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Distributional Spectral Architecture for Cyber-Physical Logistics: Trusted Telemetry and Constrained Annealed Dispatch

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Abstract—

The study develops an integrated mathematical and operational architecture in which distributional integral transforms provide the analytical layer for resilient smart logistics. The framework retains the original testing-function-space construction, its seminorm topology, Fréchet-space completeness, dual-space characterization, fractional reaction–diffusion formulation, Mittag-Leffler representation, and Talbot inversion. Around this mathematical core, the study adds a genuine logistics interpretation through two-sided FKF/FKL spectral analysis, sequential convergence, time-shift and exponential-modulation behavior, residue-based resonance extraction, and shift-invariant analysis of multi-echelon lead times. The resulting spectral state is connected to digital-twin telemetry, Internet of Things sensing, green logistics indicators, blockchain traceability with hash-chain provenance, Industry 5.0 human-centric automation, swipe authorization, anti-theft signaling, hybrid-electric mobility, and quantum-annealing QUBO models for constrained Unit Load Device dispatch. The objective is not to replace the underlying equations but to show how the same distributional machinery can support heterogeneous physical and cyber-physical supply-chain signals while preserving mathematical consistency.

Keywords— shift/modulation and resonance; spectral lead-time analysis; supply-chain resilience; digital twins; green logistics; blockchain traceability; quantum logistics, annealing and QUBO; ULD; IoT/ITS; Industry 5.0; hybrid-electric and solar-assisted mobility; battery sizing.

I. INTRODUCTION

Coupled observation, energy storage, authorization hardware, and dispatch engines now share one operational timeline, so a logistics network can no longer be treated as an isolated inventory or routing process. Their state is distributed across sensors, vehicles, warehouses, energy stores, authorization devices, digital twins, and optimization engines.

The original manuscript establishes a functional-analytic basis for generalized distributional transforms and connects it with quantum algorithms, electrochemical dynamics, smart mobility, and supply-chain resilience. Battery pack sizing for plug-in hybrid duty cycles supplies a concrete energy-store observable for that basis [7]. Automotive battery reviews then place the same observable inside materials and management practice [10].

The expanded architecture interprets the FKF/FKL family as a shift-aware spectral interface. Fourier structure represents temporal oscillation and recurring lead-time behavior, while the Kontorovich–Lebedev component supplies a scale-sensitive representation for positive operational variables. Distributional extension allows singular, impulsive, discontinuous, or sparsely sampled logistics signals to remain within a rigorous dual-space formulation. Spectral residue analysis already links those modes to hybrid-vehicle resonance [14].

A second layer is resilience. A resilient supply chain must preserve feasible operation when lead times, demand, energy availability, vehicle state, authorization status, or network connectivity deviate from nominal values. Reviews of global resilience make that requirement explicit [19]. The framework therefore separates sensed state, transformed state, constraint state, and decision state. This separation is important for digital twins, because a twin should not merely visualize telemetry; it should maintain an analytically interpretable state that can be passed to optimization [18].

The third layer is sustainability and trust. Green logistics introduces energy and emissions constraints through measurable indicators rather than slogans [16]. Blockchain integration adds tamper-evident provenance for the same transactions [17]. Internal and external complexity must be navigated together if those constraints are to remain feasible after a shock [20].

Finally, the architecture incorporates intelligent transportation and Industry 5.0. Smart mobility and intelligent transportation systems provide the operational context [12]. Human-centric automation limits fully autonomous decisions by retaining authorization and intervention points, including swipe-controller logic [9] and power-line-carrier anti-theft signaling [8]. Solar-assisted hybrid architectures add physical energy bounds to the same mobility layer [13]. Recent cell and pack technologies refine those bounds at the electrochemical interface [11]. Digital transformation of resilience supplies the research agenda that joins these layers [21]. Quantum methods enter only where discrete constrained search is required [15], including logistics-scale opportunity maps [22] and Unit Load Device annealing under disruption [23].

The resulting research question is therefore: how can a distributional spectral framework preserve mathematical rigor while becoming an actionable state representation for resilient, trusted, sustainable, and quantum-constrained logistics? The following sections answer this by retaining the original equations and expanding the surrounding theory, interpretation, and systems integration.

