



Equitable Color Class Certified Domination Number for Specific Derived Path Graphs

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Abstract—

Consider G as (V, E) to signify a connected graph. Consider a set of color denoted as CC . An equitably colorable function $\varpi: V(G) \rightarrow CC$ is assumed. A certified dominating subset D of V is termed an equitable color class certified dominating set when the quantity of certified dominating nodes across each color class remains uniform. The smallest size of an equitable color class certified dominating set of G is referred to as the equitable color class certified domination number, represented by $\gamma_{cer}^{ECC}(G)$.

Keywords— Dominating set, certified dominating set, certified dominating number, equitable color class certified dominating set, equitable color class certified dominating number.

I. INTRODUCTION

The concepts of graph theory instead of [1]. In the context of graph theory, a graph is defined as a finite, simple, and undirected graph. The sets of vertices and edges for a graph G are denoted by $V(G)$ and $E(G)$, respectively.

II. PRELIMINARIES

Definition :

Consider G as (V, E) signify a connected graph. Consider a set of color denoted as CC . An equitably colorable function $\varpi: V(G) \rightarrow CC$ is assumed. A certified dominating subset D of V is termed an

equitable color class certified dominating set when the quantity of certified dominating nodes across each color class remains uniform. The smallest size of an equitable color class certified dominating set of G is referred to as the equitable color class certified domination number, represented by $\gamma_{cer}^{ECC}(G)$.

III. EQUITABLE COLOR CLASS DOMINATION NUMBER CONCERNING SUBDIVISION, SPLITTING, AND SHADOW GRAPHS FOR ANY PATH GRAPH

Definition 3.1: [2] A typical subdivision graph is generated by adding a new node of degree 2 to every



edge of the original graph, which is represented as $S(G)$.

Theorem 3.2: The equitable color class certified domination number for the subdivision path $S_d(P_n)$ (where $n > 5$) is $2\left\lceil \frac{n}{3} \right\rceil$.

Proof: Let $S_d(P_n)$ represent the subdivision of the path with $V(S_d(P_n)) = \{v_i, u_j: 1 \leq i \leq n, 1 \leq j \leq n-1\}$. Here, b_i denotes the nodes of path P_n , and a_j represents the nodes inserted between b_i and b_{i+1} . Thus, The vertex count for $|V(S_d(P_n))| = 2n-1$. The chromatic number χ_E of $S_d(P_n)$ is 2. Define $\varpi: V(S_d(P_n)) \rightarrow CC$ as an equitably colorable function where $CC = \{1, 2\}$. Assign color 1 to the b_i nodes and color 2 to the a_j nodes.

Case(i): $n \equiv 0 \pmod{3}$

Now we choose certified dominating nodes a_1, a_{n-1}, b_3, b_4 and if $n > 6$, given the remaining vertices $a_i (4 \leq i \leq n-1)$ and $b_i (5 \leq i \leq n-1)$, we can only choose the vertices a_{3j+5} and $b_{3j+7}, 0 \leq j \leq \left\lfloor \frac{n}{3} \right\rfloor - 3$. These nodes are recognized as certified dominating all other nodes, with the number of certified dominating nodes in each color class being equal.

Case(ii): $n \equiv 8 \pmod{3}$

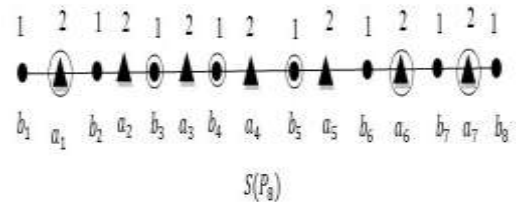
Now we choose certified dominating nodes $a_1, a_{n-1}, b_3, b_4, a_{n-2}, b_{n-3}$ and if $n > 8$, given the remaining vertices $a_i (4 \leq i \leq n-4)$ and $b_i (5 \leq i \leq n-4)$, we can only choose the vertices a_{3j+5} and $b_{3j+7}, 0 \leq j \leq \left\lfloor \frac{n}{3} \right\rfloor - 4$. These nodes are recognized as certified dominating all other nodes, with the number of certified dominating nodes in each color class being equal.

