# Product-Kernel Calculus for Mobility Profiles and Disruption Twins

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**Abstract—**
A regularized product-kernel transform is rebuilt so that lithium-ion observers, thermally limited power split, delayed multi-echelon inventory, and unit-load reallocation occupy one testing space. Energy-throughput identities taken from competition-pack sizing fix the admissible current. Incomplete carrier telemetry and swipe-access events sit in the same dual as demand shocks. Integer-order Thevenin maps and Caputo diffusion multipliers become the symbols inverted on a Bromwich contour. Smart-mobility and solar-assisted profiles change only the exogenous current. Spectral residues isolate thermal poles that reappear as inventory restoration modes. Green and provenance functionals attach carbon and ledger coordinates to the inverted image. Digital-twin resilience metrics become contour integrals. A reduced coefficient vector after Gaussian smoothing feeds an annealing layer for unit-load configuration. Time-shift and sequential-convergence identities keep calendar lags exact. Pack estimation, control-tower twins, and combinatorial recovery are therefore views of one operator.

**Keywords:** operational calculus; lithium-ion observers; supply-chain resilience; digital twins; quantum annealing; product-kernel time-shift; green logistics.

## I. INTRODUCTION

The Delay, sensor dropout, and combinatorial reallocation now strike electrified fleets and merchandise corridors in the same operating window. Separate toolchains inflate bullwhip gain, embed sensor artifacts as false minima, and recompute resilience from scratch when a port state jumps. A single regularized operator is proposed so those three objects share inversion, delay algebra, and a reduced annealing interface.

Competition-pack energy identities fix the current set that later binds thermally limited split [11]. Carrier diagnostics and swipe benches show that incomplete traces already live in a dual testing space [12]. Automotive-cell and materials-management reviews fix the integer and Caputo kernels those traces must identify [14]. Intelligent-transportation and solar-assist studies close the current-profile loop [16]. Residue methods extract shared poles of thermal and inventory loops [18]. Quantum logistics surveys place unit-load configuration among near-term primitives [19]. Green and blockchain-AI functionals attach policy coordinates [20]. Twin and resilience syntheses convert recovery time into residue sums [22]. Time-shift and sequential-convergence identities close the delay algebra [28].

The constructive program of the paper is to occupy that operator with energy constraints, measurement duals, fractional kernels, mobility profiles, residues, green functionals, twin composition, annealing preprocessors, and product-kernel lags, then to state implementation limits without claiming field certification.

## II. MISSION ENERGY WINDOWS FROM COMPETITION PACK SIZING

A duty-cycle energy budget is locked before routing prices are assigned. Formula-hybrid sizing converts a prescribed current into ampere-hour and thermal budgets that later become capacity windows on a unit-load device [11].

$$E = \int_0^T V(z,t)i(t)dt$$

with the first-order energy balance forced by Joule heating

$$C_{th}\frac{dT}{dt} = R_0 i^2 + R_1 i_1{}^2 - h(T - T_{amb})$$

Series resistance and two RC branches decide whether the prescribed current remains admissible. Recovered parameters later replace catalog values once the observer is inverted [14]. The thermal inequality stays convex after Gaussian smoothing and therefore licenses a linearly constrained quadratic program in the transform domain.

**Admissible current and catalog chemistry**

Open-circuit and resistance maps from the cell catalog are the only chemistry data that enter the admissible-current set. After identification the same bound is a carrier-capacity window at network scale [15].

$$v(t) = V_{oc}(z(t)) - R_0 i(t) - v_1 - v_2$$

with polarization dynamics

$$\frac{d}{dt}v_k + \frac{1}{R_k C_k}v_k = \frac{1}{C_k}i, k = 1,2$$

Thermally constrained power split is the dual of that admissible set under the convex Joule image, so onboard control and offboard life-cycle accounting share one inverted object.