2. ADMISSIBLE GROWTH ENVELOPES FOR HETEROGENEOUS LOGISTICS OBSERVABLES

Weighted testing spaces are introduced here so that demand pulses, lead-time paths, pack-current traces, and event logs can be compared under one family of growth and derivative controls. The testing-function space is the mathematical entry point for signals whose derivatives, growth rates, and boundary behavior must be controlled simultaneously. In logistics, such signals may represent demand pulses, lead-time trajectories, battery-current profiles, telemetry streams, or event-driven authorization records. A weighted family of seminorms makes these heterogeneous signals comparable without assuming classical smoothness alone. Plug-in hybrid battery-sizing traces are one admissible current profile for that comparison [7].

2.1 WEIGHTED REGULARITY AND ADMISSIBLE GROWTH

Let the open operational domain be understood as the state region in which time, space, scale, and system variables are observed. The original definition is retained below without modification. Its role is to bound derivatives after weighting, thereby controlling both local regularity and asymptotic behavior.

Let $\Omega \subseteq \mathbb{R}^n$ be an open domain. We define a testing function space $U_{\alpha,\beta}(\Omega)$ consisting of infinitely differentiable complex-valued functions $\phi \in C^\infty(\Omega)$ subject to specific weighted growth conditions.

Definition (The Space $U_{\alpha,\beta}$). Let $\alpha, \beta \in \mathbb{R}$ and $c > 0$ be fixed structural parameters. For every pair of multi-indices $k, m \in \mathbb{N}_0^n$, we define a family of mappings $\gamma_{k,m;\alpha,\beta}: C^\infty(\Omega) \to [0, \infty)$ by

$$\gamma_{k,m;\alpha,\beta}(\phi) = \sup_{x\in\Omega} \left| x^\alpha (1 + |x|)^\beta e^{c|x|} D^k x^m \phi(x) \right|$$

where D^k denotes the standard multi-index partial differential operator of order $|k|$,

$$D^k = \frac{\partial^{|k|}}{\partial x_1^{k_1} \cdots \partial x_n^{k_n}}, \qquad |k| = k_1 + \cdots + k_n.$$

The space $U_{\alpha,\beta}(\Omega)$ is defined as

$$U_{\alpha,\beta}(\Omega) = \{\phi \in C^\infty(\Omega) : \gamma_{k,m;\alpha,\beta}(\phi) < \infty \ \forall k, m \in \mathbb{N}_0^n\}.$$

For smart logistics, the weights can be interpreted as admissibility envelopes. Large weights penalize states that grow too rapidly, while derivative orders capture increasingly detailed fluctuations. This is useful for multi-echelon lead times because small oscillations can be separated from large structural deviations without discarding the underlying signal. Cell-level management traces from contemporary electric-vehicle packs occupy the same envelope when current and temperature are treated as weighted observables [11].

2.2 SEMINORM CONTROL AND OPERATIONAL MEANING

Seminorm axioms are recorded next because later digital-twin aggregation is legitimate only after non-negativity, homogeneity, and the triangle inequality have been verified. Automotive battery reviews justify why current, voltage, and thermal channels must be rescaled before they share one envelope [10].

Proposition (Seminorm Properties). For each fixed $k, m \in \mathbb{N}_0^n$, the functional $\gamma_{k,m;\alpha,\beta}$ is a seminorm on $U_{\alpha,\beta}(\Omega)$.

Proof. Non-negativity: By construction of the absolute value and supremum, $\gamma_{k,m;\alpha,\beta}(\phi) \geq 0$ for all $\phi \in U_{\alpha,\beta}(\Omega)$.

Absolute Homogeneity: For any scalar $\lambda \in \mathbb{C}$,

$$\gamma_{k,m;\alpha,\beta}(\lambda\phi) = \sup_{x\in\Omega} |x^\alpha (1+|x|)^\beta e^{c|x|} D^k x^m (\lambda\phi)(x)| = |\lambda|\, \gamma_{k,m;\alpha,\beta}(\phi).$$

Triangle Inequality: By linearity of differentiation D^k and subadditivity of the supremum norm,

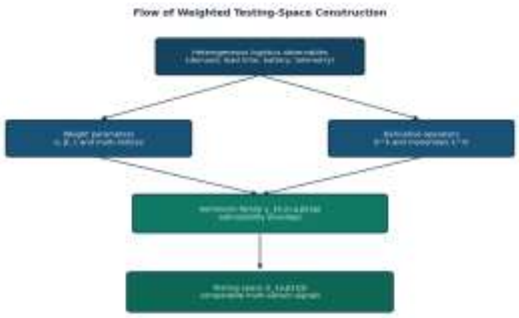
$$D^k x^m (\phi + \psi)(x) = D^k x^m \phi(x) + D^k x^m \psi(x).$$