Case(iii): $n \equiv 10 \pmod{3}$

Now we choose certified dominating nodes $a_1, a_{n-1}, b_3, b_4, b_{n-5}, b_{n-4}, a_{n-3}, a_{n-2}$ and if $n > 10$, given the remaining vertices $a_i (4 \leq i \leq n-6)$ and $b_i (5 \leq i \leq n-6)$, we can only choose the vertices a_{3j+5} and $b_{3j+7}, 0 \leq j \leq \left\lfloor \frac{n}{3} \right\rfloor - 5$. These nodes are recognized as certified dominating all other nodes, with the number of certified dominating nodes in each color class being equal.

Therefore $\gamma_{cer}^{ECC}(S_d(P_n)) = 2\left\lceil \frac{n}{3} \right\rceil$.

Example 3.3: The equitable color class certified domination number of $S_d(P_8)$.



For $n = 8$, we have $\gamma_{cer}^{ECC}(S(P_n)) = 2\left\lceil \frac{n}{3} \right\rceil$, thus $\gamma_{cer}^{ECC}(S(P_8)) = 2\left\lceil \frac{8}{3} \right\rceil = 2(3) = 6$. illustrates the selection of equitable color class certified dominating nodes for $S_d(P_8)$, which are $\{a_1, b_3, b_4, b_5, a_6, a_7\}$.

Definition 3.4: [2] The Splitting graph $S(G)$ of a graph G is formed by introducing a new vertex v' for every vertex v in G , ensuring that v' is linked to every vertex that shares an edge with v .

Theorem 3.5: The equitable color class certified domination number of the splitting graph of a path, $S(P_n)$ (for n is even) is $2\left\lceil \frac{n}{4} \right\rceil$.

Proof: Let $S(P_n)$ represent the splitting graph of the path with vertices $V(S(P_n)) = \{b_i, a_i: 1 \leq i \leq n\}$. Here, b_i denotes the nodes of path P_n , and a_i represents the newly added nodes. The vertex count for $|V(S(P_n))|$ equals $2n$. The chromatic number χ_E of $S(P_n)$ is 2. Define $\varpi: V(S(P_n)) \rightarrow CC$ as an equitably colorable function where $CC = \{1, 2\}$. Assign color 1 to the nodes b_i, a_i (for odd i) and color 2 to the nodes b_i, a_i (for even i).

Case(i): $n \equiv 0 \pmod{4}$

When we choose two vertices, a_{i+1} and a_{i+2} , between any two vertices a_i and a_{i+n} (where $1 \leq i \leq n$ and n is congruent to 0 modulo 4), the certified domination condition is fulfilled. The color classes linked to the vertices in the certified domination set are identical.

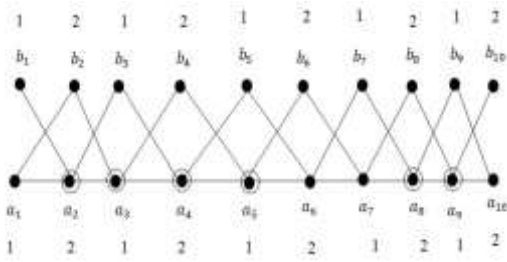
Case (ii): $n \equiv 2 \pmod{4}$

In the case where $n \equiv 2 \pmod{4}$, we have vertices a_i (for $1 \leq i \leq n$). It is essential to select four intermediate vertices: $a_{i+1}, a_{i+2}, a_{i+3}$ and a_{i+4} . These selected vertices must fulfill the certified dominating condition, and all vertices within each color class should be identical.

Subsequently, from the remaining vertices, it is adequate to choose only the two middle vertices, a_{i+1} and a_{i+2} . These selected vertices must also



satisfy the certified domination requirement and belong to the same color class.



If we choose the two vertices a_{i+1} and a_{i+2} between any two vertices a_i and a_{i+n} (where $1 \leq i \leq n$ and $n \equiv 0 \pmod{4}$), the certified domination condition is fulfilled. The color classes related to the vertices in the certified domination set are consistent.

