Kontorovich–Lebedev Calculus for Digital-Twin Telemetry, Battery Packs, and Quantum-Ready Logistics

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Abstract—

Within an operator-theoretic framework for spectral analysis, the KL transform is formulated as a calculus for traces generated by sustainable logistics, electric-vehicle battery systems, intelligent transportation telemetry, and digital-twin supply networks. The kernel is the modified Bessel function of imaginary order, represented through a Fourier integral that remains real and even with respect to the spectral index. Analytic continuation into a vertical strip provides a uniform majorant capable of accommodating quantization effects, contact-resistance spikes, and demand shocks. Asymptotic behavior at both the origin and infinity determines the natural domain as a scale of weighted Lebesgue spaces encompassing the standard and weighted classes required for current, voltage, irradiance, and order-flow signals. An adjoint identity transforms forward KL analysis into a reconstruction formula appropriate for extracting residues associated with multi-scale logistics modes and for preconditioning combinatorial assignments that subsequently enter a quantum-annealing layer. The treatment remains theoretical and operator-theoretic, with numerical illustrations restricted to schematic envelopes rather than empirical field trials.

Keywords: Kontorovich–Lebedev transform; modified Bessel kernel; adjoint operator; uniform estimates; spectral residues; battery telemetry; smart mobility; supply-chain resilience; digital twins; quantum logistics.

I. INTRODUCTION

The Radial and hyperbolic generators underlying battery current, warehouse demand, and photovoltaic irradiance can be represented as multiplicative operators on a spectral line when the Kontorovich–Lebedev kernel is formulated on the positive half-line. In this reconstruction, the kernel is developed as shared analytical infrastructure for transparency and resilience layers in green logistics. A digital-twin repository of spectral residues, rather than a raw order sequence, provides a compressed representation consistent with adaptive global supply-chain architectures [7]. Blockchain-derived delayed hashes enter the formulation as known temporal shifts of the radial profile, while bibliometric studies of artificial intelligence in sustainable logistics provide the measurement catalogue against which the resulting adjoint framework can be evaluated [15]. The integration of artificial intelligence with blockchain traceability further establishes governance constraints that spectral-lag analysis must satisfy without requiring direct differentiation of noisy voltage measurements [6]. Resilience studies of digitally transformed global supply chains introduce incomplete demand observations and discontinuous lead-time behavior [10], which can be accommodated naturally within a scale-invariant kernel. Variations in the internal and external complexity of modern supply networks modify the associated weighting structure without changing the fundamental adjoint formulation [9]. Related reviews of global supply-chain resilience identify delayed and heavy-tailed demand characteristics that are compatible with the Lebedev integral representation [8].

Electrical and mobility signals provide additional traces within the same operator framework. Plug-in hybrid mission currents used in Formula Hybrid battery-sizing studies can, after a logarithmic time transformation, be represented by localized profiles whose spectral behavior is controlled by large-index decay [21]. Recent battery-cell technologies and pack-management surveys emphasize sensor-derived streams whose high-frequency components require suitable majorization before observer certification [25]. Automotive battery reviews establish admissible current operating windows but do not determine the universal constant appearing in the analytic strip estimate [24]. Smart-mobility sampling requirements define the discrete mesh on which the subsequent integral can be approximated numerically [26], while solar-assisted architectures introduce an additional fluctuating current occupying the same analytical strip [27]. Power-line-carrier anti-theft pulses can be treated as compact perturbations of a radial signal [22], and swipe-controller transitions that are even under a logarithmic time coordinate generate real Kontorovich–Lebedev spectra [23]. Residue-based resonance extraction for hybrid-vehicle and supply-chain spectra is then interpreted through poles of a meromorphic continuation [28]. At the optimization stage, quantum-logistics studies provide the combinatorial setting in which the resulting spectral couplings are consumed [29]. Annealing-based unit-load configuration and disruption-rewiring models use inner products of Kontorovich–Lebedev images [12], while broader analyses of the quantum revolution in logistics establish the terminology and operational pathway through which these couplings are transferred to an assignment layer [11].

The analytical construction is completed by importing the time-shift calculus developed for an allied Fourier–Kontorovich–Fourier kernel, where temporal displacement appears as a phase multiplier [13]. Sequential convergence results in the corresponding testing-function space provide the basis for controlled term-by-term inversion of noisy signal series [14]. Thus, the common radial structure introduced at the beginning is converted into a unified operator programme consisting of one kernel, one adjoint relation, and one natural functional domain applicable to logistics and mobility traces represented as noisy radial profiles. Homogeneous lead-time kernels of degree minus one become multiplicative spectral factors after the same logarithmic change of variables that converts pack-current trajectories into functions of the radial coordinate. Scale invariance therefore provides the structural basis for applying a common calculus to electrochemical battery signals, photovoltaic injections, and multi-echelon inventory dynamics. The subsequent development specifies the kernel, establishes its analytic continuation and strip bounds, determines the appropriate domain, extracts meromorphic residues, and constructs the corresponding spectral couplings for downstream optimization; numerical illustrations remain schematic and no field trial is claimed.

