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Digital Twin Supply Chains: Coupled Sizing, Residual Spectra, and Resilient Allocation

[1]Gupta AA, [2]Shinde S
[1]Research Scholar, [2]Prof, Department of Mathematics
Government College of Engineering, Amravati, Maharashtra, India

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Abstract—
A supply-chain digital twin is a time-synchronized replica of plants, docks, lanes, inventories, and policies. This paper treats that replica as the network lift of a vehicle energy-store problem: pack polytopes, channel integrity, command latency, and solar harvest already define a feasible set; leftover charge after a stint is the same residual object as leftover pallets after an assignment. We write the twin state, an observer, a three-coordinate resilience vector, a disruption-aware assignment Hamiltonian, and a two-sided shift identity that moves delayed residue pulses without leaving the test space. The literature is organized as one regularized program size the store, keep the channel observable, read the residual spectrum, twin the flow, anneal the allocation, and attribute service movement so that delayed leftovers remain interchangeable with on-time ones.

Keywords: digital twin, supply-chain resilience, pack sizing, residual spectrum, ripple effect, quantum allocation, two-sided FKF transform, Shapley attribution

I. INTRODUCTION

Competition-hybrid and road-hybrid packs cannot be sized from ampere-hours alone [11], [14], [15]. Usable energy, peak discharge, mass, bit-error rate on the integrity channel [12], and human-to-controller latency [13] shrink the feasible set to a polytope rather than an interval. Once the vehicle is a node, that polytope is no longer a private energy budget: leftover charge after a stint, leftover heat, and leftover pallets after a dock assignment are instances of one residual process r(t) [18], [28], [29]. A digital twin is the object that is required to carry the same observer from cell voltages to warehouse gauges without a change of clock [22], [23], [24], [25]. Mobility policy and harvest close the same loop [16], [17]; quantum and combinatorial allocation score leftover load [19], [26], [27]; ledgers and green-logistics learners attribute the residual [20], [21].

Let E_{trac} denote traction energy on the prescribed cycle, E_{hotel} the hotel load, Δt the stint length, and E_{res} the reserve demanded by the safety case. Admissibility of usable energy E_{use} is the cut

$$E_{use} \geq E_{trac} + E_{hotel}\,\Delta t + E_{res}. \qquad (1)$$

Mass M and peak power P_{pk} add two further inequalities, so the design point is a vector rather than a scalar:

$$M_{pack} \leq M_{max}, \quad P_{pk} \leq P_{lim}. \qquad (2)$$

Channel integrity and command latency enter as coupled limits. If I(s; ŝ) is the mutual information between the true pack–controller state s and the broadcast ŝ, and I_{min} is the information needed for a valid state update, then

$$I(s;\,\hat{s}) \geq I_{min} = g(\mathrm{BER}, C), \qquad (3)$$

where BER is the bit-error rate and C is the effective capacity of the carrier. On an approximately Gaussian medium one may write the Shannon form

$$C = B\,log_2(1 + SNR), \qquad (4)$$

but power-line and radio traces used in anti-theft settings are impulsive, so a robust scale of the observer innovation should replace SNR whenever the vehicle leaves the laboratory bench. Latency τ_{cmd} of a swipe-style torque map is operationally equivalent to a missing energy quantum and, to first order, to a time shift of the residual r(t) treated later.

A compact scalarization used throughout is the weighted objective

$$J(\theta) = \alpha_1 (E_{use})^- + \alpha_2 M^- + \alpha_3 (\mathrm{BER})^- + \alpha_4 (\tau_{cmd})^- + \alpha_5 \rho^-, \quad (5)$$

with nonnegative weights summing to one and ρ a normalized leftover after the duty cycle. Minimizing J recovers the Formula-hybrid cut when the channel and latency weights vanish, and recovers the communication-aware pack when those weights are restored. Leftover capacity is not scrap; it is the seed of the residual spectrum.

The same leftover, written as a function r(t) on the duty-cycle clock, admits a Fourier-type band energy

$$E_\Omega[r] = \int_\Omega |R(\omega)|^2 \, d\omega, \quad (6)$$

where R = T{r} is a transform of the leftover. Peaks of E_Ω are treated as operational signatures fade bands, delayed shipments, leftover heat rather than as measurement noise. That identification is what allows a digital twin of a logistics network to reuse the pack observer.

Write the replica state as $X_k = \Phi(\hat{x}_k, z_k, u_k)$, where $\hat{x}_k$ is the cell observer, z_k a network disturbance (lane closure, harvest drought, dock mask), and u_k a policy or assignment. Coupling fails when Φ is updated on a different clock from the pack observer. Resilience is not mean flow; it is the vector

$$R = (A, B, \gamma), \quad (7)$$

of absorptive, recovery, and adaptive capacities. A schematic service path after a shock at t_0 is

$$s(t) = s_\infty - A\, e^{-\gamma(t-t_0)} + B\,(1 - e^{-\gamma(t-t_0)}), \quad (8)$$

for $t \geq t_0$. Digital transformation raises observability of (A, B, γ); it does not by itself raise them. Ledgers make the path auditable; twins make it forecastable; resilience reviews insist that all three coordinates be reported rather than a single uptime percentage.

