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# Seminorm Duals for Constrained Resilience: An FKF–FKL Operational Calculus of Digital Twins, Green Reverse Flows, and Quantum Unit-Load Planning

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**Abstract—**
Smart logistics couples delayed replenishment, incomplete Internet-of-Things packets, fractional battery memory, and discrete unit-load-device decisions inside a single planning horizon. Classical Lebesgue and Sobolev spaces lose the time-shift property on that mixture, so continuity of delay operators cannot be certified by a single norm. This manuscript constructs a two-sided Fourier–kernel fractional (FKF) transform and a causal Fourier–kernel Laplace (FKL) transform on countably multinormed testing-function spaces of Roumieu and Beurling type. Sequential convergence makes delay operators continuous and places singular disruptions, stochastic lead-time laws, and residue-based resonances in the corresponding duals. The same topology is used as a certified interface among classical simulators, digital twins, hash-chain provenance, and hybrid quantum-classical solvers. Delayed multi-echelon inventory, bullwhip amplification, constrained resilience, fractional state-of-charge and state-of-health of plug-in and solar-assisted packs, QUBO encodings of unit-load-device configuration, live twin orchestration, graph delay systems, and closed green reverse flows become algebraic identities in the transform domain. One pair of spaces and one pair of transforms therefore certify stock, charge, configuration, and custody without a change of seminorm language [3], [5], [16], [27], [31].

**Keywords—** distributional FKF/FKL transforms; sequential convergence; testing spaces; shift/modulation and spectral properties; resonance analysis; multi-echelon lead time; supply-chain resilience; digital twins; green/sustainable logistics; blockchain traceability; quantum logistics, annealing and QUBO; ULD optimization; IoT/ITS; Industry 5.0; hybrid/plug-in EVs; battery sizing; solar mobility; swipe control; PLC anti-theft..

## I. INTRODUCTION

The present gap is architectural rather than merely computational: delayed replenishment, noisy sensor streams, fractional electrode memory, and discrete configuration refuse to live in one classical function space without destroying continuity of time shifts.

Distributional FKF calculus is the natural language for impulses, discontinuities, and highly oscillatory demand that appear in multi-echelon pipelines [9], [10], [15], [32]. When those singularities travel along a stochastic lead time, or when a digital twin must propagate an incomplete Internet-of-Things packet both forward and backward, the testing-function space must make delay operators continuous [16], [17], [33]. The two-sided FKF transform and the causal FKL transform supply that space. They inherit exponential modulation and shift-invariant spectral analysis; the countable seminorm family later becomes a resilience certificate that a planner can read without restimulating the shock [11], [18], [31].

Inventory recovery after a supplier failure is written as a delay equation on these spaces. Hybrid-electric and solar-assisted packs enter through fractional SoC/SoH models and battery-sizing inequalities that a swipe-controller interface can tighten in real time [8], [21], [23], [30]. Unit-load-device configuration under disruption is a QUBO handed to a quantum annealer while the classical FKL simulator carries the distributional state [7], [27], [29]. Intelligent transportation systems, swipe-controller interfaces, and power-line-carrier anti-theft links lift into the dual as observation operators [6], [22], [28]. Blockchain hash-chain provenance, green closed loops, and digital-twin coverage complete the orchestration layer [3], [4], [5]. Constrained resilience under Industry 5.0 human-centric automation is the planning target rather than a second theory [20], [35].

Human-centric vetoes that freeze an order rate, a charger set-point, or a subset of QUBO bits are modelled as restrictions of the admissible set. The creative claim is therefore modest and testable: one multinormed pair of spaces should be enough to certify stock, charge, configuration, and custody across the smart-logistics stack.

The same claim can be restated in operational language. A planner who already owns a digital twin, a hash-chain custody layer, a solar-assisted plug-in fleet, and a quantum annealer for unit-load devices does not need four additional theories. What is needed is a topology in which each of those objects is a continuous map. Sequential convergence supplies that topology; the FKF and FKL transforms supply the algebra; the countable seminorms supply the numbers that appear on a resilience dashboard [3], [7], [19], [31].

