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IOT-Enabled FKF–FKL Spectral Analysis for Intelligent and Resilient Supply Chains

V. A. Sharma[1]

[1]Prof and Head, Department of Mathematics, Smt. Narsamma Arts, Commerce and Science college, Amravati 444606
vidyasharma700@gmail.com

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Abstract—
Modern multi-echelon supply chains generate heterogeneous Internet of Things (IoT) telemetry containing lead-time fluctuations, irregular sampling, transportation delays, intensity variations, and operational disruptions. This study proposes an FKF–FKL spectral framework for constrained resilience analysis that separates observational effects from genuine structural change. Fourier analysis captures temporal dynamics, while the Kontorovich–Lebedev (FKL) representation characterizes scale-dependent behavior. Shift-invariant properties distinguish temporal displacement from structural change, whereas exponential modulation and spectral differentials characterize intensity variation and localized disruptions. A distributional formulation on a testing-function space accommodates irregular IoT observations, and sequential convergence of the two-sided FKF transform supports continuous spectral-state updating inside digital twins. Compact spectral features feed resilience scoring, AI-based decision-making, green logistics and sustainable mobility planning, hash-chain (blockchain) provenance, and hybrid quantum-classical logistics optimization (quantum annealing / QUBO). The framework provides an interpretable route for detecting and classifying disturbances across procurement, production, warehousing, unit-load-device handling, and intelligent transportation systems under Industry 5.0 human-centric automation.

Keywords— FKF/FKL transforms;; sequential convergence; testing-function spaces; time-shift/modulation; residue resonance; shift-invariant spectral analysis; multi-echelon lead times; resilient supply chains; digital twins; green logistics; blockchain/hash-chain traceability; swipe control; quantum logistics/annealing; QUBO–ULD optimization; IoT; intelligent transportation; Industry 5.0; human-centric automation; hybrid-electric/solar EV mobility; battery sizing; power-line communication; anti-theft; disruption detection; anomaly diagnosis..

I. INTRODUCTION

Modern multi-echelon supply chains integrate the Internet of Things, artificial intelligence, intelligent transportation systems, Industry 5.0 human-centric automation, and emerging quantum technologies. The resulting operational fabric is highly interconnected and requires continuous monitoring together with adaptive decision-making [14,15,20]. Digital twins furnish synchronized representations of physical logistics processes and support resilience assessment, what-if simulation, and recovery planning [23,28,29]. In parallel, AI and blockchain (hash-chain provenance) strengthen sustainable logistics, supply-chain transparency, and operational intelligence [30,31].

Logistics telemetry, however, is rarely a clean, uniformly sampled waveform. Multi-echelon lead-time records contain irregular sampling, missing scans, reporting offsets, transportation delays, and scale-dependent intensity variation. Conventional time-domain metrics therefore confuse observational artifacts with genuine structural disruption. The FKF framework addresses this limitation by combining temporal Fourier analysis with the scale-sensitive FKL representation [7,8,9,11]. Its time-shift property identifies temporal displacement through spectral

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phase while preserving spectral magnitude [8,12], whereas exponential modulation characterizes systematic intensity drift [13].

Localized events missed handovers, customs delays, quality holds, capacity restrictions, and unit-load-device (ULD) misconfiguration can be modeled as spectral perturbations [9,10]. Spectral displacement together with FKL concentration then distinguishes transient shocks from residue-based resonance. For irregular IoT observations, the distributional FKF formulation and sequential convergence of windowed two-sided transforms enable robust analysis without assuming ideal continuous sampling [24,25].

The same spectral layer accommodates energy-aware and green logistics. Battery technologies, plug-in-hybrid battery sizing, solar-assisted electric mobility, and hybrid-electric vehicle architectures influence transportation capacity and constrained resilience [6,16,18,22]. Secure industrial communication including power-line-carrier anti-theft links and swipe-controller human–machine interfaces further supports reliable intelligent logistics environments [15,17,32].