Therefore $\gamma_{cer}^{ECC}(S(P_n)) = 2 \left\lceil \frac{n}{4} \right\rceil$.

Example 3.6: The equitable color class certified domination number of $S(P_{10})$

For $n = 10$, we have $\gamma_{cer}^{ECC}(S(P_n)) = 2 \left\lceil \frac{n}{4} \right\rceil$ thus $\gamma_{cer}^{ECC}(S(P_{10})) = 2 \left\lceil \frac{10}{4} \right\rceil = 2(3) = 6$. The process of choosing the equitable color class certified dominating nodes for $S(P_{10})$ is illustrated, which included the nodes $\{a_2, a_3, a_4, a_5, a_8, a_9\}$.

Theorem 3.7: The equitable color class certified domination number of the splitting graph of a path,

$$S(P_n) \text{ (for } n \text{ is odd)} \text{ is } \begin{cases} 3\lceil n/5 \rceil & \text{if } n \geq 9 \text{ and } n \text{ is odd} \\ 2n & \text{if } n = 3 \\ \lceil \frac{n}{2} \rceil & \text{if } n = 5 \\ \lceil \frac{n}{2} \rceil + 2 & \text{if } n = 7 \end{cases}$$

Proof: Let $S(P_n)$ represent the splitting graph of the path with vertices $V(S(P_n)) = \{b_i, a_i : 1 \leq i \leq n\}$. Here, b_i denotes the nodes of path P_n , and a_i represents the newly added nodes. The vertex count for $|V(S(P_n))|$ equals $2n$. The chromatic number χ_E of $S(P_n)$ is 3. Define $\varpi: V(S(P_n)) \rightarrow CC$ as an equitably colorable function where $CC = \{1, 2, 3\}$.

$n \equiv 0 \pmod{3}$:

Assign color 1 to the nodes b_i, a_i (for $i \equiv 1 \pmod{3}$) and color 2 to the nodes b_i, a_i (for $i \equiv 2 \pmod{3}$) and also color 3 to the node b_i, a_i (for $i \equiv 0 \pmod{3}$)

$n \equiv 1 \pmod{3}$:

Replace the assign color 2 instead of color 1 to vertex b_n . All the assigned color options mentioned above apply here.

$n \equiv 2 \pmod{3}$:

Replace the assign color 3 instead of color 2 to vertex b_n . All the assigned color options mentioned above apply here.

Case (i): $n \geq 9$ and n is odd

For each graph, the selected certified domination number is the same, but the vertices difference.

Subcase (i) : $n \equiv 9 \pmod{4}$

Now we choose certified dominating nodes b_{4i+2}, b_{4i+3} and b_{4i+4} where $(i = 0, 1, 2, \dots, \left\lceil \frac{n}{5} \right\rceil - 1)$. These nodes are certified dominating all other nodes and the number of certified dominating nodes in each color class are same.

Subcase (ii) : $n \equiv 15 \pmod{4}$

Now we choose certified dominating nodes a_{5i+2}, a_{5i+3} and a_{5i+4} where $(i = 0, 1, 2, \dots, \left\lceil \frac{n}{5} \right\rceil - 1)$.

Suppose $5i + 4 = n$ for $\left\lceil \frac{n}{5} \right\rceil - 1$, the replace the certified dominating vertex from a_{5i+4} to a_{n-3} . (or If $5i + 4 > n$ for $\left\lceil \frac{n}{5} \right\rceil - 1$, the replace the certified dominating vertex from a_{5i+3}, a_{5i+4} to a_{n-3}, a_{n-2}). These nodes are certified dominating all other nodes and the number of certified dominating nodes in each color class are same.

Subcase (iii) : $n = 11$

Now we choose certified dominating nodes as a_i ($2 \leq i \leq 10$). These nodes are certified dominating all other nodes and the number of certified dominating nodes in each color class are same.

Therefore, $\gamma_{cer}^{ECC}(S(P_n)) = 3 \left\lceil \frac{n}{5} \right\rceil$

Case (ii): $n = 3$

Assign color 1 to the nodes b_1, a_1 and color 2 to the nodes b_2, a_2 and also color 3 to the node b_3, a_3 . The certified domination condition is met only when all vertices in the $S(P_3)$ graph are chosen; these vertices are part of the same color class. Therefore, $\gamma_{cer}^{ECC}(S(P_n)) = 2n$.