The contribution of this article is therefore not another isolated estimator. It is a map that keeps delayed leftovers interchangeable with on-time ones, scores allocation on leftover load as well as on assignment cost, and writes every named object pack polytope, channel cut, harvest forecast, replica Φ, vector R, Hamiltonian H′, residual spectrum, and two-sided shift so that a later measurement campaign can log them on one clock. Section II reviews the source stack against that map.

II. *ENERGY-STORE SIZING AND CELL OBSERVERS*

Formula-hybrid sizing papers fix usable energy, peak discharge, and mass at once [11]. Cell-material and management reviews replace the scalar E_{use} with a state that includes fade, thermal headroom, and leftover energy after the cycle [14], [15]. Let x collect state of charge, polarization voltages, and a fade parameter. A linear observer

$$\dot{\hat{x}} = (A - KC)\,\hat{x} + Bu + Ky, \quad (9)$$

is the information geometry later reused by network twins that ingest warehouse and lane gauges [22]. The discrete Lyapunov increment of the error $e = x - \hat{x}$ contracts when (A − KC) is Schur and the process-noise covariance stays inside the design envelope:

$$V(e_{k+1}) - V(e_k) = -e_k^{\top} Q\, e_k + tr(PW), \quad (10)$$

with $Q \succ 0$. A channel that violates $I(s;\hat{s}) \geq I_{min}$ injects a packet-drop process into y and inflates the innovation. Thermal headroom $T \leq T_{max}$ is an inequality constraint, not a soft penalty, because a single over-temperature event censors the rest of the duty cycle [15].

III. MOBILITY, HARVEST, AND THE POLICY LOOP

Intelligent-transport surveys place the sized pack inside a sense–plan–act loop whose binding constraints are connectivity, safety cases, and data rights [16]. Solar-assisted architectures add a stochastic generation term G(t) to the discrete energy balance [17],

$$E_{k+1} = E_k + G(t_k) - D(t_k, \pi) - L_k, \quad (11)$$

where D is demand induced by policy π and L_k collects hotel and retransmission taxes. Pack undersizing relative to (1)–(2) is admissible only if π can forecast G and D jointly and if the channel of (3) still carries the forecast. Harvest does not cancel BER or τ_{cmd}; it only relaxes the E_{use} cut when those other limits remain feasible. A reliability statement on a horizon N is

$$\frac{1}{N}\sum_{k=0}^{N-1} 1[E_k \geq E_{res}] \geq 1 - \delta, \quad (12)$$

for a tolerance δ set by the safety case. Night stints fall back on the pack cut of Section I. Policy sensitivity can be written as a directional derivative

$$\nabla_\theta s = \frac{\partial s}{\partial \pi}\frac{\partial \pi}{\partial \theta}, \quad (13)$$

where θ parameterizes π. Large components along harvest features mean the stack is betting on G(t); those bets must be logged if solar-assist slack is to be audited. Large components along disruption masks mean the policy is already doing part of the allocator's work.

IV. DIGITAL TWINS, LEDGERS, AND THE RIPPLE EFFECT

Ivanov and co-authors define a supply-chain digital twin as a model that can represent network state at any given moment and allow complete end-to-end visibility to improve resilience and test contingency plans [32], [33]. The twin is not a visualization; it is the composition of simulation, optimization, and data analytics on a shared state. The ripple effect occurs if a disruption cannot be localized and cascades downstream into sales, service, and cost.

A minimal inventory balance inside the replica is

$$I_{i,k+1} = I_{i,k} + \sum_{j\in N_{+i}} q_{ji,k} - \sum_{\ell\in N_{-i}} q_{i\ell,k} - d_{i,k} + \xi_{i,k}, \qquad (14)$$

with shipments q, local demand d, and a disruption residual ξ. Service at node i is $S_{i,k} = \min\{1, I_{i,k}/d_{i,k}\}$ when $d_{i,k} > 0$. A first-order ripple map on a downstream path P is

$$\Delta s_P = \sum_{i\in P} \beta_i \, \xi_i \prod_{j<i} (1 - \mu_j), \qquad (15)$$

where μ_j is the local absorption (redundancy, buffer, alternative source) and β_i converts unabsorbed residue into service loss. Equation (15) is the network analogue of the leftover ρ in (5): both are signed accounts of what the official flow did not clear.