Applying the triangle inequality for complex numbers and taking the supremum yields

$$\gamma_{k,m;\alpha,\beta}(\phi + \psi) \leq \gamma_{k,m;\alpha,\beta}(\phi) + \gamma_{k,m;\alpha,\beta}(\psi).$$

Thus, $\gamma_{k,m;\alpha,\beta}$ defines a valid seminorm.

The seminorm properties provide the minimum consistency conditions needed before a transform is extended distributionally. In an operational setting, non-negativity corresponds to measurable deviation magnitude, absolute homogeneity supports rescaling across sensors, and the triangle inequality allows independent perturbations to be aggregated. These properties are especially relevant when digital-twin telemetry combines signals with different units and amplitudes [18]. Solar-assisted hybrid architectures further require that photovoltaic and traction-battery channels remain comparable after that rescaling [13].



Weighted testing-space construction from heterogeneous observables to the admissible envelope $U_{\alpha,\beta}(\Omega)$.

The section therefore closes by treating the weighted envelope as the common gate through which battery, mobility, and warehouse signals must pass before any spectral comparison is attempted.

3. FRÉCHET COMPLETENESS AND DUAL REPRESENTATION OF DISRUPTION EVENTS

Completeness is required before a live telemetry stream can be called convergent rather than merely smooth on a dashboard. A transform framework is useful only if convergence is meaningful. The Fréchet structure supplies a complete metric topology generated by countably many seminorms, while the dual space supplies continuous linear functionals capable of representing generalized signals. This section retains the original completeness and duality arguments and extends their interpretation to sequentially arriving logistics data. Intelligent transportation streams are the natural sequential source for that extension [12].

3.1 FRÉCHET STRUCTURE AND CONVERGENCE

The metric generated by the countable seminorm family is the device that turns visual twin updates into a mathematically checkable Cauchy condition. Digital transformation of supply-chain resilience treats that check as part of the operating model rather than as an afterthought [21].

Theorem (Completeness of $U_{\alpha,\beta}$). Endowed with the topology generated by the countable collection of seminorms $\gamma_{k,m;\alpha,\beta}$, the testing function space $U_{\alpha,\beta}(\Omega)$ is a Fréchet space.

Proof. Since $\mathbb{N}_0^n \times \mathbb{N}_0^n$ is countable, we can enumerate the seminorms as $(\gamma_j)_{j=1}^{\infty}$. Define the metric $d: U_{\alpha,\beta} \times U_{\alpha,\beta} \to [0, \infty)$ by

$$d(\phi, \psi) = \sum_{j=1}^{\infty} 2^{-j} \frac{\gamma_j(\phi - \psi)}{1 + \gamma_j(\phi - \psi)}.$$

Let $(\phi_\nu)_{\nu=1}^{\infty}$ be a Cauchy sequence with respect to d. Then for every $j \in \mathbb{N}$ and $\epsilon > 0$, there exists $N_j(\epsilon) \in \mathbb{N}$ such that for all $\nu, \mu \geq N_j(\epsilon)$,

$$\gamma_j(\phi_\nu - \phi_\mu) < \epsilon.$$

Fix $x \in \Omega$ and multi-indices k, m. The estimate implies that $D^k x^m \phi_\nu(x)$ is a Cauchy sequence in the uniform norm over compact subsets of Ω. By the standard completeness of $C^\infty(\Omega)$ under uniform convergence of derivatives, there exists a limit function $\phi \in C^\infty(\Omega)$ such that for every multi-index k,

$$D^k x^m \phi_\nu(x) \to D^k x^m \phi(x) \quad \text{uniformly on compact sets as } \nu \to \infty.$$

To show $\phi \in U_{\alpha,\beta}(\Omega)$, fix $j = (k, m)$ and let $\epsilon > 0$. Choose N such that $\gamma_j(\phi_\nu - \phi_\mu) < \epsilon$ for all $\nu, \mu \geq N$. For any $x \in \Omega$,

$$|x^\alpha (1+|x|)^\beta e^{c|x|} D^k x^m (\phi_\nu - \phi_\mu)(x)| \leq \epsilon.$$