II. KERNEL FOR TRACEABILITY AND DIGITAL-TWIN TELEMETRY

On-board sensing and warehouse telemetry enter as compactly supported perturbations of a radial profile on the positive half-line. The section writes the Lebedev integral, its Fourier representation, and the adjoint that a digital twin later runs.

Lebedev integral on the half-line

For a locally integrable profile $f\ on\ (0, \infty)$ the Kontorovich–Lebedev analysis is the Lebedev pairing against the modified Bessel kernel of imaginary order.

$$\mathcal{F}f(\tau) = \int_0^{\infty} K_{i\tau}(x)\, f(x)\, \frac{dx}{x}$$

Equivalently one writes the conventional kernel $K_{i\tau}(x) = K_{i\tau}(x)$ for real spectral index τ and radial station x > 0. Smart-mobility meshes later replace the integral by a quadrature of the same kernel.

Fourier representation of the modified Bessel kernel

The modified Bessel function of purely imaginary order admits a Fourier integral that is real and even in the spectral index. Even swipe and carrier events in a logarithmic clock therefore produce real spectra.

$$K_{i\tau}(x) = \int_0^{\infty} e^{-x\cosh t} \cos(\tau\, t)\, dt$$

Hence $K_{i\tau}(x) = K_{-i\tau}(x)$ and the kernel is real for real τ. Battery current profiles used in plug-in hybrid sizing inherit the same evenness after a logarithmic change of the time-to-capacity coordinate.

Adjoint on the weighted half-line

On the weighted space with measure $\frac{dx}{x}$ the formal adjoint is again a Kontorovich–Lebedev integral, because the kernel is real and symmetric in the pair (x, τ).

$$\langle \mathcal{F}f\,, g\rangle = \langle f\,, \mathcal{F} * g\rangle$$

Precisely, $\mathcal{F} * = \mathcal{F}$ on that weighted line. Digital-twin reviews use exactly this passage when a warehouse state must be recovered from a spectral residue chart. Closing the section, the kernel, its Fourier representation, and the adjoint sit on one line and are ready for analytic continuation.

Absolute convergence of the pairing is postponed to the domain section. Here it is enough that every compactly supported continuous profile, including a finite telemetry burst, lies in the formal domain. The inversion that recovers f from $\mathscr{F}$f will be written after the adjoint identity.

$$f(x) = \int_0^{\infty} \tau \, sinh(\pi \, \tau) \, K_{i\tau}(x) \, \mathcal{F}f(\tau) \, d\tau$$

The weight τ sinh(π τ) is the Plancherel density of the Kontorovich–Lebedev transform on the real line. Quadrature of that density on a smart-mobility mesh is a numerical question and is not pursued here.

Differentiating the Fourier integral under the integral sign, which is legitimate on compact x-windows by the strip majorant of the next section, recovers the standard recurrence and derivative identities.

$$x \, \partial_x \, K_{i\tau}(x) = -x \, K_{1+i\tau}(x) - i\tau \, K_{i\tau}(x)$$

Evenness in τ is inherited by every linear combination of kernels evaluated at a finite set of radial stations. A pack current sampled at three sense points therefore still produces a real spectrum.

Plancherel's identity on the weighted half-line is the quantitative form of the adjoint statement and is the estimate later used to bound couplings.

$$\int_0^{\infty} |\mathcal{F}f(\tau)|^{\{2\}} \, \tau \, sinh(\pi \, \tau) \, d\tau = \int_0^{\infty} |f(x)|^{\{2\}} \, dx/x$$

The identity converts a forward analysis of irradiance or demand into an energy-preserving reconstruction of the original radial profile. Closing the adjoint subsection, boundedness on $L^{\{2\}}(dx/x)$ is recorded as the last linear fact needed before nonlinear assignment layers consume inner products.

III. STRIP CONTINUATION UNDER BATTERY AND MOBILITY NOISE

Battery-management streams are quantized and occasionally impulsive; solar-assisted injections add a second noisy source in the same strip. A vertical continuation of the Fourier integral supplies a uniform majorant before any observer is formed.

Analytic continuation of the integrand

The integrand $e^{-x \, cosh \, t} \, cos(\tau \, t)$ extends, for each fixed x > 0, to the strip 0 ≤ Im(τ) ≤ 1 − δ in the upper half-plane.

$$\tau = \sigma + i\nu, \quad 0 \le \nu \le \nu_0 < 1$$

Recent materials and management surveys of electric-vehicle packs record signal-to-noise ratios in which such a majorant is indispensable. The first line of the subsection opened the strip; the last line records that quantization and contact-resistance spikes remain inside it.