At the decision layer, quantum annealing and hybrid quantum-classical QUBO solvers provide complementary machinery for combinatorial logistics problems such as ULD configuration and disruption-aware routing [5,19,21,26,33]. The proposed architecture therefore uses FKF/FKL analysis as an intermediate layer that converts high-dimensional operational telemetry into a compact diagnostic state

$$\psi = \left(\mu_0, \mu_1, \rho_{peak}, \omega^*\right)^T. \tag{1}$$

Here μ_0, μ_1 are low-order FKL-scale characteristics, ρ_{peak} is the dominant FKF magnitude, and ω^* is its characteristic frequency. The overall pipeline is

$$Telemetry \rightarrow FKF - FKL\ spectra \rightarrow confound\ separation \tag{2a}$$

$$\rightarrow perturbation\ diagnosis \rightarrow \psi \rightarrow digital\ twin \rightarrow AI/quantum\ decisions. \tag{2b}$$

Thus, the framework integrates FKF/FKL spectral theory [7–13,24,25] with digital twins, green logistics, blockchain traceability, secure connectivity, and quantum logistics [5,6,14–23,29–32], providing an interpretable mechanism for distinguishing translation, intensity variation, transient disturbances, and genuine structural disruption in multi-echelon supply chains.

II. OPERATIONAL CONTEXT AND RELATED BUILDING BLOCKS

A. Digital Twins and Constrained Resilience

A supply-chain digital twin is a living spectral-state machine rather than a static replica. At each update epoch the twin ingests IoT packets, recomputes windowed FKF/FKL features, and revises a resilience score subject to capacity, energy, and service-level constraints hence constrained resilience [23,29,34]. Uncertainty-aware twins can further couple the spectral state to hybrid quantum-classical robust design models [26,33]. Digital transformation of global networks therefore reduces to keeping ψ synchronized with the physical multi-echelon lead-time field while refusing to reinterpret every sampling irregularity as a new dynamical regime [24,25].

B. Green Logistics, Sustainable Mobility, and Energy Constraints

Green logistics and sustainable mobility enter the spectral picture through intensity and capacity modulation. Solar-assisted electric mobility and hybrid-electric / plug-in-hybrid architectures change the feasible transportation rate and the recovery envelope after a shock [6,16,18,22]. Battery sizing for formula-hybrid and fleet applications determines how long a disrupted lane can be substituted by an electrified alternative before the FKL trend μ_0 drifts into a new scale regime [18]. AI bibliometric evidence likewise places sustainable and green supply-chain management among the fastest-growing decision layers that can consume compact spectral features [1,3,30].

C. Hash-Chain Provenance, Blockchain Traceability, and Human-Centric Interfaces

Blockchain traceability and hash-chain provenance bind each spectral update to an immutable observation digest, supporting supply-chain transparency when multiple echelons dispute a delay attribution [31]. On the shop-floor and yard side, human-centric Industry 5.0 automation relies on trustworthy operator interfaces. Prior power-line-carrier anti-theft communication and swipe-controller designs illustrate the class of secure, low-cost industrial links that can timestamp scans feeding the distributional FKF pairing [15,17,32]. Smart-mobility telemetry from intelligent transportation systems supplies the same pairing with lane-level delay traces [20].

D. Quantum Logistics: Annealing, QUBO, and ULD Configuration

Once ψ has isolated a genuine structural disruption, the remaining task is combinatorial: re-slot unit load devices, re-sequence waves, or re-allocate constrained capacity. Quantum annealing and hybrid quantum-classical QUBO encodings are directly applicable to ULD configuration and disruption-aware logistics [5,19,21]. Multi-objective hybrid solvers further close the linear-quadratic gap that appears when resilience penalties are quadratic in spectral displacement [2,26]. The spectral layer does not replace these solvers; it supplies them with translation-robust, intensity-normalized decision variables.

III. FKF/FKL SPECTRAL ANALYSIS AND SUPPLY-CHAIN DIAGNOSIS

Shift-invariant spectral analysis of multi-echelon lead times via the FKF transform establishes that a pure clock translation is not a new operational mechanism [8]. Complementary papers develop FKF-based resilience features [9], spectral differentials of distributional FKF transforms [10], FKL key properties [11], expanded time-shift and exponential-modulation laws [12,13], residue-based resonance extraction in hybrid-vehicle and multi-scale logistics signals [7], and the distributional / sequential-convergence theory needed for irregular traces [24,25]. The remainder of this section collects those properties into a single professional equation set.

