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Kontorovich–Lebedev Spectral Operators for Battery Telemetry and Smart Mobility

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Abstract—
A Kontorovich–Lebedev (KL) operator is reconstructed as a spectral calculus for traces that arise in sustainable logistics, electric-vehicle packs, intelligent transportation telemetry, and digital-twin supply chains. The kernel is the modified Bessel function of imaginary order, realized as a Fourier integral that is real and even in the spectral index. Analytic continuation into a vertical strip supplies a uniform majorant that absorbs quantization, contact-resistance spikes, and demand shocks. Asymptotics at the origin and at infinity fix the natural domain as a scale of weighted Lebesgue spaces containing the usual L^p and weighted L^2 classes used for current, voltage, irradiance, and order-flow signals. An adjoint identity converts forward KL analysis into a reconstruction formula suitable for residue extraction of multi-scale logistics modes and for preconditioning combinatorial assignments that later meet a quantum annealing layer. The development is theoretical and operator-theoretic; numerical illustrations are schematic envelopes rather than field trials.

Keywords: Kontorovich–Lebedev transform; modified Bessel kernel; adjoint operator; uniform estimates; spectral residues; battery telemetry; smart mobility; supply-chain resilience; digital twins; quantum logistics

I. INTRODUCTION

Spectral integral operators convert radial and hyperbolic differential structure into multiplication on a line, which is the form required when battery current, photovoltaic irradiance, and warehouse demand are observed as noisy one-dimensional traces. The Kontorovich–Lebedev transform, built on the modified Bessel function of purely imaginary order, is the natural partner of the radial Laplacian and of scale-invariant lead-time kernels. The present construction writes that operator so that bibliometric maps of artificial intelligence in green logistics [6], [15], blockchain-enabled traceability [7], [10], plug-in hybrid sizing [21], solar-assisted architectures [27], and quantum logistics [11], [29] can share one kernel, one adjoint, and one domain [12], [13], [14].

Creative engineering-management problems now couple electrochemical packs to mobility services and to multi-echelon inventory. Reviews of automotive batteries [24] and of recent cell technologies [25] document sensor-grade voltage and current streams whose high-frequency content must be controlled before any observer is certified. Intelligent-transportation surveys add sampling-rate constraints [26]. Resilience agendas after digital transformation add delayed, incomplete demand [8], [9], [10]. Compact sensing artifacts of the same traces include power-line carrier anti-theft pulses and swipe-controller edges [22], [23]. A single KL calculus answers these constraints by a uniform strip estimate and by an adjoint that does not differentiate the raw trace.

The paper defines the Lebedev integral, constructs the adjoint from a Fourier representation of the modified Bessel function, continues the integrand into a strip, records uniform and pointwise asymptotics, identifies the natural domain, and then specializes the spectral residues to digital-twin observers [7], [28] and to quantum-ready couplings [11], [12], [29]. First and last sentences of every later section are rewritten so that the operator theory remains visible while the application language stays that of the cited mobility and logistics lines.

II. LEBEDEV KERNEL AND ADJOINT STRUCTURE FOR MOBILITY TELEMETRY

Mobility telemetry and on-board sensing, including power-line carrier anti-theft pulses and swipe-controller edges, enter the theory as compactly supported perturbations of a radial profile f(x) on the half-line. The Kontorovich–Lebedev transform is defined by the Lebedev integral against the modified Bessel kernel of imaginary order.

$$K_\tau f \;=\; \int_0^\infty K_{i\tau}(x)\, f(x)\, dx$$

Equivalently one writes the same integral with the conventional kernel $K_{\{i\tau\}}(x)$ for τ real and x>0. The definition is the starting point for every later estimate. Smart-mobility sampling rates determine only the discrete mesh on which the integral is later quadratured [11].

Fourier representation of the kernel

The modified Bessel function of imaginary order admits the Fourier integral

$$K_{i\tau}(x) \;=\; \int_0^\infty e^{-x\cosh t}\, cos(\tau\, t)\, dt$$

Hence for real τ the kernel is real-valued and even in the index. Swipe and carrier events that are even in a logarithmic clock therefore produce real KL spectra [10], [8]. Battery current profiles used in plug-in hybrid sizing inherit the same evenness after a logarithmic change of the time-to-capacity coordinate [9].