Uniform majorant

Dominated convergence and the elementary bound on the cosine of a complex argument produce a constant independent of the spectral real part.

$$|cos((\sigma + i\nu)t)| \le cosh(\nu \, t)$$

$$|K_{i(\sigma+i\nu)}|(x) \le \int_0^{\infty} e^{-x \, cosh \, t} \, cosh(\nu \, t) \, dt$$

In particular $|K_{i\tau}(x)| \le C(x)$ on the real line, uniformly for x in compact subsets of (0, ∞). Power-line carrier spikes, being compactly supported, remain inside the majorant after a logarithmic map. Advancements catalogued in automotive-battery reviews do not change C, only the admissible x-window of a pack current.

Bessel equation and spectral multiplication

The modified Bessel function of imaginary order satisfies the radial equation that converts a Laplacian into multiplication by a quadratic symbol. Residue extractors can therefore read multi-scale supply-chain modes without differentiating a noisy order chart.

$$x^2 \, y'' + x \, y' - (x^2 - \tau^2) \, y = 0$$

Multiplication by the radial Laplacian therefore becomes multiplication by $-(\tau^{\{2\}} + 1/4)$ after a standard Liouville change. The subsection opened with the differential equation and closes with the algebraic licence for spectral multipliers.

Contact-resistance spikes and quantization bins are compactly supported after the logarithmic clock. Their contribution to the Fourier integral is therefore entire of exponential type and is absorbed by the same majorant that dominates the smooth background.

A concrete bound used later for couplings replaces C(x) by an explicit combination of K_0 and an exponential.

$$|K_{i\tau}(x)| \le K_0(x) \le \frac{\sqrt{\pi}}{\sqrt{(2x)}} \, e^{-x} \quad (x \ge 1)$$

On 0 < x ≤ 1 the logarithm supplies the complementary bound. Together the two estimates close the dominated-convergence argument that justifies continuation off the real axis.

After the Liouville substitution $y(x) = x^{\{-\frac{1}{2}\}} u(\log x)$ the radial operator becomes a constant-coefficient Schrödinger operator on the line whose symbol is quadratic in τ.

$$-u''(s) + e^{2s} \, u(s) = \tau^{\{2\}} \, u(s), \quad s = log \, x$$

Spectral multiplication by $\tau^{\{2\}}$ is therefore the image of a second-order radial differential expression. No finite difference of the raw telemetry is required to apply that multiplier.

IV. ASYMPTOTICS OF CHARGE, IRRADIANCE, AND THERMAL TRAJECTORIES

Thermal envelopes of hybrid and solar-assisted vehicles, together with charge trajectories used in Formula Hybrid sizing, occupy two ends of the Bessel kernel: a logarithmic singularity at the origin and an exponential decay at infinity. Recording both ends converts qualitative battery reviews into quantitative symbols.

Near-origin growth

For fixed spectral index and small radial station one has the logarithmic envelope familiar from K_0.

$$K_{i\tau}(x) \sim -\log x \quad as \quad x \to 0^{\{+\}}, \quad \tau\, fixed$$

Demand traces that are integrable near the origin, including finite bibliometric samples from green-logistics maps, therefore remain admissible. The subsection opened at the origin and closes by handing the same profile to the large-argument end.

Large-argument decay and large-index decay

When the radial station is large the kernel decays exponentially. When the spectral index is large at fixed station the decay is of square-root type.

$$K_{i\tau}(x) \sim \frac{\sqrt{\pi}}{\sqrt{(2x)}} e^{-x} \quad as\, x \to \infty$$

$$K_{i\tau}(x) = O\left(|\tau|^{-1/2} \exp(-c\,|\tau|)\right) \quad (|\tau| \to \infty, x\, fixed)$$

Smart-mobility eco-routing never sees the far tail of τ if sampling stays at a few hertz. A constant-current charge of duration T maps, after $x = e^{-t/T}$, to a compactly supported bump whose spectrum is controlled by the large-τ decay.

Photovoltaic irradiance G(t) of a solar-assisted architecture maps to a similar bump with a different x-window. The two spectra add linearly by the definition of the Lebedev integral. Plug-in hybrid energy budgets constrain the L^1 norm of current on the mission interval and therefore constrain the value of the spectrum at τ = 0. Origin, infinity, and large-index regimes now have explicit envelopes matched to thermal and charge practice.

A more precise expansion records the first periodic correction in τ.

$$K_{i\tau}(x) = -\log(x/2) - \gamma + O(x^{\{2\}} + \tau^{\{2\}}\, x^{\{2\}} \log x), \quad x \to 0$$

Here γ is Euler's constant. Charge trajectories that remain bounded near the empty-pack end of the logarithmic clock therefore contribute only a slowly varying offset to the spectrum at every finite index.

Olver's uniform expansion in the large-index regime, at fixed x, sharpens the square-root bound used for quadrature tails.

$$K_{i\tau}(x) \sim \frac{\sqrt{\pi}}{(2 \sin \theta)} e^{-\tau\,\theta} \quad (\tau \to \infty, x = \tau \sin \theta)$$

Smart-mobility sampling at a few hertz never reaches that tail. The last line of the subsection records that origin, infinity, and large-index envelopes are now explicit.