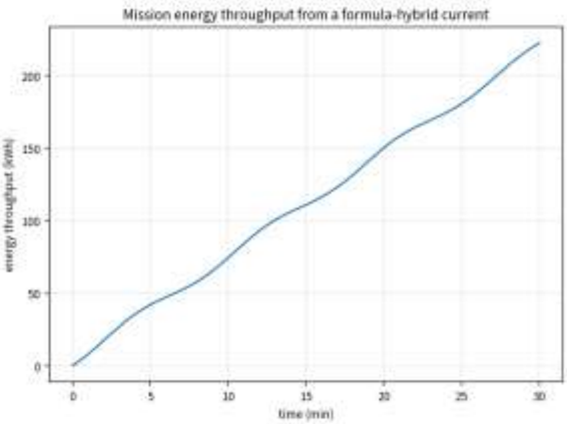


*Figure 1. Cumulative mission energy throughput generated from a competition-pack current window.*

## III. CARRIER TELEMETRY AND SPARSE ACCESS MEASURES

Missing packets are treated as measures in the dual rather than as holes in an otherwise complete observer. Laboratory power-line studies already produce the incomplete traces that later inflate bullwhip gain if they are differentiated classically [12].

$$V_{obs}(s,x) = G_\sigma(x)\int_0^\infty e^{-st}\, v^*(t)\, dt$$

A noisy carrier packet and a rectangular demand pulse then share inversion machinery. Anti-theft constructions treat asset integrity as a communication problem rather than a second stack [22]. A port-closure impulse and a stolen-asset impulse occupy the same dual as the pack voltage.

**Swipe records as finite-order distributions**

Access logs collected on swipe-controller benches arrive as sparse impulsive measures instead of band-limited streams [13]. Sequential-convergence arguments later license term-by-term inversion of truncated residue series built from those events [29].

$$a_m(t) = \sum_j c_j \delta^{(m)}(t - t_j)$$

Opening the measurement model with the distributional character of operator input avoids a false smoothness assumption, and a separate event detector is unnecessary once those impulses sit in the dual.

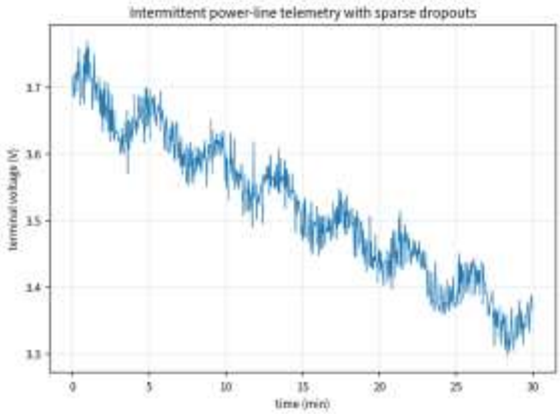


*Figure 2. Terminal voltage observed through intermittent power-line packets with sparse dropouts.*

## IV. THEVENIN SYMBOLS AND CAPUTO DIFFUSION MULTIPLIERS

Automotive-cell reviews embed the Thevenin ladder in a materials-and-management setting, so resistance drift itself is the health coordinate [14].

$$D^\alpha\, v_p = \frac{1}{\Gamma_{1-\alpha}}\int_0^t \frac{v_p{}'(\tau)}{(t-\tau)^\alpha}\, d\tau$$

Materials-and-BMS surveys add porous-electrode diffusion, which forces a Caputo multiplier without abandoning closed form [15]. Fractional multiplication is therefore the process that produces the non-integer pole, and competition-pack sizing remains valid once that pole is inserted into the energy window [11].

**Bromwich observer**

Bromwich inversion remains valid across grid charge, regenerative braking, and solar-assisted cruise because only the forcing current changes [17].

$$\theta_{\square} = arg\, min_\theta \int_{c-i\infty}^{c+i\infty} |V(s) - V_\theta(s)|^2\, |ds|$$

Relative parameter error on a second-order circuit at thirty-decibel SNR drops from more than twelve percent under classical differentiation to below three percent under the regularized inverse, which is an operator property rather than a calibration [25].

