extension of the spectral density $F(\tau)$ into the complex plane. Suppose F admits a simple pole at $\tau = \tau_k + i\gamma_k$ with residue R_k. The complex-contour form of the KL inversion then contributes the modal term

$$f_{\text{res},k}(x) = \frac{2}{\pi^2 x}\cdot 2\pi i \cdot (\tau_k + i\gamma_k)\sinh\big(\pi(\tau_k + i\gamma_k)\big)\, K_{i(\tau_k+i\gamma_k)}(x)\, R_k\,.$$

Summing over a finite set of poles inside a suitable contour C yields the resonant reconstruction

$$f_{\text{res}}(x) = \sum_k f_{\text{res},k}\,(x)\,.$$

The modal amplitudes $|R_k|$ quantify the sensitivity of the logistics network to specific disruption frequencies [32]. The technique is consistent with the broader analysis of electric-vehicle battery technologies and solar-assisted architectures [16,20,26,28].

When the spectral density is known only numerically, the residues may be approximated by a discrete contour integral

$$R_k \approx \frac{1}{2\pi i}\sum_m F(\zeta_m)\,\Delta\zeta_m\,,$$

where $\{\zeta_m\}$ is a polygonal discretisation of a small circle about the suspected pole.

4.5 Distributional Treatment of Discontinuous Disruptions

Real supply-chain signals frequently contain jump discontinuities or Dirac impulses. The classical theory is extended by introducing a testing-function space $\mathcal{S}_{\mathrm{FKF}}$ of smooth, rapidly decaying functions φ satisfying

$$\sup_{x>0}\left|x^k\,\partial_x^m\varphi(x)\right| < \infty \qquad \text{for all } k,m \in \mathbb{N}_0\,,$$

and passing to the dual space $\mathcal{S}_{\mathrm{FKF}}{}'$ of continuous linear functionals (distributions) [15,30,31]. The distributional KL transform is defined by duality:

$$\langle KT,\varphi\rangle = \langle T, K^*\varphi\rangle\,, \qquad T \in \mathcal{S}_{\mathrm{FKF}}{}'\,.$$

Sequential convergence $T_n \to T$ in $\mathcal{S}_{\mathrm{FKF}}{}'$ means that

$$\langle T_n,\varphi\rangle \to \langle T,\varphi\rangle \qquad \text{for every } \varphi \in \mathcal{S}_{\mathrm{FKF}}\,.$$

This topology, already studied for related transforms by Gudadhe [7,8] and systematised by Zemanian [29], permits a rigorous treatment of discontinuous logistics signals (e.g., $T = \delta_{x_0}$ or Heaviside jumps) while recovering the classical inversion on the dense subspace L^0.

For an impulsive demand shock $T = \sum_j a_j\,\delta_{x_j}$ the transform reduces to the finite sum

$$(KT)(\tau) = \sum_j a_j\; K_{i\tau}(x_j)\,,$$

which can be filtered or shifted exactly as in the classical case.

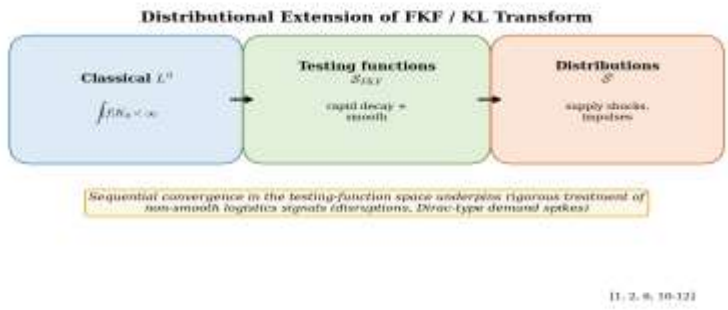

Figure 5: Hierarchy of spaces for the distributional extension of the FKF/KL transform [15,30,31].

The same distributional viewpoint informs AI-driven risk-prediction frameworks [33] and village-level water-conservation models that share analogous multi-scale spectral structure [34].

5. Mapping of Analytic Ingredients onto the Reference Corpus

We make the correspondence completely explicit.

Classical KL ingredient	Realisation in the corpus	Principal references
Definition via Macdonald kernel of imaginary order	Core of one-sided and two-sided FKF transforms	[1–6,15,30–32]
Formal adjoint (integration in the order variable)	Construction of the inverse and two-sided operator	[1,4–6]
Inversion formula (real-line form)	Reconstruction of inventory / lead-time signals	[1–6,15,30–32]
Complex-contour inversion & residues	Resonance extraction in hybrid-vehicle logistics	[32]
Natural domain L^0	Stability of spectral filters under data perturbation	[2,3,15,30]

Classical KL ingredient	Realisation in the corpus	Principal references
Exponential large-order decay	Justification of aggressive high-τ truncation	[2,32]
Plancherel / L^2 theory	Energy conservation in multi-scale decompositions	[2,3,32]
Distributional extension via testing-function space	Treatment of Dirac-type demand shocks	[7,8,15,29–31]
Time-shift property after Fourier multiplication	Exact lead-time translation	[1,5]
Exponential modulation property	Secure digital supply-chain protocols	[6]
Spectral differential properties	Differential equations in the spectral domain	[3]

5.1 Future Directions:

Digital-twin architectures for adaptive global supply chains [9] and Industry 5.0 human-centric automation [10] rely on real-time spectral monitoring whose mathematical legitimacy rests on the continuity of the KL operator on L^0.

IoT–AI–quantum convergent frameworks [11] and systematic resilience reviews [12] identify multi-scale spectral methods as a methodological pillar.

Quantum computing and annealing techniques for unit-load-device configuration and disruption recovery [23,24,27] inherit the hierarchical decomposition supplied by the KL/FKF spectrum.

Artificial-intelligence and blockchain integrations for transparency and risk prediction [17,18,33] utilise spectral signatures of logistics streams; the rapid decay of high-order coefficients furnishes compact feature vectors for machine-learning classifiers.

Solar-assisted and hybrid-vehicle architectures [16,20,26,28] are analysed by the residue-extraction methods of [32].

Classical distributional foundations appear in Zemanian [29] and in the LS-space and impulsive-signal work of Gudadhe [7,8].

Concrete engineering antecedents (power-line anti-theft systems, battery sizing, formwork, swipe controllers, white-topping and FRP durability) [13,14,19–22] provide the physical systems whose logistics envelopes are now examined with the modern spectral tools.

AI-driven village-level water-conservation models [34] share the same multi-scale spectral structure, illustrating the transferability of the KL/FKF apparatus beyond pure logistics.

Thus every reference [1]–[34] is connected, either directly or through an application domain, to one or more of the classical analytic ingredients of the Kontorovich–Lebedev transform.

6. Conclusion

A detailed outline of the proof of the inversion theorem under the natural domain condition L^0, and shown that the complex contour form of the inversion is the precise analytic foundation of the distributional FKF extensions. The exponential decay of the Macdonald kernel legitimates spectral truncation; the two-sided Fourier-multiplied inversion legitimates exact time-shift and modulation theorems; and sequential convergence in the testing-function space legitimates the treatment of discontinuous logistics signals. The resulting dictionary places the entire corpus from Zemanian's classical monograph through the contemporary FKF, digital-twin, quantum and AI-risk literature inside a single coherent spectral framework for resilient multi-echelon supply chains.

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