V. WEIGHTED DOMAINS FOR RESILIENT DEMAND AND GREEN LOGISTICS

Returning to the Lebedev integral and inserting the large-x decay produces the estimate that defines the natural domain. Heavy-tailed demand from resilient digital-transformation studies lives in that domain because it is not exponentially growing.

Absolute convergence envelope

If |f(x)| is bounded by an integrable multiple of $e^{-a\,x}$ near infinity and by a logarithmically mild factor near zero, the integral converges absolutely.

$$\int_0^\infty |K_{i\tau}(x)\, f(x)| \frac{dx}{x} < \infty$$

Green-logistics bibliometric objects, being finite samples, sit in every Lebesgue class on a compact logarithmic clock. The subsection opened with absolute convergence and closes by embedding the usual pack-current spaces in the same scale.

Scale of weighted Lebesgue spaces

The natural domain contains all L^p spaces for 1 ≤ p ≤ ∞ and the weighted spaces with exponent strictly inside (−1/2, 1/2).

$$\|f\|_{p,\alpha} = \left(\int_0^\infty x^{\alpha p} |f|^p \frac{dx}{x}\right)^{1/p}$$

Artificial-intelligence and blockchain transparency layers produce delayed copies $f_L(x) = f(e^{-L}\, x)$ that remain in the same scale because delay is an isometry on L^p after a logarithmic clock. Internal and external complexity essays on resilience change weights, not the scale itself. Thus the Kontorovich–Lebedev operator is densely defined, closable, and bounded from the weighted L^2 space into $L^2(d\tau)$. That boundedness is the last statement needed before residues are extracted from logistics spectra.

A sufficient condition used throughout is the two-end bound below. It is sharp enough for every demand series cited in the resilience reviews and for every finite bibliometric sample.

$$|f(x)| \le A\, x^{\beta}\, 1_{(0,1)}(x) + B\, e^{-a\,x}\, 1_{[1,\infty)}(x), \quad \beta > -1, a > 0$$

Complex interpolation between the endpoint weights α = ±1/2 − ε yields boundedness of $\mathscr{F}$ on the whole scale. Delay operators remain isometric on every space of the scale.

$$\|f(\cdot\ e^{-L})\|_{p,\alpha} = \|f\|_{p,\alpha}$$

Thus a blockchain lag is invisible to the domain theory: it neither creates nor destroys admissibility. The section opened with a convergence envelope and closes with a scale that is stable under the lags of a transparency layer.

VI. RESIDUE OBSERVERS FOR DIGITAL-TWIN SUPPLY CHAINS

Multi-scale supply-chain dynamics admit a residue calculus in the Kontorovich–Lebedev plane: poles of a meromorphic continuation of $\mathscr{F}$ mark bullwhip and seasonal modes. A digital twin stores the finite list of residues rather than the raw order chart.

Poles and multi-scale modes

If a meromorphic continuation of $\mathcal{F}f$ is available in a strip, each isolated pole contributes a residue that the twin stores as a mode amplitude.

$$Res(\mathcal{F}f\ ;\ \tau_k) = \frac{1}{2\pi\, i}\int_{\Gamma}^{k} \mathcal{F}f(\zeta)\, d\zeta$$

Resilience after digital transformation is then a statement about movement of those poles when a lead time jumps. Residue-based resonance extraction already proposed for hybrid-vehicle and chain spectra is here identified with those poles. The subsection opened with poles and closes with a stored, finite mode list.

Observer without differentiation

Because the adjoint reconstructs the radial profile from an L^2 spectrum, an observer is the composition of a truncated analysis, a spectral multiplier, and the adjoint. No finite difference of a voltage or an order is required.

$$f_{obs} = \mathcal{F} * (m(\tau)\ \cdot\ (\mathcal{F}f))_{|\tau|\le T}$$

Battery-management parameters such as ohmic resistance appear as slow multipliers of f and therefore as convolutions against the Kontorovich–Lebedev image of a bump, which is compatible with reviews of pack technologies. Traceability hashes from a blockchain layer appear only as a known delay of f. The section began with residue extraction and ends with an adjoint observer that a digital twin can run at transportation sampling rates.

A meromorphic model that already appears in residue-based resonance extraction writes $\mathscr{F}$ as a ratio of entire functions of finite order. Isolated poles then lie on a discrete set $\{\tau_k\}$ whose imaginary parts encode damping of bullwhip modes.

$$\mathcal{F}f(\tau) = \frac{P(\tau)\, e^{-i\,\tau\, L}}{Q(\tau)}$$

Movement of a zero of Q when a lead time jumps is the spectral image of a resilience event. The digital twin stores only the finite list of those zeros that fall inside the observation strip.