AI-plus-ledger designs treat shipment and state-of-charge traces as append-only records so that substitution events remain auditable [21]. A ledger entry at time k is a tuple $(\hat{x}_k, q_k, \mathrm{stamp}_k)$ with a hash chained to k−1. Digital-twin reviews argue that the chain is not sufficient: a live replica must carry the observer of (9) up to lane and warehouse coordinates [22]. Systematic reviews of digital-era resilience list that joint observation as an open agenda item rather than as a solved interface [23]–[25]. Recent stage-based surveys map twin services onto preparedness, resistance, rebound, and growth, and list data quality, partner unwillingness to share, skill shortages, and governance as binding barriers.

V. COMBINATORIAL ALLOCATION AND LEFTOVER-AWARE HAMILTONIANS

Unit-load configuration and disruption recovery are set-partition problems whose classical neighbourhood search misses deep local minima [27]. Quantum-logistics notes collect annealing, amplitude amplification, and hybrid variational candidates [19], [26]. A compact assignment energy for loads ℓ on devices d is

$$H(q) = \sum_{\ell,d} c_{\ell d}\, q_{\ell d} + \sum_m \lambda_m \big(g_m(q)\big)^2, \qquad (16)$$

with binary $q_{\ell d}$ and linear penalties g_m for capacity and disruption masks. Scoring annealers only on H hides leftover inventory parked on a soon-to-fail node. A disruption-aware cost used here is

$$H'(q,z) = H(q) + \mu \sum_d w_d(z)\, \rho_d(q), \qquad (17)$$

where $\rho_d(q)$ is leftover inventory on device d and $w_d(z)$ rises with the disruption mask. That leftover is the discrete analogue of ρ in (5) and of r(t) in (6). Connecting H′ to the mobility policy is straightforward on paper and rare in deployments: the assignment becomes part of the feature that π re-solves.

VI. TWO-SIDED FKF MAPS FOR DELAYED RESIDUE PULSES

Multi-scale residue extraction on hybrid-vehicle and supply traces is a transform-domain claim, not only a filter design [18]. The two-sided FKF transform T acts on a test-function space in which a time shift τ becomes a structured multiplier [28]. If r(t) is leftover capacity, delayed shipment, or leftover charge, then

$$T[r(\cdot - \tau)] = M_\tau\, T[r], \qquad (18)$$

lets a delayed leftover be moved through the same pipeline as an on-time pulse. Sequential convergence in the test-function space justifies interchanging limit and pairing when residue pulses arrive one shock at a time [29]. If r_n is a sequence of incremental leftovers and $r_n \to$ r in the test topology, then

$$\langle Tr, \psi\rangle = lim_{n\to\infty} \langle Tr_n, \psi\rangle, \qquad (19)$$

for test functions ψ used as network probes. A derivative identity of the schematic form $T\{\dot{r}\}$ = N T{r} lets fade, as a rate of leftover charge, ride the same pipeline. An integral identity moves unmatched assignment mass from H′ into a queue that ψ can probe. Annealed assignments enter as piecewise-constant q(t); a jump in q is a Heaviside step whose transform must be subtracted before E_Ω is read as a fade signature, otherwise a dock event is attributed to chemistry.

Attribution on the coupled map

Green-logistics bibliometrics show that supervised and ensemble learners already dominate published flow-prediction work [20]. Once a model f maps route, pack, and disruption features to energy, delay, or service, Shapley credits

$$\varphi_i = \sum_{S \subseteq N \setminus [i]} \frac{|S|!\,(n-|S|-1)!}{n!}[v(S \cup [i]) - v(S)], \quad (20)$$

attribute the residual to pack mass, harvest, latency, or disruption masks without breaking additivity. Tree ensembles common in the logistics corpus are the natural host for TreeSHAP; twins supply the background measure against which v(S) is estimated. A reporting convention consistent with the stack is to publish, for each explained instance, the vector φ beside the resilience vector R. Large φ_τ with small φ_E says the interface, not the pack, moved service. Large φ_q with small H says the annealer cheapened assignment while parking leftover load.

Synthesis and measurement gaps

Three gaps follow from the cited chain. First, pack sizing and cell-management reviews still publish few open duty-cycle tables that a twin could ingest without hand alignment [11], [14], [15], [22]. Second, quantum allocation notes rarely close the loop to an FKF residual, so annealed assignments are scored on H and not on E_Ω or R [19], [26], [27], [28]. Third, resilience vectors are seldom estimated with the same observer used for pack state, which breaks the information geometry of (9) [14], [23], [25]. A joint agenda is therefore: publish coupled energy–flow corpora with channel-quality stamps; score annealers on residue spectra and on R; and attach Shapley heads to twin updates so that adaptive capacity is inspectable.

None of those items requires a new physical layer. They require that the objects already named in (1)–(20) be logged on one clock and scored with one residual language. Until that happens, each layer can be locally correct and globally incomparable. The remainder of a full paper would walk a single stint through the stack pack state, channel quality, harvest forecast, assignment from a low-energy band of H′, ledger stamp, T{r}, optional shift $\mathbf{M}_\tau$, pairing against a feeder probe ψ, update of R, and storage of φ beside R so that failure modes remain localizable.

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