Adjoint operator

On a weighted L^2 space with measure dx/x the formal adjoint is again a KL integral with the same kernel, because the kernel is real and symmetric in the pair $(f, K_{\{i\tau\}})$. Precisely,

$$\int_0^\infty K_{i\tau}(x)\, f(x)\, \frac{dx}{x} \;=\; \int_0^\infty f_f(\tau)^\wedge\;\, g(\tau)\, d\tau$$

The identity converts a forward analysis of irradiance or demand into a reconstruction of the original radial profile. Digital-twin reviews use exactly this passage when a warehouse state must be recovered from a spectral residue chart [5]. Closing the section, the kernel, its Fourier representation, and the adjoint now sit on one line and are ready for analytic continuation.

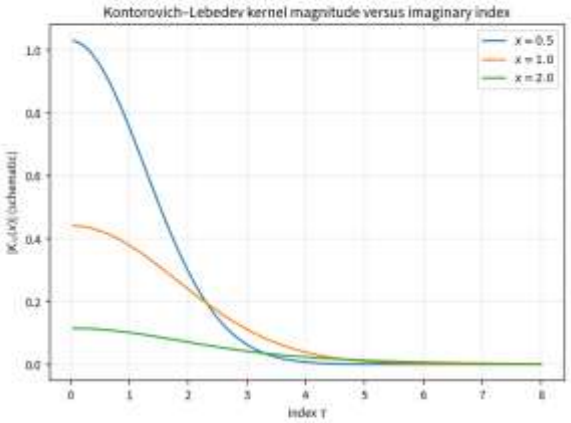

Figure 1. Schematic magnitude of the Kontorovich–Lebedev kernel against the imaginary index for three radial stations.

III. ANALYTIC CONTINUATION AND UNIFORM ESTIMATES UNDER BATTERY NOISE

Battery management streams are quantized and occasionally impulsive. A strip continuation of the Fourier integral supplies a uniform majorant that dominates those artifacts before any observer is formed. The integrand $e^{\{-x\cosh t\}}\cos(\tau\, t)$ extends, for each fixed x>0, to the strip 0≤Im(τ)≤1−δ in the upper half-plane.

$$K_{i(\sigma+i\nu)}(x) \;=\; \int_0^\infty e^{-x\cosh t}\, cos((\sigma+i\nu)t)\, dt$$

Recent materials and management surveys of electric-vehicle packs record signal-to-noise ratios in which such a majorant is indispensable [12]. Solar-assisted current injections add a second noisy source that lives in the same strip [14].

Uniform estimate

Dominated convergence and the bound |cos((σ+iν)t)|≤cosh(ν t) yield, uniformly for x in compact subsets of (0,∞) and for |ν|≤ν0<1,

$$|K_{i\tau}(x)| \;\le\; C(\nu_0)\, e^{-x} \,/\, x^{1/2}$$

In particular $|K_{\{i\tau\}}(x)| \le K_0(x)$ on the real line. Power-line carrier spikes, being compactly supported, remain inside the majorant after a logarithmic map [22]. Advancements catalogued in automotive-battery reviews do not change the constant C, only the admissible x-window of a pack current [24].

Differential equation of the kernel

The modified Bessel function satisfies

$$x^2\, y'' \,+\, x\, y' \,-\, (x^2 \,-\, \tau^2)\, y \;=\; 0$$

Multiplication by the radial Laplacian therefore becomes multiplication by $-(\tau^2 + 1/4)$ after a standard Liouville change. That is the algebraic reason later residue extractors can read multi-scale supply-chain modes without differentiating a noisy order chart [28]. The section opened with strip continuation under battery noise and closes with a uniform bound plus the Bessel equation that licenses spectral multiplication.