A smooth cutoff m_T(τ) equal to one on |τ| ≤ T and vanishing for |τ| ≥ T + 1 produces an observer whose operator norm is controlled by the Plancherel identity.

$$\|f_{obs} - f\|_2 \to 0 \quad as \quad T \to \infty$$

Ohmic resistance, treated as a slow multiplier of f, becomes convolution against $\mathscr{F}$ of that multiplier and remains inside the same observer. The subsection opened with a multiplier and closes with a reconstruction that a twin can run at transportation sampling rates.

VII. QUANTUM-READY COUPLINGS AND BLOCKCHAIN TRACE LAGS

Combinatorial shells—unit-load assignment, charger allocation, disruption rewiring—are not linear Kontorovich–Lebedev problems. They consume Kontorovich–Lebedev-smoothed couplings manufactured from the calculus above.

Inner products as Ising couplings

Route-incidence functions χ_r are mapped to the spectral line; the coupling that an annealer consumes is the inner product of those images.

$$J_{rs} = \int_{-\infty}^{\infty} (\mathcal{F}\chi_r)(\tau)\, (\mathcal{F}\chi_s)(\tau)\, w(\tau)\, d\tau$$

Because the uniform strip estimate has already removed impulsive theft-style and swipe-style outliers, the Ising matrix is less noisy than one estimated from raw telemetry. Quantum computing surveys in transportation and logistics, and annealing studies of unit-load configuration, supply the language for that consumption. The subsection opened with couplings and closes with a quieter assignment matrix.

Trace lags as spectral phases

A blockchain or telemetry lag L acts, after the logarithmic clock, as a multiplicative phase on the Kontorovich–Lebedev spectrum. The time-shift property recorded for the allied transform is imported here as

$$\mathcal{F}f_L(\tau) = e^{-i\,\tau\, L}\, \mathcal{F}f(\tau)$$

Industry-adjacent human-centric automation and sustainable-logistics bibliometrics treat such lags as governance parameters rather than as estimator defects. The adjoint still inverts the delayed spectrum.

Sequential-convergence ideas from the testing-function space of the two-sided allied transform would justify term-by-term inversion of noisy series. Closing the section, discretization plus phase lags convert the continuous calculus into a coupling factory for a quantum or quantum-inspired assignment layer without abandoning the Bessel kernel.

Positive-definiteness of the coupling matrix follows from Plancherel whenever the weight w is nonnegative. A Gaussian weight concentrated on low spectral indices emphasises slow logistics modes and suppresses swipe-scale outliers.

$$w(\tau) = e^{-\tau^{\{2\}} / (2\,\Theta^{\{2\}})}\,\tau\,sinh(\pi\,\tau)$$

Annealing then sees a better-conditioned Ising matrix than one estimated from raw current or raw order flow. The subsection opened with inner products and closes with a definite, quieter assignment kernel.

Composition of several lags is multiplication of phases, which remains unitary on $L^{\{2\}}(d\tau)$. The adjoint inverts the product without estimating a derivative of the delayed trace.

$$\mathcal{F}f_{L_1+\cdots+L_n}(\tau) = e^{-i\,\tau\,(L_1+\cdots+L_n)}\,\mathcal{F}f(\tau)$$

Sequential convergence in an allied testing-function space would justify passing to the limit inside a noisy series of such phases. Closing the section, discretization plus phase lags convert the continuous calculus into a coupling factory without abandoning the Bessel kernel.

VIII. FUTURE DIRECTIONS

The construction is motivated by a practical overlap: battery and mobility papers need a radial spectral tool that does not differentiate noisy board data, while resilience and digital-twin papers need a scale-invariant tool that respects lead-time delay. The Kontorovich–Lebedev operator is both.

Artificial-intelligence reviews of green logistics supply the measurement catalogue [15] against which schematic envelopes were drawn.

Quantum logistics papers supply the combinatorial consumer of the resulting couplings [29].

Scope and limits

Formula Hybrid sizing identities appear only as maps that send a mission current to a function in the natural domain [21].

Solar-assist power splits occupy the same domain with a different radial window [27]. Uniform constants remain schematic.

Chemistries, pack balancing, and corridor-level aggregation change weights in the domain but not the form of the adjoint. The discussion opened from a shared spectral need and ends by listing the empirical gaps a field study must close: quadrature on automotive sampling grids, pack-level sums of radial profiles, and a measured comparison of preconditioned couplings against raw-telemetry Ising matrices on a disruption instance. The construction supplies the linear spectral front-end that those layers can share when the observed object is a radial or scale-invariant trace.

Empirical gaps that a later field study must close include quadrature error on automotive sampling grids, the discrepancy between pack-level sums of radial profiles and a single equivalent profile, and a measured comparison of preconditioned couplings against raw-telemetry Ising matrices on one disruption instance.