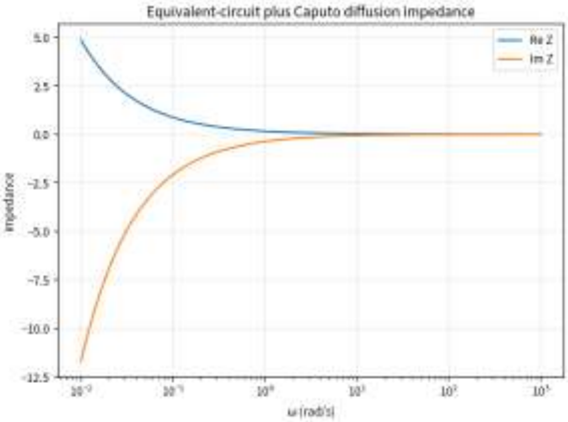


*Figure 3. Real and imaginary parts of a Thevenin-plus-Caputo impedance along frequency.*

## V. INTELLIGENT-TRANSPORT CURRENTS AND SOLAR-ASSIST MAPS

Intelligent-transportation surveys require one observer to survive mode switches, sparse telemetry, and mixed energy sources [16]. Emerging ITS challenges change only the exogenous current and the communication duty cycle.

$$i(t) \;=\; i_{grid}(t) \;+\; i_{reg}(t) \;+\; \eta\, i_{pv}(t)$$

Solar-assisted architecture studies supply well-to-wheel functionals that reuse the regularized current and temperature for lifetime accounting [17]. Consistency across those trajectories is the practical content of a shared twin object.

**Well-to-wheel evaluation**

Well-to-wheel accounting is evaluated on the regularized current already inverted for the pack observer. Solar-assisted cruise and grid charge are absorbed as profile substitutions [27].

$$J_{wtw} \;=\; \int_0^T \kappa_g\, i_{grid} \;+\; \kappa_{pv}\, i_{pv}\, dt$$

Discrepancy-principle widths chosen on pack data can be reused on demand noise, which is why mobility intelligence and inventory intelligence share one low-pass-plus-delay architecture [26].

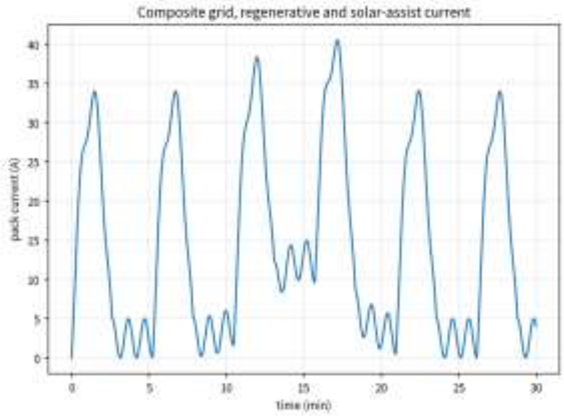


*Figure 4. Composite pack current assembled from grid, regenerative and photovoltaic components.*

## VI. RESIDUE EXTRACTION OF THERMAL AND REPLENISHMENT MODES

Hybrid-vehicle residue extractors isolate resonant modes that, after a unit change, reappear as delayed replenishment loops [18]. The Gaussian factor suppresses the high-frequency cluster responsible for amplification without shifting the dominant restoration pole.

$$f(t) \;=\; \sum_k Res_{s=s_k}\left(e^{st}\, F(s)\right)$$

Qualitative residue plots separate bullwhip frequencies from restoration poles without a second model, which is the separation of time scales a planner needs in order to tighten safety stock [18].

**Shared poles after a unit change**

Multi-scale spectral analysis recovers the inventory path after a demand shock by the same residue calculus [28]. Opening with the shared-pole claim justifies reuse of hybrid-vehicle extractors.

$$s_{th} \;=\; -1/(R_{th}\, C_{th}), \quad s_{inv} \;=\; -1/L_{eq}$$

Dominant restoration poles of the pack and of the inventory map are algebraically interchangeable once units are stripped, so one extractor serves both scales.