Taking the pointwise limit as $\mu \to \infty$ yields

$$|x^\alpha (1+|x|)^\beta e^{c|x|} D^k x^m (\phi_\nu - \phi)(x)| \leq \epsilon.$$

Taking the supremum over $x \in \Omega$ confirms that $\gamma_j(\phi_\nu - \phi) \leq \epsilon$ for all $\nu \geq N$. By the triangle inequality, $\gamma_j(\phi) \leq \gamma_j(\phi_\nu) + \epsilon < \infty$, proving $\phi \in$

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$U_{\alpha,\beta}(\Omega)$ and $\phi_\nu \to \phi$ in topology. Hence, $U_{\alpha,\beta}(\Omega)$ is complete and metric-inducible, i.e., a Fréchet space.

Sequential convergence has a direct digital-twin interpretation. A stream of telemetry snapshots can be treated as a sequence in the testing space, provided every required seminorm stabilizes. This prevents a visually smooth dashboard from being mistaken for mathematical convergence: convergence must hold across the selected derivative and growth controls. Hybrid-vehicle spectral studies already treat such sequential stabilization as a precursor to residue extraction [14].

3.2 DUAL-SPACE REPRESENTATION OF SINGULAR EVENTS

Dual functionals are now introduced so that an interruption, a current spike, or an authorization edge can enter the same calculus as a smooth lead-time path. Power-line-carrier anti-theft events are one such singular record [8].

Theorem (Structure of the Dual Space $U_{\alpha,\beta}'$). A linear functional f on $U_{\alpha,\beta}(\Omega)$ belongs to the dual space $U_{\alpha,\beta}'(\Omega)$ if and only if there exist a positive constant $C > 0$ and a non-negative integer $N \in \mathbb{N}_0$ such that

$$|\langle f, \phi \rangle| \le C \max_{|k|,|m|\le N} \gamma_{k,m;\alpha,\beta}(\phi) \qquad \forall \phi \in U_{\alpha,\beta}(\Omega).$$

Proof. Assume the inequality holds. If a sequence $\phi_\nu \to 0$ in $U_{\alpha,\beta}(\Omega)$, then $\gamma_{k,m;\alpha,\beta}(\phi_\nu) \to 0$ for all $|k|, |m| \le N$. Consequently, $|\langle f, \phi_\nu \rangle| \le C\max \gamma_{k,m;\alpha,\beta}(\phi_\nu) \to 0$, establishing continuity at the origin. By linearity, f is continuous on $U_{\alpha,\beta}(\Omega)$.

Conversely, let $f \in U_{\alpha,\beta}'(\Omega)$. By functional continuity, $f^{-1}\{z \in \mathbb{C} : |z| < 1\}$ is an open neighborhood of $0 \in U_{\alpha,\beta}(\Omega)$. Thus, there exist $\delta > 0$ and an integer $N \ge 0$ such that the set

$$V = \left\{ \phi \in U_{\alpha,\beta}(\Omega) : \max_{|k|,|m|\le N} \gamma_{k,m;\alpha,\beta}(\phi) < \delta \right\}$$

satisfies $f(V) \subseteq \{z \in \mathbb{C} : |z| < 1\}$. For any arbitrary $\phi \in U_{\alpha,\beta}(\Omega) \backslash \{0\}$, define

$$\psi(x) = \frac{\delta}{2 \max_{|k|,|m|\le N} \gamma_{k,m;\alpha,\beta}(\phi)} \phi(x).$$

Clearly, $\psi \in V$, which implies $|\langle f, \psi \rangle| < 1$. By linearity,

$$\left\langle f, \frac{\delta \phi}{2\max \gamma_{k,m;\alpha,\beta}(\phi)} \right\rangle < 1 \Longrightarrow |\langle f, \phi \rangle| \le \frac{2}{\delta} \max_{|k|,|m|\le N} \gamma_{k,m;\alpha,\beta}(\phi).$$

Setting $C = 2/\delta$ completes the proof.

The dual space is particularly important for disruption analysis. A sudden shipment interruption, a sensor spike, a battery-current impulse, or an authorization transition may not behave like an ordinary smooth function. A continuous functional on the testing space provides a principled way to represent such events while retaining compatibility with transform operations. Swipe authorization edges are recorded in the same dual class [9]. Weaving internal and external complexity into one disruption account then uses those functionals as event markers rather than as narrative labels [20].