A. Forward FKF Transform

Let $\ell(t)$ denote a multi-echelon lead-time, inventory-flow, or logistics-demand signal defined on $\mathbb{R}$. The forward Fourier–FKF transform maps the time-domain signal into a joint spectral representation in which temporal oscillations and positive scale/intensity characteristics appear simultaneously:

$$\mathcal{F}_{FKF}[\ell](\omega,\nu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \ell(t)\, e^{-i\omega t}\, K_{i\nu}(e^{t})\, dt$$

where $\omega \in \mathbb{R}$ is the Fourier frequency, $\nu > 0$ is the Kontorovich–Lebedev spectral parameter, and $K_{i\nu}(\cdot)$ is the modified Bessel function of the second kind (Macdonald function) of imaginary order $i\nu$. When a positive scale variable $r > 0$ is explicit, the operator takes the scale-invariant measure dr/r:

$$\mathcal{F}_{FKF}[\ell](\omega,\nu) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty} \ell(t,r)\, e^{-i\omega t}\, K_{i\nu}(r)\, \frac{dr}{r}\, dt$$

The kernel therefore factors as

$$K_{\omega,\nu}(t,r) = \frac{1}{2\pi} e^{-i\omega t}\, K_{i\nu}(r).$$

The Fourier factor resolves periodicity, delay cycles, and lead-time fluctuation; the Macdonald factor resolves positive scale, magnitude, and logistical intensity. Coupling scale to time through $r = e^{t}$ recovers the single-integral form used for time-only traces. The Macdonald kernel satisfies

$$r^2 \frac{d^2 K_{i\nu}(r)}{dr^2} + r\frac{d\, K_{i\nu}(r)}{dr} - (r^2 - \nu^2)K_{i\nu}(r) = 0.$$

Consequently, the FKF spectrum may be read as

$$FKF\ spectrum = (temporal\ Fourier\ structure) \times (positive - scale\ KL\ structure).$$

A compact implementation used downstream factors the kernel into an oscillatory carrier and a stabilizing window $W(t)$:

$$\mathcal{F}_{FKF}[\ell](\omega) = \int_{-\infty}^{\infty} \ell(t)\, K_{FKF}(t,\omega)\, dt,$$

$$K_{FKF}(t,\omega) = W(t)\, e^{-j\omega t}.$$

The complementary FKL transform [11] uses a kernel L with a distinct localization profile, so that the pair $(\mathcal{F}_{FKF}, \mathcal{F}_{FKL})$ separates slow trend from resonant oscillation:

$$\mathcal{F}_{FKL}[\ell](\omega) = \int_{-\infty}^{\infty} \ell(t)\, K_{FKL}(t,\omega)\, dt.$$

B. Time-Shift Property

Let $\tau \in \mathbb{R}$ be a pure translation of the operational clock: a shipment hold, reporting offset, batch-release delay, or digital-twin synchronization lag. The translated signal is $\ell_\tau(t) = \ell(t-\tau)$. Substituting into (C.1) and changing variables $u = t - \tau$ yields the expanded time-shift law [12]:

$$\mathcal{F}_{FKF}[\ell_\tau](\omega,\nu) = e^{-i\omega\tau}\, \mathcal{F}_{FKF}[\ell](\omega,\nu). \quad (C.1)$$

Because the prefactor is unimodular,

$$|\mathcal{F}_{FKF}[\ell_\tau]| = |\mathcal{F}_{FKF}[\ell]| \ \forall(\omega,\nu). \quad (C.1b)$$

Phase therefore carries the translation, while the FKF spectral envelope carries the mechanism [8]. Explicitly, $arg\, \mathcal{F}_{FKF}[\ell_\tau] = arg\, \mathcal{F}_{FKF}[\ell] - \omega\tau$, so the delay is the phase slope

$$\tau = -\frac{\partial\, arg\, \mathcal{F}_{FKF}[\ell_\tau](\omega,\nu)}{\partial\omega} \quad (after\ unwrapping).$$

This invariance is the reason a translated lead-time pattern must not be diagnosed as a new failure mode inside a digital twin: the magnitude spectrum is shift-invariant, and only the phase orbit is displaced.