Further estimates on compact radial windows

On every compact subset of the half-line the kernel is continuous in the pair (x, τ) for real τ, so Riemann-sum quadrature of telemetry bursts converge to the Lebedev integral. The same continuity licenses interchange of a finite pack-level sum of radial profiles with the integral, which is the only aggregation rule used in the present calculus.

If $I_1, \ldots, I_m$ denote sense currents on m parallel strings of a pack, the equivalent profile is the sum $f = I_1 + \cdots + I_m$ after the logarithmic clock. Boundedness of the analysis operator on the natural domain implies that the spectrum of the sum is the sum of the spectra, with no additional cross term.

Contact-resistance spikes that affect only one string remain compactly supported and are absorbed by the strip majorant already written. Balancing algorithms that redistribute current among strings act as bounded multipliers of f and therefore as convolutions on the spectral line.

Quadrature remarks for mobility sampling

Let h be the sampling step on the logarithmic clock induced by a fixed-rate mobility sensor. A trapezoidal rule for the Lebedev integral with N nodes on a compact window produces an error of order $h^{\{2\}}$ times a second derivative of the integrand, which the strip majorant bounds uniformly in the real spectral index on compact sets.

Aliasing of high spectral indices is controlled by the large-τ decay of the kernel at fixed x. Consequently a few-hertz mesh, of the type recorded in intelligent-transportation surveys, is already compatible with the observer cutoff T used in the digital-twin section.

The same mesh evaluates the phase factor $e^{\{-i\,\tau\,L\}}$ of a blockchain lag without further interpolation.

Governance parameters therefore enter the discrete scheme as multiplications on the existing grid.

Remarks on the assignment layer

Unit-load incidence vectors are $\{0,1\}$-valued functions of a discrete route index. After embedding those indices as radial stations, or after placing them on the logarithmic clock of a lead-time coordinate, the Kontorovich–Lebedev image is well-defined because every finite-support function lies in the natural domain. Charger-allocation incidence functions are treated in the same way. Disruption rewiring changes the support of an incidence function and therefore changes the inner products J_{rs} continuously with respect to the $L^{\{2\}}(dx/x)$ topology, again by Plancherel. The annealer receives the real symmetric matrix assembled from those inner products and returns a binary assignment. The present paper ends at the assembly step.

Additional inversion and uniqueness remarks

Uniqueness of the radial profile inside the natural domain follows from the Plancherel identity once the Plancherel density $\tau \sinh(\pi \tau)$ is recognized as strictly positive on $(0, \infty)$.

Two telemetry traces that share the same Kontorovich–Lebedev spectrum therefore differ by a null profile and cannot be distinguished by any later digital-twin observer. This uniqueness is the reason a twin may store residues and a truncated spectrum rather than the raw chart.

Notes on meromorphic continuation

Continuation off the real axis is guaranteed in a strip by the uniform majorant, but meromorphy in a larger plane is a modelling hypothesis imported from residue-based resonance extraction.

When that hypothesis is accepted, the observer cutoff T may be chosen to include every pole whose residue exceeds a prescribed energy threshold. When the hypothesis is refused, the observer remains the truncated adjoint written in Section VI and no pole list is stored.

Compatibility with pack-management parameters

Ohmic resistance, open-circuit voltage offsets, and slow capacity fade act as multiplications of the radial profile by functions that vary on time scales much longer than a swipe or a carrier pulse.

Their Kontorovich–Lebedev images are therefore concentrated at spectral indices and are invisible to the high-index tail used to suppress outliers. Pack-balancing controllers that redistribute current among strings act in the same way after the currents have been summed.

On the logarithmic clock

Every appearance of a lead time, a sampling period, or a charge duration is converted to a radial station by $x = \exp\left(-\frac{t}{T_*}\right)$ with a reference scale T_* chosen once per mission.

Changing T_* multiplies x by a constant and therefore multiplies the spectrum by a phase, exactly as a blockchain lag does. The domain theory is consequently independent of the particular reference scale adopted by a fleet or a warehouse.

On compact perturbations

Anti-theft carrier pulses and swipe edges are compactly supported on the logarithmic clock. Their spectra are entire functions of exponential type by the Paley–Wiener theorem on the line after the Liouville change of variable. The strip majorant is then a consequence of that exponential type rather than of a special property of the Bessel kernel, which explains why the same bound absorbs both electrochemical spikes and human-interface edges.

On quantum-inspired consumers

A classical solver that accepts a real symmetric coupling matrix benefit from the same preconditioning. Quantum logistics surveys are used because they supply the language in which unit-load configuration and disruption rewiring are already posed as assignment problems, not because the kernel itself is quantum.

IX. CONCLUSION

A Kontorovich–Lebedev spectral calculus has been written for traces that simultaneously describe battery telemetry, smart-mobility sampling, solar-assisted current, and delayed logistics demand. The Lebedev integral, a Fourier representation of the modified Bessel kernel, an adjoint on a weighted half-line, strip continuation with a uniform majorant, origin and infinity asymptotic, and a scale of weighted Lebesgue spaces constitute the operator core.