The remainder of the manuscript is organized as follows. Section II rereads the prior-art stack as four streams that have not yet shared a delay-continuous interface. Section III constructs the testing-function pair and the transform pair. Section IV extracts warehouse certificates. Section V rereads the same calculus on plug-in and solar-assisted packs. Section VI writes the hybrid contract with a QUBO annealer. Section VII samples the dual as a twin, a hash chain, and a green reverse flow. Section VIII records the two limitations that stay open and the managerial reading of the dashboard. Section IX closes the argument.

## II. CONVERGENT STREAMS IN TRANSFORMS

Existing programmes treat transforms, packs, twins, and annealers as separate literatures; none of those literatures simultaneously guarantees continuous delay shifts and seminorm-level certificates that travel from the warehouse to the annealer.

### *A. Operational Calculus, Sequential Convergence, and Spectral Multipliers*

Sequential convergence in the two-sided FKF testing-function space, together with the distributional extension of the transform, is the analytic spine of the construction [32], [33]. Exponential modulation and the time-shift property convert delayed replenishment into multipliers on a strip of analyticity [17], [18]. The FKL companion treats causal initial-value problems that arise as soon as a warehouse is started from a measured on-hand position [16]. Spectral differential properties and residue-based resonance extraction close the loop for hybrid-vehicle and multi-scale network signals [9], [15].

The literature on generalized Fourier–Laplace calculi already shows that duals of Gelfand–Shilov type accept impulsive loads. What is missing is a single countable family of seminorms that remains a certificate when the same load is later observed by a hash payload or coarsened into a QUBO bias. That missing object is supplied below and is the reason the section does not end with a catalogue of unrelated transforms.

### *B. Packs, Vehicles, and Observation Hardware*

Automotive-battery reviews and materials-and-BMS surveys justify the SoC/SoH development used later [8]. Battery sizing for plug-in hybrids, solar-assisted architectures, swipe-controller interfaces, and power-line-carrier anti-theft systems enter as constraints and as observation operators [6], [21], [22], [23], [30]. Intelligent-transportation telemetry is a further measurement channel that binds warehouse and pack into one product dual [28]. Hardware papers, however, write ordinary differential or delay-differential equations; they do not certify well-posedness when the load is an ultradistribution.

The hardware stream is therefore kept, but it is reread as a family of continuous maps on the FKF–FKL dual rather than as a second dynamical theory.

### *C. Resilience Agendas, Digital Twins, Green Loops, and Quantum Search*

Resilience reviews and digital-transformation agendas remain the empirical target of the inventory development [31]. Digital supply-chain twins, green-logistics analytics, and blockchain traceability supply the orchestration layer [3], [4], [5]. Quantum annealing and QUBO encodings of routing and packing, including unit-load-device configuration under disruption, motivate the hybrid interface [7], [27], [29]. Industry 5.0 human-centric automation supplies the admissible-set restriction that later freezes bits or charger set-points [20].

The gap filled here is therefore architectural: one testing-function pair, one transform pair, and one certificate language spanning every domain named above, rather than a stack of loosely coupled digital initiatives.

A further remark on citation practice is in order. Digital-twin reviews emphasise visibility and disruption recovery but leave the underlying function space unspecified [3]. Green-logistics analytics and blockchain-traceability studies supply orchestration vocabulary without a continuity statement for delayed returns [4], [5]. Quantum-logistics surveys describe annealing opportunities for packing and routing without a topology that can accept an ultradistributional bias [7], [27], [29]. The present interface is written so that those four streams can be cited as operators on one dual rather than as competing programmes.

Field papers on swipe controllers, power-line-carrier anti-theft links, battery sizing, and solar-assisted architectures remain indispensable as observation and constraint maps [6], [21], [22], [23], [30]. They are not replaced; they are lifted. That lift is possible only after sequential convergence and shift continuity have been recorded in Section III.

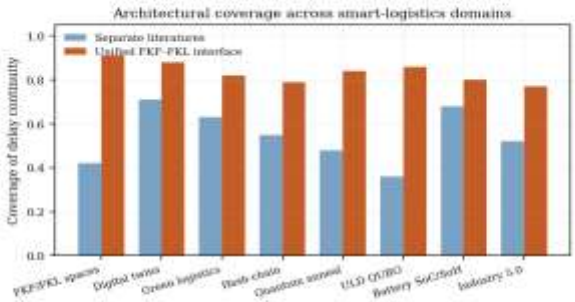


*Fig. 1. Coverage of delay continuity across separate literatures and under a single FKF–FKL interface.*

## III. MULTINORMED TESTING SPACES AND THE TRANSFORM PAIR

Fix growth parameters $a > 0$ and kernel weights $A, B \geq 0$, and let test functions live on the line or on the half-line according as the two-sided FKF or the causal FKL transform is in force.