C. Exponential Modulation and Intensity Change

Not every amplitude movement is structural. Intensity drift demand amplification, sensor-gain wander, throughput growth, or accumulation is modeled by the modulated signal $\ell_\alpha(t) = e^{\alpha t}\,\ell(t)$. Multiplication by $e^{\alpha t}$ shifts the Fourier frequency into the complex plane [13]:

$$\mathcal{F}_{FKF}[\ell_\alpha](\omega,\nu) = \mathcal{F}_{FKF}[\ell](\omega + i\alpha,\nu). \quad (C.2)$$

A uniform gain G (throughput surge, sensor recalibration, unit change) is the special case $\alpha = 0$ with a multiplicative constant:

$$\mathcal{F}_{FKF}[G\ell] = G \cdot \mathcal{F}_{FKF}[\ell],\ \ arg\ unchanged\ if\ G > 0. \quad (C.2b)$$

Equations (C.4)–(C.7) therefore classify three observational confounds: translation (phase only), intensity (amplitude or complex-frequency displacement), and genuine structural change (reorganization of the spectral support). A monitoring rule first estimates τ and α, normalizes G, and only then inspects the residual spectrum.

D. Spectral Differential of a Lead-Time Perturbation

Let the perturbed lead-time be $\ell + \pi$, where π is a localized shock (missed handover, quality hold, customs spike, ULD jam). Linearity gives the spectral differential [10]:

$$\mathcal{F}_{FKF}[\ell + \pi] - \mathcal{F}_{FKF}[\ell] = \mathcal{F}_{FKF}[\pi].$$

A first-order resilience diagnostic is the relative spectral displacement

$$\Delta_{spec}(\omega) = \frac{\|\mathcal{F}_{FKF}[\pi](\omega,\cdot)\|_{KL}}{\|\mathcal{F}_{FKF}[\ell](\omega,\cdot)\|_{KL} + \varepsilon_0}.$$

The floor $\varepsilon_0 > 0$ stabilizes quiet bands. Resonance is declared only where $\Delta_{spec}(\omega) > \theta_*$ and the FKL complement remains peaked, indicating that the shock is not absorbed by the slow trend [7,9]. The corresponding residue-based resonance indicator is

$$R_{res}(\omega) = 1_{\{\Delta_{spec} > \theta_*\}} \cdot 1_{\{FKL\,peak\}}.$$

Three qualitative regimes follow: (i) small Δ_{spec} and weak FKL concentration buffered recovery; (ii) large Δ_spec but diffuse FKL mass observable yet non-resonant shock; (iii) large Δ_{spec} with concentrated FKL residue critical resonant coupling into intrinsic multi-echelon dynamics.

E. Distributional Extension and Sequential Convergence

Operational traces contain uneven sampling, missing scans, and bursty IoT packets. The signal is therefore treated as a distribution against a rapidly decreasing testing-function space $\mathcal{S}$ (Schwartz class) [24]. For every $\varphi \in \mathcal{S}$,

$$\langle \mathcal{F}_{FKF}[\ell], \varphi \rangle = \langle \ell, \mathcal{F}_{FKF}{}^{*}[\varphi] \rangle,$$

where $\mathcal{F}_{FKF}{}^{*}$ is the adjoint kernel operator. An asynchronous scan train is the finite atomic combination $\ell = \sum_k a_k \delta_{t-t_k}$, so missing observations are simply absent atoms and bursty packets are local clusters—no interpolation is required. Let W_n be expanding operational windows with $W_n \subset W_{n+1}$ and union equal to the operational horizon. Sequential convergence in the testing-function topology of the two-sided FKF transform [25] asserts

$$\mathcal{F}_{FKF}[\ell_{W_n}] \rightarrow \mathcal{F}_{FKF}[\ell] \quad if \quad sup_n |\langle \ell_{W_n}, \varphi \rangle| < \infty \ \forall \varphi \in \mathcal{S}.$$

Uniform boundedness of the testing-function pairing is the license to update spectral features online without re-interpreting every new finite window as a new dynamical regime. Only when pairings cease to remain bounded should the twin declare a regime transition.

F. Multi-Echelon Feature Vector

From the pair of transforms one extracts the finite diagnostic vector used by the digital twin:

$$\psi = (\mu_0, \mu_1, \rho_{peak}, \omega^*)^T.$$

Low-order FKL moments summarize slow-scale trend. With $E(\nu)$ an FKL energy density,

$$\mu_0 = \frac{\int_0^\infty E(\nu)\,d\nu}{\int_0^\infty d\nu}, \quad \mu_1 = \frac{\int_0^\infty \nu\,E(\nu)\,d\nu}{\int_0^\infty E(\nu)\,d\nu}.$$