Completeness of the testing space and dual representation of singular logistics events.

Convergence across seminorms and representation of singular events together give the twin a stopping rule and an event calculus before any spectral transform is applied.

4. TWO-SIDED SPECTRAL DECOMPOSITION OF MULTI-ECHELON OPERATIONAL SIGNALS

Fourier structure and the Kontorovich–Lebedev factor are paired in this section so that recurring delay and positive operational scale can be read from one transformed state. The central spectral contribution of the reconstructed framework is the use of Fourier–Kontorovich–Lebedev structure for shift-sensitive and scale-sensitive logistics signals. The Fourier factor resolves temporal oscillations, periodic demand, and recurring lead-time cycles; the KL factor, through Macdonald-function behavior, provides a positive-scale representation suited to intensity, distance, and other non-negative operational variables. Residue-based extraction of those modes has already been stated for hybrid-vehicle systems [14].

4.1 TWO-SIDED TRANSFORM ARCHITECTURE

The two-sided construction is kept so that a delay of origin can be separated from a change of shape before any resilience score is computed. Digital twins are the natural host for that separation because they must compare profiles that do not share a common time origin [18].

The distributional extension permits the transform to act on generalized functions rather than only classical integrable signals. In practical terms, this allows the same analytical layer to process smooth telemetry, sparse events, impulses, and idealized disruption signals.

The time-shift property provides a direct mechanism for comparing lead-time profiles that begin at different operational origins. Exponential modulation provides a complementary mechanism for attenuation or growth in transformed coordinates. Together, they support shift-invariant spectral analysis in which timing changes are distinguished from genuine structural changes. Intelligent transportation traces supply the delayed profiles against which that invariance is tested [12].

4.2 RESONANCE AND MULTI-ECHELON LEAD-TIME INTERPRETATION

Resonance is treated as a monitor of persistent replenishment cycles rather than as a purely analytic curiosity. Global resilience reviews already insist that average lead time is an incomplete health statistic [19].

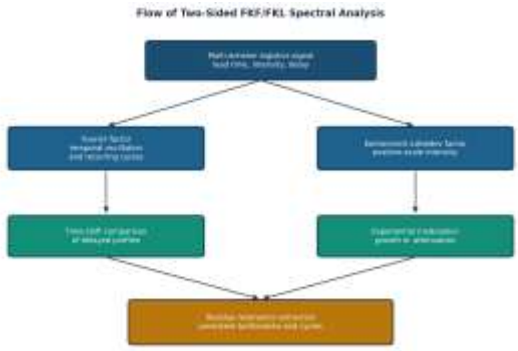
Residue-based resonance extraction identifies dominant spectral contributions associated with repeated operational patterns. In a multi-echelon network, resonance can indicate persistent replenishment cycles, synchronization between facilities, or recurrent bottlenecks. A resilience monitor can therefore track not only average lead time but also the stability of its spectral modes.

A compact operational spectral model may be expressed as a superposition of shifted modes, with each mode carrying an echelon-dependent amplitude and phase,

$$X(\omega,\kappa)=\sum_{e} A_e\,(\omega,\kappa)\,e^{-i\omega\tau_e}.$$

The corresponding resonance indicator is obtained by isolating poles or dominant modal contributions in the transformed representation, conceptually through an energy-type assembly $R(\omega)\sum_e |X_e(\omega)|^2$.

These added relations are intentionally unnumbered and are presented as professional Word equations. They do not alter any equation already present in the source manuscript. Solar-assisted hybrid duty cycles contribute additional modal content when traction and photovoltaic power share the same time base [13]. Quantum opportunity maps for logistics then inherit those modes as features of a later discrete search [22].



Two-sided FKF/FKL decomposition from a multi-echelon signal to residue resonance.

Shift, modulation, and residue analysis therefore leave the twin with a spectral state in which delay, scale, and recurrent bottleneck structure are separately visible.

5. HEREDITARY TRANSPORT, BATTERY MEMORY AND NON-LOCAL RECOVERY DYNAMICS

Fractional operators enter because recovery after a shock, and ageing inside a cell, both depend on accumulated history rather than on an instantaneous slope alone. Fractional models are useful when current system state depends on accumulated history rather than only an instantaneous derivative. This is relevant to diffusion, battery degradation, delayed transport, and resilience recovery. The original fractional PDE formulation is retained exactly and placed within the broader logistics interpretation. Recent electric-vehicle battery technology reviews motivate the electrochemical reading of that memory [11].