Case (iii): $n = 5$

Assign color 1 to the nodes b_1, a_1, b_4, a_4 and color 2 to the nodes b_2, a_2, a_5 . Additionally color 3 should be designated for the node b_3, a_3 . Replace the assign color 3 instead of color 2 to vertex b_5 .

In this graph, it is essential to select the three central vertices - a_2, a_3 and a_4 . These vertices meet the certified domination requirement and are part of the color class same.



Therefore, $\gamma_{cer}^{ECC}(S(P_n)) = \left\lfloor \frac{n}{2} \right\rfloor$.

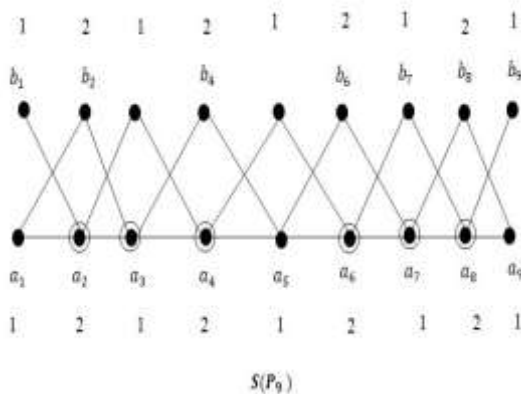
Case (iv): $n = 7$

Assign color 1 to the nodes b_1, a_1, b_4, a_4, a_7 and color 2 to the nodes b_2, a_2, b_5, a_5 . Additionally color 3 should be designated for the node b_3, a_3, b_6, a_6 . Replace the assign color 2 instead of color 1 to vertex b_7 .

Now we choose certified dominating nodes a_i, b_i where $(i \equiv 0 \pmod{2})$. These nodes are certified dominating all other nodes and the number of certified dominating nodes in each color class are same.

Therefore, $\gamma_{cer}^{ECC}(S(P_n)) = \left\lfloor \frac{n}{2} \right\rfloor + 2$.

Example 3.8: The equitable color class certified domination number of $S(P_9)$



For $n = 9$, we have $\gamma_{cer}^{ECC}(S(P_n)) = 3 \left\lfloor \frac{n}{5} \right\rfloor$, thus

$\gamma_{cer}^{ECC}(S(P_9)) = 3 \left\lfloor \frac{9}{5} \right\rfloor = 3(2) = 6$. the process of choosing the equitable color class certified dominating nodes for $S(P_9)$ is illustrated, which the included the nodes $\{a_2, a_3, a_4, a_6, a_7, a_8\}$.

Definition 3.9: [2] The Shadow graph $D_2(G)$ of a connected graph G is created by duplicating G into two copies, G' and G'' , and connecting each vertex u' in G' to the neighbors of the corresponding vertex u'' in G'' .

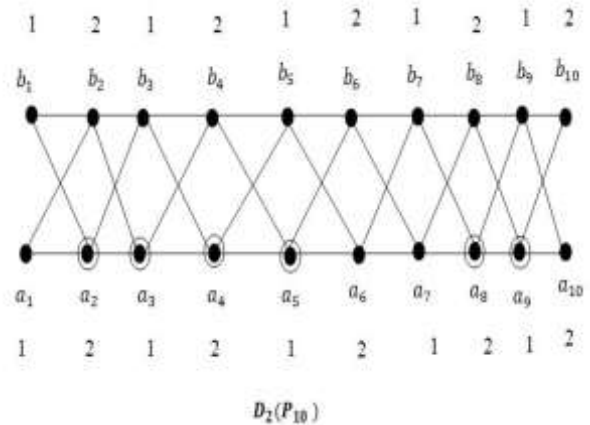
Corollary 3.10: The equitable color class certified domination number of the shadow graph of path $D_2(P_n)$ (for n is even) is $2 \left\lfloor \frac{n}{4} \right\rfloor$.