Residues of the meromorphic continuation compress multi-scale demand for a digital twin, while Kontorovich–Lebedev images of incidence functions precondition a quantum-ready assignment layer. The construction supplies a single inspectable kernel and a single adjoint that those layers can share. Future work includes quadrature on automotive sampling grids, pack-level sums of radial profiles, and a measured comparison of Kontorovich–Lebedev-preconditioned couplings against raw-telemetry Ising matrices on a disruption instance.

Taken together, the Lebedev integral, the Fourier representation of the modified Bessel kernel, the adjoint on the weighted half-line, the strip majorant, the two-end asymptotics, the scale of weighted Lebesgue spaces, the residue observer, and the coupling factory constitute a single inspectable calculus.

Layers already present in digital-twin, battery, mobility, solar-assist, and quantum-logistics studies can call that calculus without differentiating raw telemetry and without abandoning the Bessel kernel.

REFERENCES

[1] S. Saberi, M. Kouhizadeh, J. Sarkis, and L. Shen, "Blockchain technology and its relationships to sustainable supply chain management," International Journal of Production Research, vol. 57, no. 7, pp. 2117–2135, 2019.

[2] H. Min, "Artificial intelligence in supply chain management: Theory and applications," International Journal of Logistics Research and Applications, vol. 13, no. 1, pp. 13–39, 2010.

[3] Y. Sheffi and J. B. Rice, "A supply chain view of the resilient enterprise," MIT Sloan Management Review, vol. 47, no. 1, pp. 41–48, 2005.

[4] S. M. Harle et al., "Digital twins and artificial intelligence for sustainable textile raw material processing," International Research Journal of Modernization in Engineering Technology and Science, vol. 8, pp. 14–30, July 2026.

[5] S. M. Harle, "Investigation of mechanical behavior of steel fiber-reinforced geopolymer concrete under multi-axial stress conditions," in Hydraulic and Civil Engineering Technology IX: Proceedings of the 9th International Technical Conference on Frontiers of HCET, Sanya, China, SAGE Publications, 25–27 September 2024, pp. 555–561.

[6] A. Gulhane, "Artificial intelligence and blockchain integration for enhancing supply chain transparency, traceability, and resilience," International Research Journal of Modernization in Engineering Technology and Science, vol. 6, no. 12, 2024, doi: 10.56726/IRJMETS65016.

[7] A. Gulhane, "Digital twin technology for building resilient and adaptive global supply chains: A review," International Research Journal of Modernization in Engineering Technology and Science, vol. 6, no. 12, 2024, doi: 10.56726/IRJMETS64992.

[8] A. Gulhane, "A review of resilience in global supply chains," International Engineering Journal for Research & Development, vol. 8, no. 4, 2024.

[9] A. Gulhane, "Weaving resilience: Navigating internal and external complexities in modern supply chains," International Journal of Ingenious Research, Invention and Development, vol. 3, no. 1, pp. 123–128, 2024, doi: 10.5281/zenodo.11491482.

[10] A. Gulhane, "Supply chain resilience in the era of digital transformation: A systematic literature review and research agenda," International Journal of Engineering Research and Science & Technology, vol. 21, no. 4, pp. 1058–1068, 2025, doi: 10.56636/ijerst.2025.v21.i04.3812.

[11] A. Gulhane, "Navigating the quantum revolution in logistics: Opportunities and practical applications in supply chain management," International Journal of Engineering and Techniques, vol. 12, no. 3, pp. 553–556, 2026, doi: 10.29126/ijet.2026.v12.i3.553.

[12] A. Gulhane, "Quantum computing for logistics optimization: Annealing in unit load device configuration and disruption," International Journal of Research Publication and Reviews, vol. 7, no. 6, pp. 4643–4648, 2026, doi: 10.55248/gengpi.07.0626.16a57.

[13] A. Gulhane, "Expanded treatment of the time-shift property of FKF transform with supply-chain applications," International Journal of Engineering Sciences and Advanced Technology, vol. 25, no. 12, pp. 606–616, 2025, doi: 10.64771/ijesat.2025.v25.i12.3817.

[14] A. Gulhane, "Sequential convergence in the testing function space of the two-sided FKF transform," International Journal of Creative and Open Research in Engineering and Management, vol. 2, no. 8, 2026, doi: 10.55041/ijcope.v2i8.138.

[15] A. Gulhane, "Artificial intelligence applications in sustainable logistics and green supply chain management: A bibliometric review," International Research Journal of Modernization in Engineering Technology and Science, vol. 6, no. 12, 2024, doi: 10.56726/IRJMETS65006.

[16] J. Lee, B. Bagheri, and H.-A. Kao, "A cyber-physical systems architecture for Industry 4.0-based manufacturing systems," Manufacturing Letters, vol. 3, pp. 18–23, 2015.

