



Fluid Dynamics as 1st & 2nd Order Differential Equations

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Introduction: Let us consider the system of physical quantities that influence the dynamics. The dynamics problems that takes the variable mass:

Among many applications of Newton's second law of motion giving $F = ma$ where F is the force applied on a body of mass m and due to which there is an acceleration a in the particles of the body or the whole body.

Abstract: The ratio and proportion concept will be upgraded to an equation in which the proportion will be replaced by proportionality constant. Also, the change in one variable in this equation influences the change in the other variable in certain proportion which is designated by the proportionality constant. This rate of change is with respect to the other is observed to be the derivative. Depending on this, the first order differential equation is created and the rate of change in already changing variable is treated as the second order derivative. For the first and second order derivatives and the suitable proportionality constants, two laws are taken as examples which are shown as differential equations and the 2nd order differential equation is solved to give a meaningful conclusion.

Newton's 2nd law and its application:

In reality, it is obtained from the change in the velocity v of the body of mass m is equal to the force applied on the body. It can be abbreviated as ' mv ' is the mass multiplied with m multiplied with the

velocity v that indicates the motion in the body.

The rate of change in the motion of the body with respect to time is $\frac{d}{dt}(mv) \dots \dots (1.1)$

This is nothing but the acceleration in the body's motion nothing but the force applied on the body. $F = \frac{d}{dt}(mv) = \frac{dm}{dt}v + m\frac{dv}{dt} \dots \dots (1.2)$

But, m is the mass constant with respect to the time. So, $\frac{dm}{dt} = 0$ and change in velocity is acceleration given by $\frac{dv}{dt} = a$ and (1.2) becomes $F = ma$.

Note that when a rocket is launched, with respect to time, the fuel in the rocket gets exhausted. This shows that the mass m also changes with respect to time in certain cases. so, applying the Newton's law for the moving body with variable mass, it would be a matter of concern that momentum produced by the force F produced jointly by the body from an external and an internal force. So, if w is the relative velocity to m and so, the velocity with mass is $v + w$ added to m at the rate $\frac{dm}{dt}$, then the increasing momentum is given by

$$(v + w) \frac{dm}{dt} + F = \frac{d}{dt}(mv) \dots \dots (1.3)$$

One should not be carried away by the right side of (iii) and (ii) assuming that they are equal.



In fact, (1.2) is relating constant mass and (1.3) is dealing with momentum.

The set of experiments will supplement this idea.

In the case of rocket that burns fuel experiment, if the structural mass of the rocket is m_1 and the fuel of initial mass m_2 projected normal to the earth's surface, then the acceleration is obviously $\frac{dm}{dt} = -a$. In this, m is the variable mass of the rocket given by change in the structural mass + change in the fuel.

That is, $0 + \frac{dm_2}{dt}$ and the rocket is moving at a constant velocity b .

There is buoyance force on the rocket which may be neglected with respect to its weight. If it is the case of a parachute, then with respect to the weight of a person, the forces other than gravitational force will matter greatly. So, depending on the situation itself, we have to consider the other forces than the gravitational force while calculating the momentum.

In the present experiment, we usually calculate the burnout velocity and burnout height that are nothing but, the velocity at which the total fuel is spent out and the total height the rocket reaches.

In the case of falling the raindrop, the raindrop of the size 0 radius will keep collecting the water vapor in the atmosphere as it falls to the ground under the gravitational force and at a steady rate, the size of the raindrop increases and as a result, more mass gains more momentum and acceleration due to gravity which is almost equal to $\frac{g}{4}$. Also, if the initial radius is r_0 , then (1.3) becomes $\frac{g}{4} (1 + \frac{3r_0^4}{r^4})$
..... (1.4)

Observe that, the rain drop initially starts from rest from the clouds with almost of negligible radius and so, $F = 0$ in (1.3).

In this experiment, we have seen that increasing radius leads to increasing mass and the increasing surface area attracts increasing gravitational force. This is what the theory of relativity means and so, due to Einstein, the special theory of relativity is mentioned to be $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ in which the velocity of light is c and

the mass initially at rest denoted by m_0 .
..... (1.5)

Observe that the momentum in the space is in the opposite direction of the source and so, $w = -v$ in (iii) and as $t \rightarrow \infty$, the momentum becomes equal to the velocity of light. That is, $v \rightarrow c$.

This results in $M = m - m_0$ is the change in the mass of the particle, the relative increase in energy is considered as E and so, the total work done by the force F is equal to the energy E given by $E = \int_0^v F dx$ where x is the distance.