Proof: Let $D_2(P_n)$ represent the shadow graph of the path with vertices $V(D_2(P_n)) = \{b_i, a_i: 1 \leq i \leq n\}$. The nodes v_i are the vertices of path P_n , while a_i are the vertices of the duplicate path P_n . Each node b_i is linked to the neighbors of the corresponding node a_i . The vertex count for $|V(D_2(P_n))|$ equals $2n$. The chromatic number χ_E of $D_2(P_n)$ is 2. Define $\varpi: V(D_2(P_n)) \rightarrow CC$ as an equitably colorable function

where $CC = \{1, 2\}$. The process of assigning colors to the nodes and selecting dominating nodes is derived from theorem 3.5.

Therefore $\gamma_{cer}^{ECC}(S'(P_n)) = 2 \left\lfloor \frac{n}{4} \right\rfloor$.

Example 3.11: The equitable color class certified domination number of $D_2(P_{10})$



For $n = 10$, we have $\gamma_{cer}^{ECC}(D_2(P_n)) = 2 \left\lfloor \frac{n}{4} \right\rfloor$, thus

$\gamma_{cer}^{ECC}(D_2(P_{10})) = 2 \left\lfloor \frac{10}{4} \right\rfloor = 2(3) = 6$, the process of choosing the equitable color class certified dominating nodes for $D_2(P_{10})$ is illustrated, which the included the nodes $\{a_2, a_3, a_4, a_5, a_8, a_9\}$.

Corollary 3.12: The equitable color class certified domination number for any the shadow graph of path $D_2(P_n)$ (for n is odd) is

$$\begin{cases} 3 \left\lfloor \frac{n}{5} \right\rfloor & \text{if } n \geq 9 \text{ and } n \text{ is odd} \\ 2n & \text{if } n = 3 \\ \left\lfloor \frac{n}{2} \right\rfloor & \text{if } n = 5 \\ \left\lfloor \frac{n}{2} \right\rfloor + 2 & \text{if } n = 7 \end{cases}$$

Proof: Let $D_2(P_n)$ represent the shadow graph of the path with vertices $V(D_2(P_n)) = \{b_i, a_i: 1 \leq i \leq n\}$. The nodes b_i are the vertices of path P_n , while a_i are the vertices of the duplicate path P_n . Each node b_i is linked to the neighbors of the corresponding node a_i . The vertex count for $|V(D_2(P_n))|$ equals $2n$. The chromatic number χ_E of $D_2(P_n)$ is 3. Define $\varpi: V(D_2(P_n)) \rightarrow CC$ as an equitably colorable function where $CC = \{1, 2, 3\}$. The process of assigning colors to the nodes and selecting dominating nodes is derived from theorem 3.7.

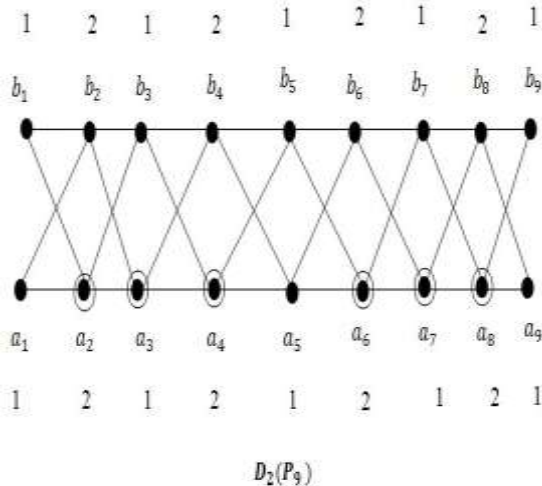


Therefore

$$\gamma_{cer}^{ECC}(D_2(P_n)) =$$

$$\begin{cases} 3\lceil n/5 \rceil & \text{if } n \geq 9 \text{ and } n \text{ is odd} \\ 2n & \text{if } n = 3 \\ \lceil \frac{n}{2} \rceil & \text{if } n = 5 \\ \lceil \frac{n}{2} \rceil + 2 & \text{if } n = 7 \end{cases}$$

Example 3.13: The equitable color class certified domination number of $D_2(P_9)$



For $n = 9$, we have $\gamma_{cer}^{ECC}(D_2(P_n)) = 3\lceil \frac{9}{5} \rceil$, thus

$\gamma_{cer}^{ECC}(D_2(P_9)) = 3\lceil \frac{9}{5} \rceil = 3(2) = 6$, the process of choosing the equitable color class certified dominating nodes for $D_2(P_9)$ is illustrated, which included the nodes $\{a_2, a_3, a_4, a_6, a_7, a_8\}$.