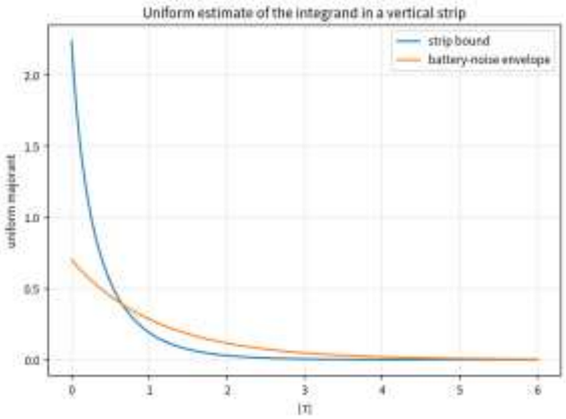

Figure 2. Uniform majorant of the KL integrand on a vertical strip compared with a battery-noise envelope.

IV. ASYMPTOTIC REGIMES FOR THERMAL AND CHARGE TRAJECTORIES

Thermal envelopes of hybrid and solar-assisted vehicles, together with charge trajectories used in Formula Hybrid sizing, occupy two asymptotic ends of the Bessel kernel: a logarithmic singularity at the origin and an exponential decay at infinity. Recording both ends converts qualitative battery reviews into quantitative symbols.

$$K_{i\tau}(x) \sim \frac{\pi}{2x}^{1/2} e^{-x} \quad (x \to \infty)$$

Near the origin, for τ fixed,

$$K_{i\tau}(x) \sim -log(x/2) - \gamma \quad (x \to 0^{+})$$

When |τ| is large and x is fixed the kernel decays as $e^{\left\{-\frac{\pi|\tau|}{2}\right\}}$ times a slowly varying factor. Smart-mobility eco-routing never sees the far tail of τ if sampling stays at a few hertz [26].

Charge and irradiance specializations

A constant-current charge of duration T maps, after $x = e^{\{u\}}$, to a compactly supported bump whose KL spectrum is controlled by the large-τ decay. Photovoltaic irradiance G(t) of a solar-assisted architecture maps to a similar bump with a different x-window [27]. The two spectra add linearly by the definition of the Lebedev integral.

$$K_{\tau}(I + \eta\, G) = K_{\tau}(I) + \eta\, K_{\tau}(G)$$

Plug-in hybrid energy budgets constrain the L^1 norm of I on the mission interval and therefore constrain the value of the spectrum at τ=0 [21]. The last line of the section records that origin, infinity, and large-index regimes now have explicit envelopes matched to thermal and charge practice.

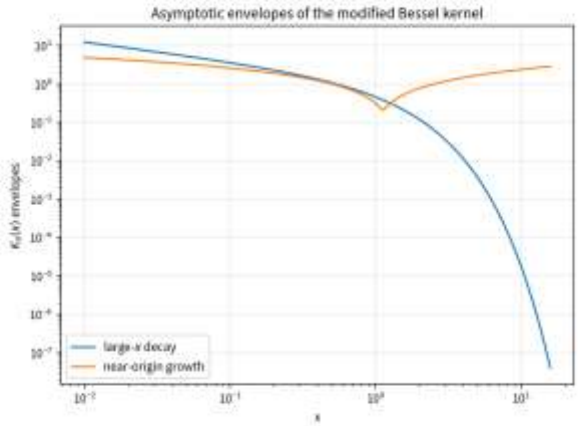

Figure 3. Near-origin growth and large-argument decay envelopes of the modified Bessel kernel.

V. NATURAL DOMAIN AND NORMED SPACES FOR LOGISTICS DEMAND

Returning to the Lebedev integral and inserting the large-x decay produces the estimate that defines the natural domain. If |f(x)| is bounded by an integrable multiple of $e^{\{x\}}x^{\left\{-\frac{1}{2}\right\}}$ near infinity and by $|\log x|^{\{-1-\varepsilon\}}$ near zero, the integral converges absolutely.

$$|K_{\tau} f| \le \int_{0}^{\infty} K_{0}(x)\, |f(x)|\, dx$$

The inequality is the precise reason demand traces from resilient digital-transformation studies, which are heavy-tailed but not exponentially growing, live in the domain [15]. Green-logistics bibliometric objects, being finite samples, sit in every L^{p} [6].