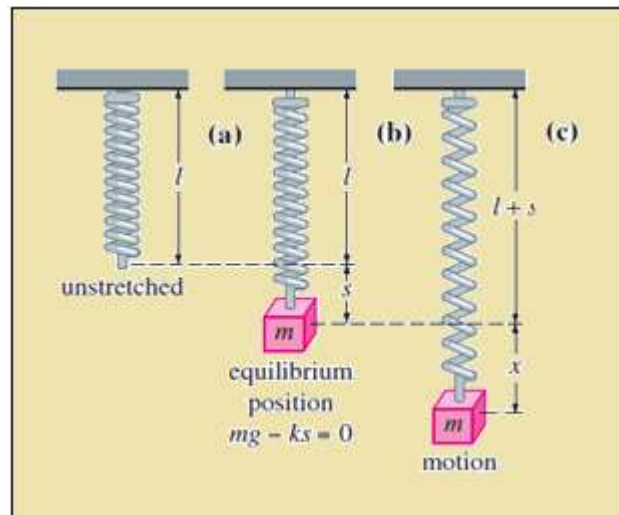
Using (ii), this further becomes $E = \int_0^v m a \cdot dx$

$$= \int_0^v \frac{d}{dt}(mv) dt = \int_0^v v d(mv) = \lim_{t \rightarrow \infty} (m - m_0) v^2 = M c^2$$

..... (1.6)

Let us slowly increase the first order differential $\frac{d}{dt}$ in the above discussion to the 2nd order derivative experimentally.

This idea is extended by Hooke's law and Newton's second law as follows



Hooke's law: let us take a spring that is suspended vertically from a beam with mass m at the free end. Usually the elongation of the spring depends on the mass. That is, different weights will stretch the spring to different levels. So, the usual or internal force or the restoration force F is already present in the spring which will be observed when the mass m is removed from the spring. It is identified above as $F = ks$ where k is the spring constant. If the mass has the weight 10 pounds, and the stretch is $\frac{1}{2}$ feet, then the spring constant is $10 = k(\frac{1}{2}) = 20 \text{ lb./ft.}$ under free conditions and only gravitational force applies, then the force is equal to the weight of the body stated by Hooke's law. (2.1)

The position of the mass at the distance s from the original free end of the spring is called the equilibrium position. At this stage, the weight of the object or mass is equal to the restoring force given by $W = ks$.

On the other hand, the actual equation of weight is $W = mg$ where g is the gravitational constant $= 32 \text{ ft./s}^2$, or 9.8 m/s^2 . The weight W is measured in slugs or kilograms. Putting the above equations together, we get $mg = ks$ (2.2)

at the equilibrium position.

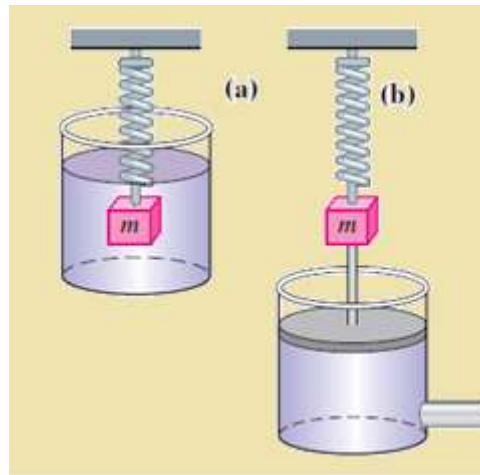
Suppose an external force displaced the mass by x from the equilibrium position, then the required restoring force is $k(s + x)$. When there are no retardation forces in the path of the motion or the free motion, then the Newton's second law for the net or the resultant force to restore the equilibrium position by the weight is

$$m \frac{d^2x}{dt^2} = -k(s + x) + mg$$

Using (2.2) in this, it becomes $m \frac{d^2x}{dt^2} = -kx$ (2.3)

The situation where no other retardation forces are existing on this motion can be termed as the free damped motion and this leads to the simple harmonic motion given by $\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x$ and writing $\frac{k}{m} = \omega^2$, the 2nd order differential equation suitable to the undamped motion $\frac{d^2x}{dt^2} = \omega^2 x$

Its auxiliary equation is $\lambda^2 + \omega^2 = 0$ and so, the Eigen values $\pm i\omega$ and the solution followed as $x(t) = c_1 \cos \omega t + c_2 \sin \omega t$ and the time period is measured in $T = \frac{2\pi}{\omega}$ seconds. The frequency is given by $f = \frac{1}{T}$ that says the number of cycles completed by the motion in one second. A cycle is the time elapsed between the lowest point to the highest point of the motion.



The figure (a) shows the spring mass system undamped motion and (b) is damped motion. The rate of change ratio and proportionality equation is

$$m \frac{d^2x}{dt^2} = -kx - \beta \frac{dx}{dt} \text{ where } \beta > 0 \text{ is the damping constant.}$$

The damping forces acting on a body are proportional to a power of the instantaneous velocity.