IV. EQUITABLE COLOR CLASS CERTIFIED DOMINATION NUMBER CONCERNING TOTAL, MIDDLE, AND CENTRAL GRAPHS FOR ANY PATH GRAPH

Definition 4.1: [2] The total graph of a graph G , indicated as $T(G)$, is defined with a vertex set that consists of $V(G) \cup E(G)$. In $T(G)$, two vertices x and y are adjacent if one of these conditions is satisfied: (i) both x and y are in $V(G)$ and x is adjacent to y in G . (ii) both x and y are in $E(G)$ and they are adjacent in G . (iii) x is in $V(G)$, y is in $E(G)$, and they are incident in G .

Theorem 4.2: The ECC Certified domination number for the total graph of a path is given by $3\lceil \frac{2n-1}{5} \rceil$.

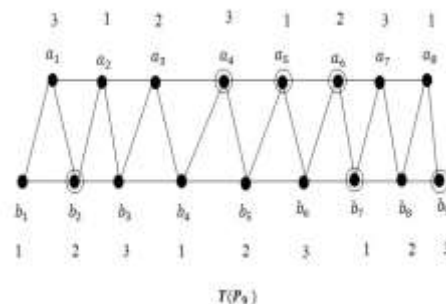
Proof: Let $T(P_n)$ represent the total graph of the path, where $V(T(P_n)) = \{b_i, a_j: 1 \leq i \leq n, 1 \leq j \leq n -$

$1\}$ Here, b_i denotes the nodes of the path P_n , and a_j are the newly introduced nodes. The vertex count for $|V(T(P_n))|$ is $2n - 1$. The edge set $E(T(P_n))$ consists of $\{b_i b_{i+1}: 1 \leq i \leq n - 1, a_j a_{j+1}: 1 \leq j \leq n - 2, b_i a_j: i = j, b_i a_j: 2 \leq i \leq n, j = i - 1\}$. The chromatic number χ_E of $T(P_n)$ is 3. We define $\varpi: V(T(P_n)) \rightarrow CC$ as an equitably colorable function, where $CC = \{1, 2, 3\}$.

Assign color 1 to the nodes b_i and a_j where $(i \equiv -2 \pmod{3} \text{ and } j \equiv -1 \pmod{3})$, color 2 to the nodes b_i and a_j where $(i \equiv -1 \pmod{3} \text{ and } j \equiv 0 \pmod{3})$, and color 3 to the nodes b_i and a_j where $(i \equiv 0 \pmod{3} \text{ and } j \equiv -2 \pmod{3})$. We then select certified dominating nodes as b_{5k+2} , for $k = 0, 1, \dots, \lceil \frac{2n-1}{15} \rceil + \lceil \frac{2n-1}{30} \rceil$ and a_{5k+4} , for $k = 0, 1, \dots, \lceil \frac{2n-1}{15} \rceil + \lceil \frac{2n-1}{30} \rceil - i$,

Where $i = 0$ if $\lceil \frac{2n-1}{15} \rceil$ is even and $i = 1$ if $\lceil \frac{2n-1}{15} \rceil$ is odd. If $n > 5k + 2$ for some k , we replace b_{5k+2} with b_n, b_{n-1} or b_{n-2} , using an equivalent CC vertex. If $n > 5k + 4$ for some k , we replace a_{5k+4} with $a_{n-1}, a_{n-2}, a_{n-3}$, or a_{n-4} , using an equivalent CC vertex. If $2n \equiv -3 \pmod{15}$, we select the final certified dominating node as a_n instead of b_{5k+2} . If $2n \equiv -2 \pmod{15}$, we select the final certified dominating node as b_{n-1} instead of a_{5k+4} . These selected nodes certified dominate all other nodes, and the number of certified dominating nodes in each color class is equal to $\lceil \frac{2n-1}{15} \rceil$. Therefore, we conclude that $\gamma_{cer}^{ECC}(T(P_n)) = 3\lceil \frac{2n-1}{15} \rceil$.