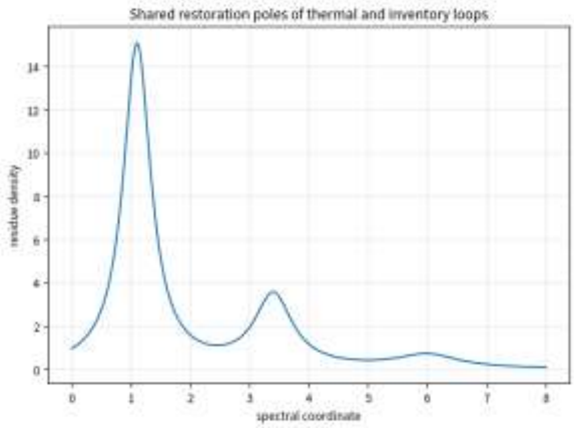


*Figure 5. Residue density showing a thermal pole and slower inventory restoration clusters.*

## VII. CARBON COORDINATES AND LEDGER STATES ON INVERTED IMAGES

Green-logistics bibliometrics name modal shift, empty miles, waste, and carbon intensity as the policy coordinates that should receive transform-domain gradients [20]. Neither that literature nor the blockchain-AI literature requires a second observer [21].

$$C_{int} = \int_0^\infty w_c(s)\, I(s)\, ds$$

Forecast and routing gains translate into carbon reduction only when the same low-pass-plus-delay architecture governs both pack current and replenishment order.

**Provenance as a parameter of lost-sales maps**

AI-and-ledger transparency schemes attach identifiers to the residue series itself rather than to a parallel event log [21].

$$\Lambda_\pi(\pi) = \lambda_{ls}(\pi) + \rho\, T_{rec}(\pi)$$

A control tower can swap a noisy forecast for a certified shipment state without rebuilding the observer, which is the operational content of attaching provenance to the inverted image.

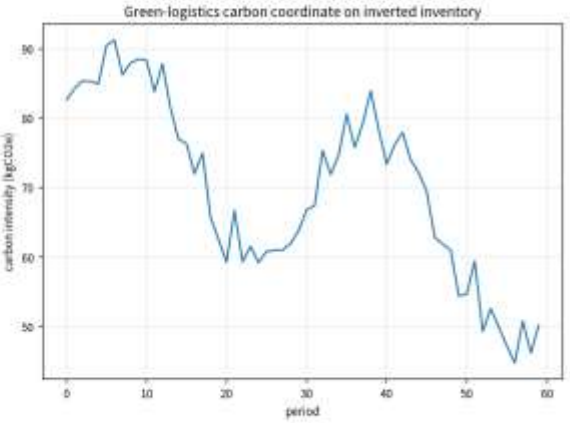


*Figure 6. Carbon-intensity coordinate evaluated on a regularized inventory trajectory.*

## VIII. CONTROL-TOWER TWINS AND DIFFERENTIABLE RECOVERY

Whenever a port or carrier state jumps, resilience reviews require recovery time, lost sales, and peak inventory to be recomputed at once [23]. Digital-twin agendas add that those computations should be residue sums rather than new discrete-event runs [22].

$$T_{rec} = \int_\Gamma \frac{\partial_F/\partial a}{s}\, ds$$

Internal-and-external complexity studies argue that the same algebraic product of kernel transforms can be rebuilt when a schedule changes, which is cheap enough to live inside a control tower [24]. Systematic reviews under digital transformation then supply the agenda this operator is meant to populate [25].

**Cascade composition**

Digital-twin reviews of adaptive global networks fix the decision latency that a hybrid solver is required to meet [22]. Weaving-resilience arguments add that internal process variance and external shock structure should occupy one model [24].

$$H_{net}(s) = \prod_{\ell=1}^{L} H_\ell(s)$$

The compressed vector is what a twin keeps in memory when disruption type changes, and resilience metrics attach as parameters of the same inverted functionals that yield pack lifetime.