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x - \frac{\beta}{m} \frac{dx}{dt} \text{ in which the negative signs show that the forces are acting against each other.}$$

Thus, the suitable differential equation is $\frac{d^2x}{dt^2} + 2\lambda \frac{dx}{dt} + \omega^2 x = 0$ and 2λ is just the algebraic convenience.

The characteristic roots are $L = -\lambda \pm \sqrt{\lambda^2 - \omega^2}$

Depending on $\lambda^2 - \omega^2 > 0$ or $\lambda^2 - \omega^2 < 0$, the motion of the spring is treated as the over damped or under damped respectively.

Extending the discussion to circuits, if L is the inductance, i is the current, R is the resistance, C is the capacitance, then the voltage drop or the energy change in the circuit at the given time is given by the 1st order differential equation

$$L \frac{di}{dt} + Ri + \frac{1}{C}q = E(t) \text{ and the charge } q \text{ is the change in the inductance at the time given by } \frac{dq}{dt} = i(t).$$

This gives the 2nd order differential equation $L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C}q = E(t)$ suitable to the LRC circuit. Depending on the discriminant of the quadratic equation, the flow of current in the circuit is a damped motion or critically damped or under damped depending on $R^2 - \frac{4L}{C} > 0$ or $= 0$ or < 0 respectively.

Application: consider the boundary value problem: when $L = 1$ hertz, $R = 20$ ohms (Ω), $C = 0.005$ farads (f), energy in the system is $E(t) = 150$ volts(V), taking the boundary conditions $t > 0$, $q(0)=0$, $i(0) = 0$, what the current $t(t)$?

Substituting this information into the 2nd order differential equation

$$1 \frac{d^2q(t)}{dt^2} + 20 \frac{dq(t)}{dt} + \frac{1}{0.005}q(t) = 150$$

This is a non homogeneous 2nd order linear differential equation with constant coefficients. Applying the Laplace transforms on both sides,

$$L(q'') + 20L(q') + 200L(q) = L(150)$$

$$s^2Q(s) + s q(0) + q'(0) + 20sQ(s) - 20q(0) + 200Q(s) = 150L(1)$$

$$\text{Observe that } \frac{dq}{dt}(0) = q'(0) = i(0) = 0 \text{ and } q(0) = 0$$

$$\text{Using these in the equation, it becomes } (s^2 + 20s + 200)Q(s) = \frac{150}{s}$$

$$Q(s) = 150 \frac{1}{s(s^2 + 20s + 200)}$$



$$Q(s) = 150 \frac{1}{s((s+10)^2 + 10^2)}$$

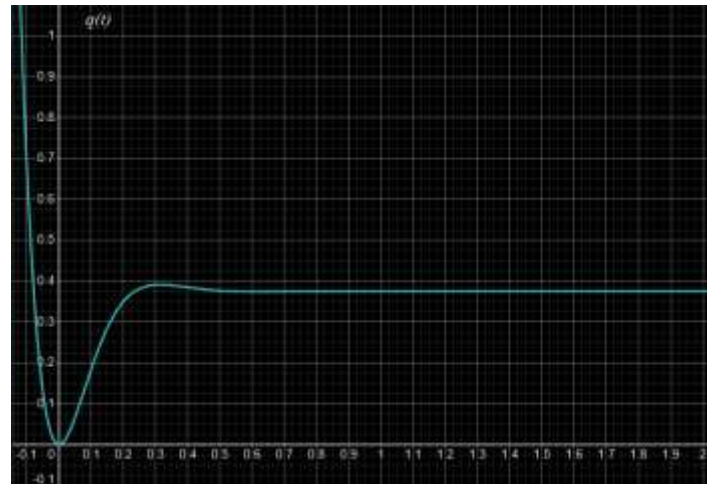
$$Q(s) = 15 \frac{10}{s((s+10)^2 + 10^2)}$$

$$Q(s) = \frac{3}{8s} - \frac{3}{8} \left\{ \frac{s+10}{(s+10)^2 + 10^2} \right\} - \frac{3}{8} \left\{ \frac{10}{(s+10)^2 + 10^2} \right\}$$

Applying inverse Laplace on both sides under the 1st shifting theorem, it gives

$$q(t) = \frac{3}{8} \{1 - e^{-10t}(\sin 10t + \cos 10t)\} \text{ volts.}$$

The graph of the wave obtained as the solution of the differential equation is



Outcome: by the given input figures of inductance, capacitance, reluctance and others, there is a vibration in the string or resonance through which the electricity is passing and this vibration will take the shape of the waves as long as the charge $q(t)$ is supplied through the wire and at a given point t , the wave appears to be which will reach back to stability when the $t \rightarrow \infty$ on $[a, b]$.

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