Example 4.3: The equitable color class certified domination number of $T(P_9)$.



For $n = 9$, we have $\gamma_{cer}^{ECC}(T(P_n)) = 3\lceil \frac{2(9)-1}{15} \rceil$, thus $\gamma_{cer}^{ECC}(T(P_9)) = 3\lceil \frac{2(9)-1}{15} \rceil = 3(2) = 6$, the process of choosing the equitable color class certified dominating nodes for $T(P_9)$ is illustrated, which included the nodes $\{b_2, b_7, b_9, a_4, a_5, a_6\}$



Definition 4.4: [2] The middle graph $M(G)$ associated with a graph G is characterized by a node set that consists of $V(G) \cup E(G)$. Nodes are deemed adjacent if they are connected in G or if one node is a vertex of G and the other is an edge linked to it.

Theorem 4.5: The ECC certified domination number for the middle graph of a path when $n > 5$ is expressed as $3 \left\lceil \frac{n}{6} \right\rceil$.

Proof: Let $M(P_n)$ represent the middle graph of the path, with $V(M(P_n)) = \{b_i, a_j : 1 \leq i \leq n, 1 \leq j \leq n-1\}$. Here, v_i denotes the nodes of path P_n , and b_j represents the newly introduced nodes. Thus, $V(M(P_n))$ consists of $2n-1$ nodes. The edge set $E(M(P_n))$ is defined as $\{b_i a_j : i = j, b_i a_j : j = i-1, 2 \leq i \leq n, a_j a_{j+1} : 1 \leq j \leq n-2\}$. The chromatic number χ_E of $M(P_n)$ equals 3. We define $\varpi : V(M(P_n)) \rightarrow CC$ as an equitably colorable function where $CC = \{1, 2, 3\}$. We assign color 1 to nodes b_i and a_j where $(i \equiv 0 \pmod 3 \text{ and } j \equiv -2 \pmod 3)$, color 2 to nodes b_i and a_j where $(i \equiv -1 \pmod 3 \text{ and } j \equiv 0 \pmod 3)$, and color 3 to nodes b_i and a_j where $(i \equiv -2 \pmod 3 \text{ and } j \equiv -1 \pmod 3)$. We will now select certified dominating nodes based on two scenarios.

Case 1: When n is odd

Select certified dominating nodes as a_{6i+k} and a_{n-k} (where $k = 1, 3, 5$ and $i = \{0, 1, \dots, \left\lceil \frac{n}{6} \right\rceil - 2\}$).

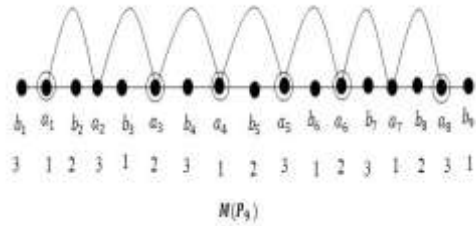
Case 2: When n is even

Select certified dominating nodes as a_{6i+k} (where $k = 1, 3, 5$ and $i = \{0, 1, \dots, \left\lceil \frac{n}{6} \right\rceil - 1\}$). For $i = \left\lceil \frac{n}{6} \right\rceil - 1$, if $6i + k > n$ when $k = 5$, replace the node a_{6i+k} with a_{n-2} . If $6i + k > n$ when $k = 3$ and 5 , replace the nodes a_{6i+3} and a_{6i+5} with a_{n-2} and a_{n-6} respectively.

In both scenarios, the selected nodes certified dominate all other nodes, and the count of certified dominating nodes in each color class remains consistent.

Therefore, we conclude that $\gamma_{cer}^{ECC}(M(P_n)) = 3 \left\lceil \frac{n}{6} \right\rceil$

Example 4.6: The equitable color class certified domination number of $M(P_9)$.