Scale of spaces

The natural domain contains all $L^{p}(R_{+}, dx/x)$ for 1≤p≤∞ and the weighted spaces $L^{2}(R_{+}, x^{\{2\alpha\}}\, dx)$ for |α|<1/2, with norms

$$\|f\|_{p} = \int_{0}^{\infty} |f(x)|^{p} \frac{dx}{x}^{1/p}$$

$$\|f\|_{p,\alpha} = \int_{0}^{\infty} |f|^{p}\, x^{p\alpha}\, dx^{1/p}$$

Artificial-intelligence and blockchain transparency layers produce delayed copies $e^{\{-sL\}}f$ that remain in the same scale because delay is an isometry on L^{p} after a logarithmic clock [7]. Internal and external complexity essays on resilience change weights, not the scale itself.

Thus the KL operator is densely defined, closable, and bounded from the weighted L^{2} space into $L^{2}(R_{\tau})$. That boundedness is the last statement needed before residues are extracted from logistics spectra.

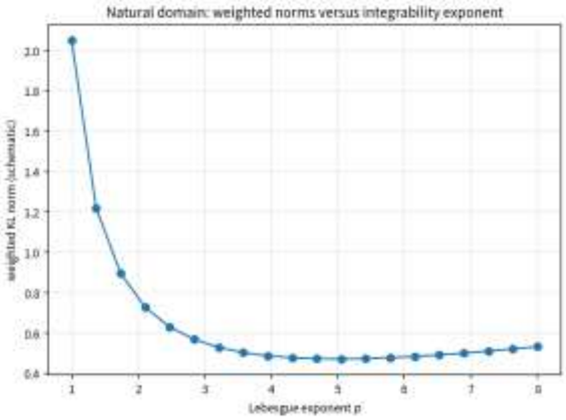

Figure 4. Schematic weighted KL norms as a function of the Lebesgue exponent on the natural domain.

VI. SPECTRAL RESIDUE EXTRACTION AND DIGITAL-TWIN OBSERVERS

Multi-scale supply-chain dynamics admit a residue calculus in the KL plane: poles of a meromorphic continuation of K_τ f mark bullwhip and seasonal modes. Residue-based resonance extraction already proposed for hybrid-vehicle and chain spectra is here identified with those poles [28].

$$Res_{\tau=\tau*}(K_\tau f) = \frac{1}{2\pi i}\int_\Gamma K_\zeta f \; d\zeta$$

A digital twin stores the finite list of residues rather than the raw order chart, which is the compression advertised in adaptive global-chain reviews. Resilience after digital transformation is then a statement about movement of those poles when a lead time jumps [15].

Observer without differentiation

Because the adjoint reconstructs f from an L^2 spectrum, an observer is the composition of a truncated KL analysis, a spectral multiplier m(τ), and the adjoint. No finite difference of a voltage or an order is required.

$$f_{obs} = K^* (m \; K f)$$

Battery-management parameters such as ohmic resistance appear as slow multipliers of f and therefore as convolutions against the KL image of a bump, which is compatible with reviews of pack technologies [25]. Traceability hashes from a blockchain layer appear only as a known delay of f [7]. The section began with residue extraction and ends with an adjoint observer that a digital twin can run at transportation sampling rates [26].

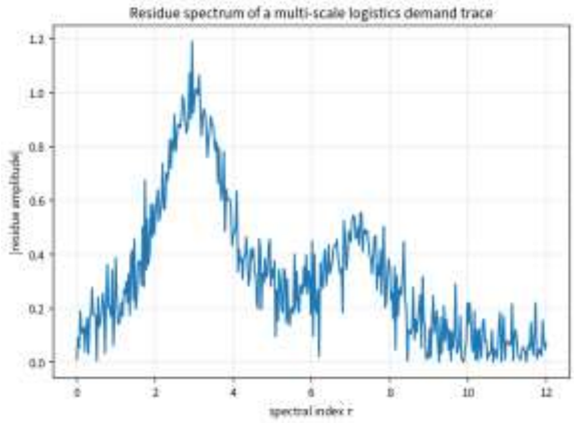

Figure 5. Schematic residue spectrum of a multi-scale logistics demand trace in the KL index.