5.1 DISTRIBUTIONAL REACTION–DIFFUSION FORMULATION

The Caputo operator is retained so that an initial pack or network state remains meaningful while hereditary transport is still encoded. Battery sizing studies already treat initial energy as an operational datum that later dynamics must respect [7].

To illustrate the operational utility of the testing function space $U_{\alpha,\beta}(\Omega)$ and its dual $U_{\alpha,\beta}'(\Omega)$, consider the non-local time-fractional reaction-diffusion PDE

$$\frac{\partial^\gamma u(x,t)}{\partial t^\gamma}=D_x^\mu u(x,t)+f(x,t), x\in\Omega,\ t>0,$$

subject to $0<\gamma\le 1$, $1<\mu\le 2$, and initial condition $u(x,0)=u_0(x)\in U_{\alpha,\beta}'(\Omega)$.

The operator $\partial^\gamma/\partial t^\gamma$ represents the Caputo fractional derivative

$${}^{C}D_t^\gamma g(t)=\frac{1}{\Gamma(1-\gamma)}\int_0^t (t-\tau)^{-\gamma}\,g'(\tau)\,d\tau.$$

The Caputo derivative captures hereditary behavior while remaining compatible with initial-value descriptions. In battery systems, this supports the physical interpretation of memory in electrochemical processes; in logistics, the same mathematical structure can represent recovery trajectories whose present state depends on the accumulated history of disruption. Automotive battery surveys collect the empirical ageing signatures that such a memory term is intended to carry [10].

5.2 MITTAG-LEFFLER RESPONSE AND NUMERICAL INVERSION

Closed-form Mittag-Leffler kernels and Talbot inversion are kept so that a hereditary response can be evaluated even when the forcing term has no elementary Laplace image. Resilience after digital transformation is precisely such a hereditary process [21].

Derivation of the Distributional Solution.

Distributional Laplace Transformation: Applying the Laplace transform $\mathcal{L}$ on $U_{\alpha,\beta}{}'$ with respect to $t \to s$ converts the differential equation into

$$s^{\gamma} U(x,s) - s^{\gamma-1} u_0(x) = D_x^{\mu} U(x,s) + F(x,s).$$

Eigenmode Separation in Dual Space: Expanding $U(x,s)$ over spatial fractional eigenfunctions $\Phi_\lambda(x) \in U_{\alpha,\beta}{}'$ satisfying $D_x^{\mu}\Phi_\lambda(x) = -\lambda^{\mu}\Phi_\lambda(x)$ yields

$$U_\lambda(s) = \frac{s^{\gamma-1} u_{0,\lambda} + F_\lambda(s)}{s^{\gamma} + \lambda^{\mu}}.$$

Inversion via Two-Parameter Mittag-Leffler Kernels: Recalling the standard inverse Laplace transform identity

$$\mathcal{L}^{-1}\left\{\frac{s^{\gamma-\delta}}{s^{\gamma} + a}\right\} = t^{\delta-1} E_{\gamma,\delta}(-a t^{\gamma}),$$

where $E_{\gamma,\delta}$ is the Mittag-Leffler function

$$E_{\gamma,\delta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\gamma k + \delta)}.$$

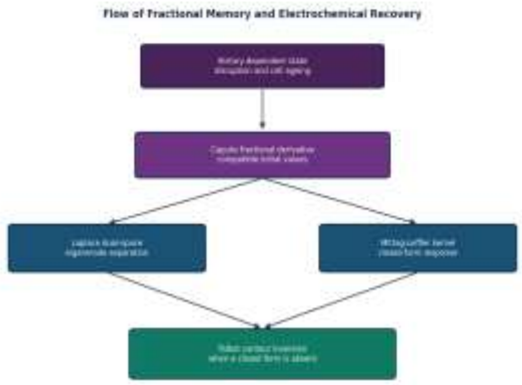
Closed-Form Representation: Taking the inverse transform gives the exact distributional solution

$$u_\lambda(t) = u_{0,\lambda} E_{\gamma,1}(-\lambda^{\mu} t^{\gamma}) + \int_0^t (t-\tau)^{\gamma-1} E_{\gamma,\gamma}(-\lambda^{\mu}(t - \tau)^{\gamma}) f_\lambda(\tau)\, d\tau.$$