The dominant FKF modulus and its location are (C.8)

$$\rho_{peak} = max_\omega |\mathcal{F}_{FKF}[\ell](\omega)|, \quad \omega^* = arg\,max_\omega |\mathcal{F}_{FKF}[\ell](\omega)|.$$

(C.9)

Shift invariance of the modulus keeps ψ stable under pure translation; (C.8)–(C.9) make ψ sensitive to genuine perturbation. A weighted feature displacement

$$\delta_\psi = (\psi_t - \psi_{t-})^T \Lambda (\psi_t - \psi_{t-}).$$

with $\Lambda \succeq 0$ a feature-scale weighting matrix, is the scalar handed to anomaly detectors, resilience scorers, and QUBO objectives. Using only the FKF peak would miss slow intensity drift; using only FKL moments would miss a rapidly emerging periodic disruption. The four-component state therefore preserves both background operational level and dominant dynamic behavior.

Theoretical stability under time translation follows at once from (C.4)–(C.5). For a translated signal the FKF transform acquires only a unimodular phase, so both ρ_{peak} and ω^* remain unchanged. Consequently $\psi(\ell_\tau) = \psi(\ell)$ provided the FKL moments are computed from translation-consistent windows. A reporting offset, synchronized batch delay, or uniform shipment hold therefore leaves the diagnostic state invariant exactly the property required of a digital twin that must not raise a new-mechanism alarm every time (C.10) a clock is corrected [8,12].

Sensitivity to genuine perturbations is complementary. From (C.8)–(C.9), a localized shock can move the dominant modulus, displace the characteristic frequency, or both, and can simultaneously tilt the FKL center of gravity μ_1 when the disturbance leaks into slower scale-dependent inventory dynamics. The twin therefore tests the hypothesis of a mere observation confound against the alternative of a structural spectral-state transition. A small, transient δ_ψ is logged as incremental information; a persistent increase accompanied by residue-based resonance is (C.11) escalated to the optimization layer [9,10,7].

IV. DOWNSTREAM DIGITAL-TWIN, PROVENANCE, AND OPTIMIZATION ARCHITECTURE

At each update interval the twin stores ψ_t together with a hash-chain digest of the contributing observation atoms. The chain provides blockchain-style provenance without requiring every echelon to operate a full public ledger: each window hash (C.12) commits the testing-function pairing that produced ψ_t [31]. When a dispute arises—customs delay versus carrier hold the committed spectrum can be recomputed independently from the archived atoms.

Resilience scoring maps δ_ψ and R_{res} onto a constrained feasible set that encodes service level, carbon intensity of the active mode (green-logistics budget), battery state-of-charge on electrified legs, and ULD slot availability [6,19,22,30]. If the residual after confound compensation remains large, a hybrid quantum-classical solver is invoked on a QUBO whose linear terms are current costs and whose quadratic terms penalize simultaneous activation of spectrally resonant lanes [5,19,21]. Industry 5.0 operators retain override authority through swipe-controller and related HMI paths, so automation remains human-centric [15,32].

Secure yard connectivity can ride existing power-line-carrier infrastructure originally demonstrated for anti-theft supervision [17], reducing the need for additional radios in dense metallic environments. Smart-mobility feeds from intelligent transportation systems close the loop between urban congestion spectra and warehouse outbound appointment books [20]. In this sense the FKF–FKL layer is not an isolated transform paper: it is the shared representation that lets digital twins, green logistics, blockchain traceability, and quantum logistics speak a common, shift-invariant language.

A. Online Digital-Twin Update Loop

A practical twin executes the following sequence on every incoming IoT block. The procedure is intentionally conservative: observational confounds are removed before any structural claim is made.

Step 1. Represent the new scans as a distributional increment of atomic observations and form the updated pairing against a fixed testing-function dictionary. Missing packets simply omit atoms; bursty arrivals add local clusters.

Step 2. Compute windowed FKF and FKL transforms. Test sequential convergence: if the testing-function pairings remain uniformly bounded, treat the window as an update of the existing spectral state rather than a new regime.

Step 3. Estimate a candidate translation τ from the unwrapped phase slope and a candidate intensity α from complex-frequency displacement. Compensate both, then normalize any uniform gain G. The residual spectrum is the only quantity admitted as evidence of structural change.

Step 4. Evaluate Δ_{spec} and the FKL concentration test. If both exceed calibrated thresholds, mark residue-based resonance and raise δ_ψ. Otherwise log a transient.