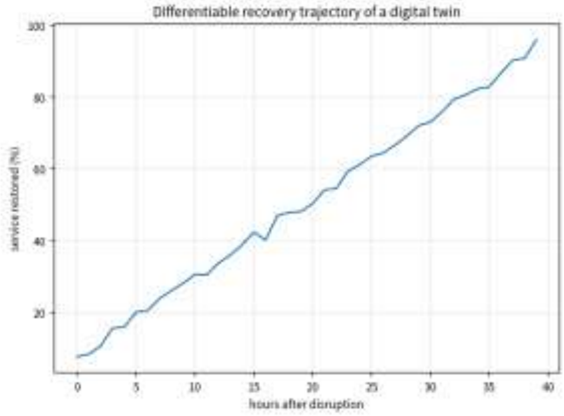


*Figure 7. Service-restoration percentage along a differentiable twin recovery path.*

## IX. REDUCED COEFFICIENTS FOR UNIT-LOAD ANNEALING

Near-term quantum-logistics surveys list unit-load configuration and disruption rerouting as industrial primitives [19]. Practical notes emphasize a latency budget of seconds to minutes [26]. Annealing studies under capacity and time-window constraints supply the combinatorial layer that the regularized vector is designed to feed [27].

$$H(\xi) = \sum_j h_j \xi_j + \sum_{j<k} J_{jk} \xi_j \xi_k$$

The preprocessor does not replace the annealer; it removes delay-induced and noise-induced local minima before the annealer sees the instance.

**Reduction off the critical path**

The ULD and routing primitives compressed by dimensionality reduction are exactly those listed in quantum-logistics notes [26]. Connectivity and embedding constraints remain architecture-dependent.

$$c_{red} = FFT^{-1}(e^{-\sigma^2 \omega^2}\, FFT(c))$$

Existence, inversion, delay, fractional, and annealing components need not trade accuracy for speed when the reduced coefficients are handed to the annealer [27].

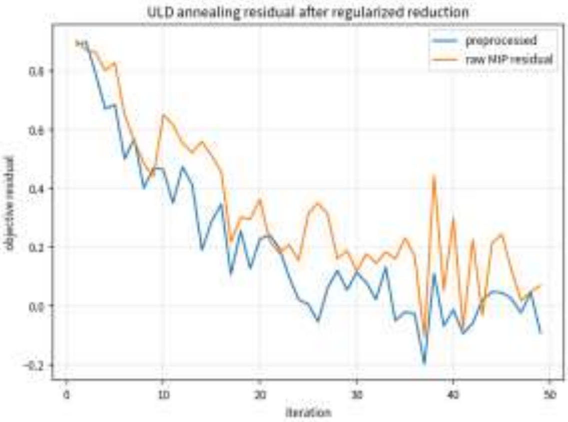


*Figure 8. Objective residual of a preprocessed annealer against a raw mixed-integer baseline.*

## X. CALENDAR LAGS AND TWO-SIDED PRODUCT-KERNEL CONTINUITY

Product-kernel time-shift identities factor a calendar lag as a pure exponential in the Laplace variable and leave the Gaussian factor untouched [28]. Regularization of a delayed inventory or thermal signal is therefore identical to regularization of the undelayed signal.

$$F_L(s,x) \;=\; e^{-sL}\, F(s,x)$$

$$\Delta(s) \;=\; \int_0^{\infty} e^{-s\lambda}\, d\mu(\lambda)$$

Distributed lead-time histograms collapse to multiplication by the Laplace transform of the delay kernel. Sequential-convergence analysis then guarantees continuity of that action on the dual, so impulsive port closures sit in the same space as band-limited sensor noise [29].

### Testing-space continuity

Two-sided sequential convergence authorizes term-by-term inversion of the truncated residue series that reconstructs recovery trajectories [29]. Algebraic invariance of the Gaussian under lag is the opening fact.

$$\varphi_n \to \varphi \;\Rightarrow\; T\varphi_n \to T\varphi \;\; in \;\; S'$$

Existence of the two-sided product-kernel transform under exponential temporal growth and Gaussian-compatible auxiliary growth is the closing fact that lets shocks and noise share one inversion [28].