Numerical Evaluation: When $f(x,t)$ lacks a closed-form Laplace transform, the numerical inversion of $U(x,s)$ along the modified Talbot contour K is computed as

$$u(x,t) = \frac{1}{2\pi i} \int_K e^{st}\, U(x,s)\, ds \approx \frac{r}{M} \sum_{k=0}^{M-1} \mathrm{Re}\left(\gamma_k e^{s_k t} U(x, s_k)\right).$$

This analytical framework provides exact solutions for non-local transport phenomena, fractional degradation dynamics in electrochemical cells, and perturbed diffusion processes in complex dynamic systems. Green logistics indicators can be attached to the same recovered trajectory when energy or emissions intensity is the monitored output [16].



Fractional memory path from a hereditary state through Caputo, Mittag-Leffler, and Talbot inversion.

Hereditary transport therefore supplies the twin with a present state that still remembers disruption and cell history when dispatch is later constrained.

6. SYNCHRONIZED TWINS, GREEN CONSTRAINTS AND HASH-CHAINED PROVENANCE

The transformed state is useful only after it is locked to live telemetry, scored against energy and emissions limits, and written into an auditable chain. The mathematical state becomes operationally useful when it is synchronized with a digital representation of the physical supply chain. A digital twin receives telemetry, estimates hidden state, applies analytical transformations, and exposes a controlled state to downstream optimization. The twin is therefore treated here as an analytical state manager rather than only a visualization platform [18].

6.1 IOT TELEMETRY AND DIGITAL-TWIN SYNCHRONIZATION

Telemetry synchronization is the step that turns a Fréchet-convergent sequence into an operational twin snapshot. Smart mobility stacks already produce the location, load, and route channels that the twin must ingest [12].

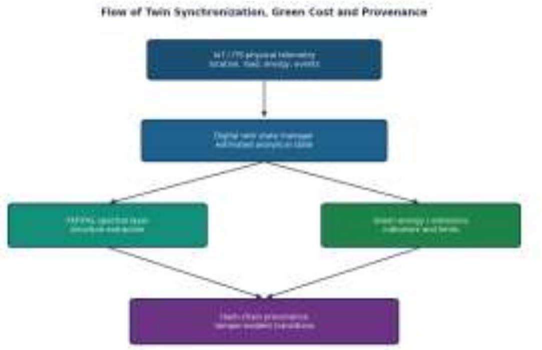
Telemetry can include location, temperature, load, battery state, route status, authorization state, power-line-carrier events, and shipment timestamps. The transform layer converts these heterogeneous streams into a common spectral representation. Sequential convergence then becomes a criterion for whether the digital twin has stabilized after a new observation sequence. Anti-theft signaling on the power line is one event channel that must appear in that snapshot [8]. Plug-in hybrid energy state is another [7].

6.2 GREEN LOGISTICS AND PROVENANCE INTEGRITY

Green cost and hash-chain provenance are added as constraints on the twin rather than as decorative labels. Artificial-intelligence reviews of sustainable logistics justify energy and emissions as first-class indicators [16]. Integration of artificial intelligence with blockchain supplies the corresponding integrity layer [17].

Green logistics is represented through measurable energy and emissions indicators rather than as a generic label. A route can be considered operationally preferable when it satisfies service constraints while reducing energy use or emissions intensity. Blockchain traceability adds a second constraint: operational records should remain auditable and tamper-evident through chained provenance.

A hash-chain model is conceptually simple: each record contains its transaction data and the digest of the previous record. Changing an earlier transaction breaks the consistency of subsequent links. This makes provenance a complement to the digital twin: the twin describes current operational state, while the provenance layer supports historical trust. Industry-scale resilience reviews treat that historical trust as part of recovery quality [19]. Internal–external complexity accounts warn that a trusted record still fails if the surrounding network geometry is ignored [20].



Twin loop linking physical telemetry, spectral state, green limits, and hash-chain provenance.

The combined layer therefore creates a closed loop: IoT observation updates the twin; the FKF/FKL representation extracts structure; green constraints evaluate environmental cost; and provenance records provide traceability for state transitions.