For $n = 9$, we have $\gamma_{cer}^{ECC}(M(P_n)) = 3 \left\lceil \frac{n}{6} \right\rceil$, thus $\gamma_{cer}^{ECC}(M(P_9)) = 3 \left\lceil \frac{9}{6} \right\rceil = 3(2) = 6$, the process of choosing the equitable color class certified dominating nodes for $M(P_9)$, is illustrated, which included the nodes $\{a_1, a_3, a_5, a_8, a_6, a_4\}$.

Definition 4.7: [2] The central graph $C(G)$ of a graph G is created by subdividing each edge of G once and connecting all non-adjacent nodes in G .

Theorem 4.8: The ECC certified domination number for the central graph of a path is $\left\lceil \frac{n}{2} \right\rceil$.

Proof: Let $C(P_n)$ denote the central graph of the path, where $V(C(P_n)) = \{b_i, a_j : 1 \leq i \leq n, 1 \leq j \leq n-1\}$. The nodes v_i correspond to the vertices of the path P_n , while u_j represents the newly added vertices. The vertex count for $C(P_n)$ is $|V(C(P_n))| = 2n-1$. The edges of $C(P_n)$ are given by $E(C(P_n)) = \{b_i a_j : i = j, b_i a_j : j = i-1, 2 \leq i \leq n\} \cup \{b_i b_k : b_i \text{ and } b_k \text{ are non adjacent nodes in } P_n\}$. The chromatic number χ_E of $C(P_n)$ equals $\left\lceil \frac{n}{2} \right\rceil$. We define $\varpi : V(C(P_n)) \rightarrow CC$ as an equitably colorable function where $CC = \{1, 2, 3, \dots, \left\lceil \frac{n}{2} \right\rceil\}$.

Assign color k ($1 \leq k \leq \left\lceil \frac{n}{2} \right\rceil$) to the nodes b_i ($k = \left\lceil \frac{i}{2} \right\rceil$) and assign colors to the u_i nodes according to the following two cases.

Case 1: If n is odd

Assign color k ($2 \leq k \leq \left\lceil \frac{n}{2} \right\rceil$) to the nodes a_i ($1 \leq i \leq n-2$) where $k = \left\lceil \frac{i}{2} \right\rceil + 1$ for odd i and $k = \left\lceil \frac{i}{2} \right\rceil + 2$ for even i . Assign color 1 to the node a_{n-1} .

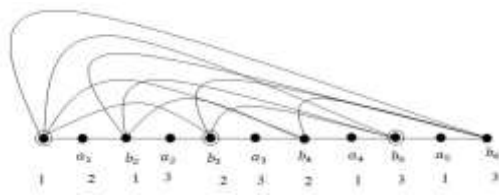
Case 2: If n is even



Assign color k ($2 \leq k \leq \lceil \frac{n}{2} \rceil$) to the nodes u_i ($1 \leq i \leq n-3$) where $k = \lceil \frac{i}{2} \rceil + 1$ for odd i and $k = \lceil \frac{i}{2} \rceil + 2$ for even i . Assign color 1 to both nodes $a_{(n-1)}$ and $a_{(n-2)}$

Next, select certified dominating nodes as b_i (where i is odd). These nodes dominate all other nodes, and the count of certified dominating nodes in each color class remains consistent. Thus, $\gamma_{cer}^{ECC}(C(P_n)) = \lceil \frac{n}{2} \rceil$.

Example 4.9: The equitable color class certified domination number of $C(P_6)$.



For $n = 6$, we have $\gamma_{cer}^{ECC}(C(P_n)) = \lceil \frac{n}{2} \rceil$, thus $\gamma_{cer}^{ECC}(C(P_6)) = \lceil \frac{6}{2} \rceil = 3$, the process of choosing the equitable color class certified dominating nodes for $C(P_6)$ is illustrated, which included the nodes $\{b_1, b_3, b_5\}$.

V. CONCLUSION

This paper provides a comprehensive analysis of the Equitable Color Class Certified Domination Number across various derived graphs of the path graph, highlighting the distinct characteristics and calculations for each type. The findings contribute valuable insights into the field of graph theory, particularly in understanding the complexities of domination numbers in different graph structures.

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