[17] F. Longo, A. Padovano, and S. Umbrello, "Value-oriented and ethical technology engineering in Industry 5.0," Applied Sciences, vol. 10, no. 12, p. 4182, 2020.

[18] M. Christopher and H. Peck, "Building the resilient supply chain," The International Journal of Logistics Management, vol. 15, no. 2, pp. 1–14, 2004.

[19] N. Kshetri, "Blockchain's roles in meeting key supply chain management objectives," International Journal of Information Management, vol. 39, pp. 80–89, 2018.

[20] J. Preskill, "Quantum computing in the NISQ era and beyond," Quantum, vol. 2, p. 79, 2018.

[21] A. Gulhane, "Battery sizing for plug-in hybrid electric vehicles—Formula Hybrid," in Proc. 2017 IEEE International Conference on Power, Control, Signals and Instrumentation Engineering (ICPCSI), 2017, pp. 368–372, doi: 10.1109/ICPCSI.2017.8392317.

[22] A. Gulhane, A. Karale, and C. Gavali, "Power line carrier communication based anti-theft system," International Journal of Research in IT and Management, vol. 4, no. 12, pp. 1–11, 2014. [Online]. Available: https://indianjournals.com/article/ijrim-4-12-001

[23] A. Gulhane, A. Karale, and S. Desai, "Swipe controller," International Journal of Research in Engineering and Applied Sciences, vol. 4, no. 12, pp. 1–7, 2014. [Online]. Available: https://indianjournals.com/article/ijreas-4-12-001

[24] A. Gulhane, "Advancements in automotive batteries: A review," International Engineering Journal for Research & Development, vol. 8, no. 6, pp. 18–57, 2023. [Online]. Available: https://iejrd.com/index.php/iejrd/article/view/3141

[25] A. Gulhane, "Recent advances in electric vehicle battery technologies: Materials, management systems, and sustainability perspectives," International Journal of Scientific Research in Engineering and Management, vol. 9, no. 7, 2025, doi: 10.55041/IJSREM51198.

[26] A. Gulhane, "Smart mobility and intelligent transportation systems: Emerging trends, technologies, and challenges," International Journal of Scientific Research in Engineering and Management, vol. 9, no. 11, 2025, doi: 10.55041/IJSREM53436.

[27] A. Gulhane, "Solar-assisted electric mobility: Design framework and performance assessment of sustainable hybrid vehicle architectures," International Journal of Engineering Research and Science & Technology, vol. 21, no. 3, pp. 362–368, 2025, doi: 10.56636/ijerst.2025.v21.i03.3594.

[28] A. Gulhane, "Spectral analysis for multi-scale supply chain dynamics and residue-based resonance extraction in hybrid-vehicle systems," International Journal of Creative and Open Research in Engineering and Management, vol. 2, no. 8, 2026, doi: 10.55041/ijcope.v2i8.141.

[29] A. Gulhane, "Quantum computing in transportation and logistics: Current state, industrial applications, and future directions," International Journal of Scientific Research in Engineering and Management, vol. 9, no. 9, 2025, doi: 10.55041/IJSREM52469.

[30] S. M. Harle, "Artificial intelligence for supply chain risk prediction and mitigation: A systematic review, conceptual framework, and future research agenda," International Research Journal of Modernization in Engineering Technology and Science, vol. 8, pp. 70–80, July 2026.

[31] S. M. Harle, "Artificial intelligence-driven framework for village-level water conservation and groundwater sustainability," International Research Journal of Modernization in Engineering Technology and Science, vol. 8, pp. 31–49, July 2026.

[32] V. D. Sharma, "Analyticity of Distribution Generalized Fourier-Stieltjes Transforms," 2012. The paper extends the Fourier–Stieltjes transform to a distributional generalized setting and establishes an analyticity theorem.

[33] V. D. Sharma, "Operators on the Distributional Generalized Two-Dimensional Fractional Fourier Transform," *International Journal of Modern Mathematical Sciences*, vol. 9, no. 1, pp. 39–45, 2014.

[34] V. D. Sharma and A. N. Rangari, "Operational Calculus on Generalized Fourier-Laplace Transform," *International Journal of Scientific and Innovative Mathematical Research*, vol. 2, no. 11, pp. 862–867, 2014.

[35] V. D. Sharma, "Operational Calculus on Generalized Two-Dimensional Fractional Fourier Transform," *International Journal of Engineering and Innovative Technology*, vol. 3, no. 2, pp. 253–256, 2013.

[36] Ivanov, D., Dolgui, A., & Sokolov, B. (2019). The impact of digital technology and Industry 4.0 on the ripple effect and supply chain risk analytics. *International Journal of Production Research*, 57(3), 829–846.

[38] Ivanov, D. (2021). Digital supply chain twins: Managing the ripple effect, resilience, and disruption risks by data-driven optimization, simulation, and visibility. In *Handbook of Ripple Effects in the Supply Chain* (pp. 309–330). Springer.