7. ANNEALED DISPATCH UNDER MOBILITY, ENERGY AND HUMAN-CENTRIC GATES

Quantum search is admitted only after the logistics task has been written as a discrete constrained choice, never as a substitute for authorization or energy physics. Quantum logistics is introduced only where a discrete constrained optimization problem genuinely arises. Unit Load Device configuration, vehicle assignment, route selection, charging decisions, and disruption-aware dispatch can be encoded as binary decisions [23]. A Quadratic Unconstrained Binary Optimization model combines operational cost with penalty terms that enforce capacity, assignment, timing, and energy feasibility. Surveys of quantum computing in transportation and logistics locate that encoding inside a still-classical operating stack [15].

7.1 QUBO AND QUANTUM-ANNEALING DISPATCH

The annealer is used as a candidate generator whose output remains subject to a classical feasibility audit. Opportunity-oriented accounts of quantum logistics support that restricted role [22].

Let binary variable x_i indicate selection of a feasible configuration. A generic constrained objective can be written as a cost term plus squared penalty terms. The annealer searches the resulting energy landscape, while a classical feasibility check remains responsible for interpreting the returned configuration against real operational constraints. An explicit penalty form is

$$Q(x) = x^T Q x + \lambda_1 P_{\text{capacity}}(x) + \lambda_2 P_{\text{time}}(x) + \lambda_3 P_{\text{energy}}(x).$$

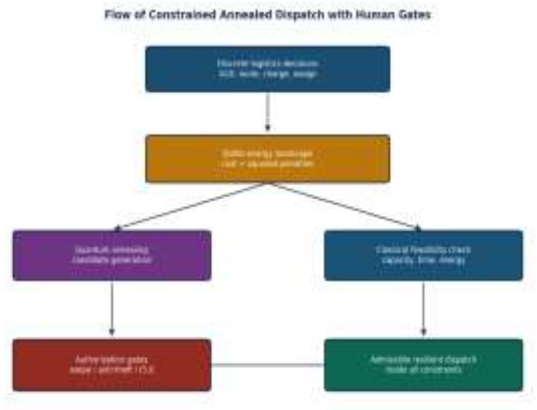
Energy penalties draw their numerical bounds from pack sizing and available traction or solar energy [13]. Digital transformation agendas for resilience treat the same penalties as part of admissible recovery, not as optional extras [21].

7.2 HUMAN-CENTRIC CONTROL AND CONSTRAINED RESILIENCE

Human gates remain outside the annealer so that a low-energy configuration cannot override authorization, theft signaling, or Industry 5.0 oversight. Swipe-controller logic is one such gate [9].

A practical architecture does not allow optimization to bypass authorization. Swipe-controller logic, power-line-carrier anti-theft signaling, Industry 5.0 human oversight, and energy limitations form explicit gates around dispatch. The hybrid-electric and solar-assisted mobility layer supplies physical feasibility bounds, including battery sizing and available energy [11].

Constrained resilience can therefore be defined as the ability to recover or continue operation while remaining inside admissible security, energy, service, and provenance constraints. This is stronger than resilience measured only as recovery speed: a fast recovery that violates authorization or energy limits is not an admissible recovery. Weaving those internal controls with external disruption is the operational meaning of constrained resilience [20].



Annealed dispatch path from binary logistics choices through QUBO search to gated admissible action.

The hybrid architecture is intentionally conservative: quantum annealing is used as a candidate-generation mechanism, whereas the digital twin, provenance layer, and human-centric authorization gates remain responsible for feasibility, trust, and accountability.

CONCLUSION

The construction leaves the testing-space, duality, FKF/FKL, fractional, Mittag-Leffler, and Talbot apparatus unchanged and uses it as a shared state layer for logistics signals. Seminorms and sequential convergence stabilize digital-twin telemetry; the dual space admits sparse or impulsive events; Fourier structure tracks recurring lead times; the Kontorovich–Lebedev factor reads positive scale; and time-shift, modulation, and residue analysis separate delay from structural change. Fractional memory then carries disruption and electrochemical history into the present state.

That state is admitted to dispatch only through digital-twin estimates, green energy and emissions limits, hash-chain provenance, human authorization, anti-theft signaling, and hybrid-electric or solar energy bounds. Quantum annealing may propose Unit Load Device configurations; it does not override those gates. Resilience is therefore feasible continuation not merely fast recovery inside security, energy, service, and trust constraints.

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