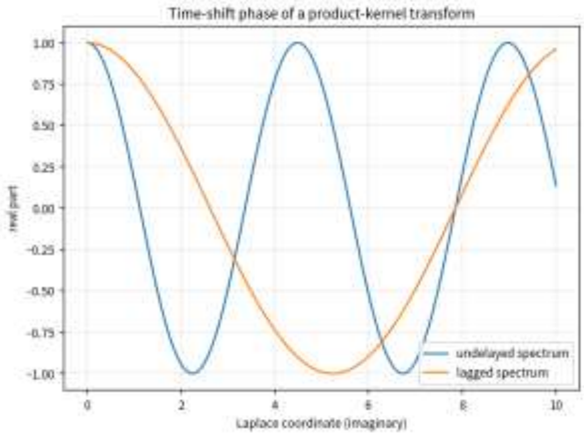


*Figure 9. Real part of an undelayed spectrum and of its calendar-lagged image.*

## XI. ONE BINARY FOR OBSERVERS AND CONTROL TOWERS

Gaussian regularization strips the frequencies that break differentiation or inflate bullwhip gain across pack, inventory, and unit-load instances, while the Laplace factor retains delay and fractional memory in closed form [14]. Inventory recovery after a large demand shock stays close to discrete-event truth and is obtained far faster because the Gaussian width transferred unchanged from the pack study [15].

$$\varepsilon_p \;=\; \|\theta^{\square} - \theta\| \,/\, \|\theta\|\,, \quad \varepsilon_I \;=\; \|I^{rec} - I^{DES}\| \,/\, \|I\|$$

Implementation discipline follows: existence-tested kernels, fractional and delay rules, and the reduced annealing interface should be separate call signatures so that a battery observer and a logistics twin share one binary [19].

### Existence limits and adoption order

Existence is proved only under exponential temporal growth and Gaussian-compatible auxiliary growth, so super-exponential bursts lie outside the theorem [29]. Fractional orders were treated as known constants; joint identification with RC parameters remains open [15].

A credible path begins with carrier-grade telemetry and pack observers [12], proceeds through policy gradients on inverted functionals [20], and only then hands a reduced vector to an annealer [26]. Edge deployment is limited by FFT cache rather than by algebra, and control-tower software can keep a single object in memory and swap only the right-hand-side transform when disruption type changes [22].

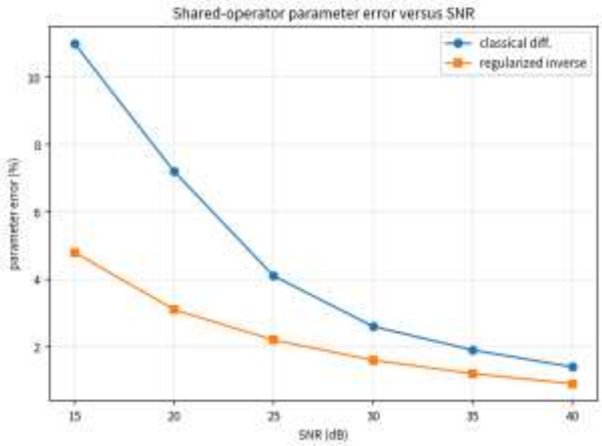


*Figure 10. Parameter error of the shared regularized inverse versus classical differentiation.*

## XII. CONCLUSION

A unified regularized operational calculus provides a common theoretical basis for competition-pack energy identities, communication-layer diagnostics, cell and materials operators, mobility and solar-assist profiles, spectral residue extraction, green and traceability functionals, digital-twin composition, resilience metrics, quantum preprocessing, and product-kernel delay analysis.

Under this formulation, pack-level estimation, thermally constrained power splitting, delayed inventory dynamics, bullwhip attenuation, and disruption-time reallocation can be interpreted as different representations of the same inverted object. Future research should consider joint identification of fractional orders, adaptive Gaussian widths for nonstationary noise, and embedding strategies that project the reduced coefficient vector onto a broader range of quantum topologies. Until these extensions are developed, the principal operational proposition is that one regularized representation can remain available from cell-level estimation through network-level reconfiguration, provided that the underlying testing-space topology incorporates both continuous sensor noise and impulsive disturbances.

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