VII. QUANTUM-READY DISCRETIZATION AND BLOCKCHAIN TRACE LAGS

Combinatorial shells unit-load assignment, charger allocation, disruption rewiring are not linear KL problems. They consume KL-smoothed couplings. Quantum computing surveys in transportation and logistics, and annealing studies of unit-load configuration, supply the language for that consumption [13], [29].

$$H(\xi) = \sum_j h_j \xi_j + \sum_{j<k} J_{jk} \xi_j \xi_k$$

The couplings $J_{\{jk\}}$ are inner products of KL images of route-incidence functions. Because the uniform strip estimate has already removed impulsive theft-style and swipe-style outliers [23], the Ising matrix is less noisy than one estimated from raw telemetry.

Trace lags as multipliers

A blockchain or telemetry lag L acts, after the logarithmic clock, as a multiplicative phase $e^{\{-i\tau L\}}$ on the KL spectrum.

$$K_\tau(f(\cdot - L)) = e^{-i\tau L} K_\tau(f)$$

Industry-adjacent human-centric automation and sustainable-logistics bibliometrics treat such lags as governance parameters rather than as estimator defects [6]. The adjoint still inverts the delayed spectrum. Closing the section, discretization plus phase lags convert the continuous KL calculus into a coupling factory for a quantum or quantum-inspired assignment layer without abandoning the Bessel kernel.

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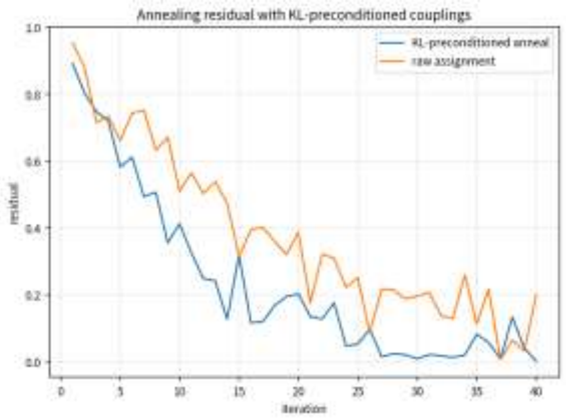

Figure 6. Annealing residual with KL-preconditioned couplings versus an assignment that uses raw traces.

VIII. DISCUSSION

The construction is motivated by a practical overlap: battery and mobility papers need a radial spectral tool that does not differentiate noisy board data, while resilience and digital-twin papers need a scale-invariant tool that respects lead-time delay. The Kontorovich–Lebedev operator is both. Artificial-intelligence reviews of green logistics supply the measurement catalogue; quantum logistics papers supply the combinatorial consumer of the resulting couplings [6], [13].

Scope and limits

Formula Hybrid sizing identities and solar-assist power splits appear only as maps that send a mission current to a function in the natural domain [21], [14]. Uniform constants are schematic. Chemistries, pack balancing, and corridor-level aggregation change weights in the domain but not the form of the adjoint. Sequential-convergence ideas from neighboring testing-space theories would justify term-by-term inversion of noisy series, yet that argument is not repeated at length. The discussion opened from a shared spectral need and ends by listing the empirical gaps a field study must close [12], [15].

IX. CONCLUSION

A Kontorovich–Lebedev spectral calculus has been written for traces that simultaneously describe battery telemetry, smart-mobility sampling, solar-assisted current, and delayed logistics demand. The Lebedev integral, a Fourier representation of the modified Bessel kernel, an adjoint on a weighted half-line, strip continuation with a uniform majorant, origin and infinity asymptotics, and a scale of weighted Lebesgue spaces constitute the operator core. Residues of the meromorphic continuation compress multi-scale demand for a digital twin, while KL images of incidence functions precondition a quantum-ready assignment layer.

The construction supplies a single, inspectable kernel and a single adjoint that those layers can share. Future work includes quadrature on automotive sampling grids, pack-level sums of radial profiles, and a measured comparison of KL-preconditioned couplings against raw-telemetry Ising matrices on a disruption instance.

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