Step 5. Commit the observation atoms and the resulting feature vector to the hash-chain provenance log so that any later dispute can recompute the same spectrum.

Step 6. If resonance persists across two or more windows, pass ψ and the resonant-lane mask to the hybrid quantum-classical QUBO layer for ULD re-slotting, mode shift (including electrified or solar-assisted legs), or capacity reallocation under the green-logistics carbon budget.

B. Energy, Mobility, and Constrained Feasible Sets

Constrained resilience means that recovery actions must lie in a feasible set that is not defined by cost alone. Battery state-of-charge on plug-in-hybrid and solar-assisted electric vehicles bounds how long a disrupted diesel lane can be substituted [6,16,18,22]. Battery sizing performed for formula-hybrid duty cycles provides a template for translating kilowatt-hour headroom into an admissible intensity envelope on μ_0 [18]. When the spectral twin predicts a prolonged resonant delay, the optimizer may shift volume onto electrified legs only up to that envelope, thereby coupling green logistics with spectral diagnosis rather than treating sustainability as an after-the-fact report.

Human-centric Industry 5.0 constraints are encoded the same way. A swipe-controller acknowledgement is a hard precondition for releasing an automated recovery wave, so the QUBO contains indicator variables that stay frozen until the operator interface returns a signed hash [15,32]. Power-line-carrier anti-theft heartbeats supply an independent liveliness signal for trailers and ULDs in metallic yards where wireless IoT is unreliable [17]. Both signals enter the distributional pairing as additional atoms with their own timestamps, and therefore inherit the same shift-invariance and sequential-convergence guarantees as ordinary lead-time scans.

V. CONCLUSION

The proposed FKF–FKL framework supplies a unified spectral methodology for representing, monitoring, and diagnosing multi-echelon lead-time dynamics under irregular and disrupted operating conditions. The forward FKF transform separates temporal behavior from positive-scale characteristics. Its time-shift property ensures that pure translation, reporting offsets, and shipment delays do not generate artificial structural changes in the spectral envelope. Exponential modulation and uniform gain distinguish intensity drift and measurement scaling from genuine changes in system dynamics.

The distributional extension on a testing-function space, together with sequential convergence of the two-sided FKF transform, licenses continuous digital-twin updating from uneven IoT scans. Spectral differentials and residue-based resonance indicators then measure whether a localized shock has coupled into persistent lead-time modes. The compact state ψ is structure-preserving: FKF modulus is translation-robust, spectral differentials are perturbation-sensitive, and FKL moments retain slow-scale intensity. Downstream, the same vector supports constrained resilience scoring, green-logistics mode choice, hash-chain provenance, and hybrid quantum annealing / QUBO decisions for ULD and disruption management. Future work includes statistically calibrated resonance thresholds, uncertainty-aware twins, data-driven feature selection, and hybrid FKF–AI predictors of disruption and recovery. Because equations are stored in Office Math (professional) layout with automatic conversion enabled, subsequent editorial revisions preserve subscripts, superscripts, fractions, integrals, and summation limits.

VI. NOTATION AND PROFESSIONAL EQUATION LAYOUT

Principal symbols used in (1)–(C.12) are collected below so that subsequent Word editing can keep every expression in professional format. In Microsoft Word, open File → Options → Proofing → AutoCorrect Options → Math AutoCorrect and enable “Automatically convert expressions to professional format.” The same instruction is stored in this file’s Equation Options title and document variables, so further edits retain subscripts, superscripts, stacked fractions, and n-ary operators (integrals and sums) with limits.

$\ell(t)$ — multi-echelon lead-time or inventory-flow signal. ω — Fourier frequency. ν — Kontorovich–Lebedev scale parameter. $K_{i\nu}$ — Macdonald kernel. τ — operational-clock translation. α — exponential intensity. G — uniform gain. π — localized perturbation. Δ_{spec} — relative spectral displacement. ε_0 — quiet-band floor. R_{res} — residue-based resonance indicator. $\mathcal{S}$ — testing-function space. W_n — expanding observation window. μ_0, μ_1 — FKL moments. ρ_{peak}, ω^* — dominant FKF modulus and its frequency. ψ — compact diagnostic state consumed by the digital twin, hash-chain provenance log, green-logistics feasible set, and quantum-annealing / QUBO layer.

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