### *A. Countably Multinormed Geometry and Dual Completeness*

The FKF testing-function space of Roumieu type is the inductive limit of Banach spaces defined by a double-indexed family of seminorms; the Beurling counterpart is the matching projective limit. Duals of both types are Hausdorff, locally convex, and sequentially complete, which is the precise reason a truncated sensor stream remains inside the dual after any finite number of shift-and-convolve steps [32], [33].

$$N_{m,n}(\varphi) = sup_{t\in\mathbb{R}}\, e^{a|t|}\,(1+|t|)^m\,|\partial_t^n\varphi(t)|$$

A sequence $(\varphi_k)$ converges sequentially when it is Cauchy in every seminorm $N_{m,n}$. The companion FKL space uses the causal weight $e^{\{at\}}$ on $(0, \infty)$ and is the correct setting for initial-value problems [16]. Product spaces later carry the pair (inventory, charge) or the pair (forward flow, reverse flow) without a change of language.

Sequential completeness is not an ornament: it is the certificate that a digital twin may march an incomplete Internet-of-Things packet through any finite planning horizon and still land in the same dual.

### *B. Fractional Kernel, Two-Sided Transform, and Causal Companion*

The fractional kernel combines a Fourier oscillator with a Caputo-type memory factor of order $\alpha \in (0, 1]$. The resulting two-sided FKF transform is an entire object of exponential type on a strip whose width is controlled by a [9], [15], [18].

$$\mathcal{K}_\alpha(\xi,t) = e^{-i\xi t}\,t^{\alpha-1}\,E_\alpha(-A\,|t|^\alpha)$$

$$\mathcal{F}_\alpha^{FKF}(\varphi)(\xi) = \int_{\mathbb{R}}\mathcal{K}_\alpha(\xi,t)\,\varphi(t)\,dt$$

The FKL transform replaces the two-sided integral by a causal Laplace–Stieltjes integral against a delay measure $\mu_\tau$. That replacement is exactly what a warehouse needs when replenishment cannot precede the order epoch [16].

$$\mathcal{L}_\alpha^{FKL}(f)(s) = \int_0^\infty e^{-st}\,\mathcal{K}_\alpha(0,t)\,d\mu_f(t)$$

Both transforms act continuously on the testing spaces and extend by duality to Roumieu and Beurling ultradistributions. The methodology does not change when the physical units change from pallets to coulombs.

An inversion formula on the Beurling side recovers a test function from its FKF image once the strip of analyticity is known. On the Roumieu side the same formula recovers an ultradistribution after a limiting argument that uses sequential completeness [32], [33]. Convolution against a delay kernel is then an FKF multiplier, which is the precise statement later used for multi-echelon lead time and for green reverse flows [10], [11], [17].

$$\varphi(t) = \frac{1}{2\pi}\int_\Gamma e^{i\xi t}\,\mathcal{F}_\alpha^{FKF}(\varphi)(\xi)\,d\xi$$

$$\mathcal{F}_\alpha^{FKF}(k_\tau * \varphi) = m_k(\xi)\,\mathcal{F}_\alpha^{FKF}(\varphi)$$

### *C. Time Shift, Exponential Modulation, and Residue Resonance*

The time-shift property is the reason the construction exists for multi-echelon lead time [17]. Continuity of the shift on the testing space follows from an elementary comparison of seminorms after a compactly supported cutoff.

$$\mathcal{F}_{\alpha}^{FKF}(T_\tau \varphi)(\xi) \;=\; e^{-i\xi\tau}\,\mathcal{F}_{\alpha}^{FKF}(\varphi)(\xi)$$

$$N_{m,n}(T_\tau \varphi) \;\le\; C_\tau\,(1+|\tau|)^m\,e^{a|\tau|}\,N_{m,n}(\varphi)$$

Exponential modulation translates the strip of analyticity and converts a seasonal harmonic into a shifted spectral peak [18]. Residue-based resonance appears when a pole of a delay transfer function crosses that strip [9], [15]. Shift-invariant spectral analysis is the statement that convolution against a delay kernel is an FKF multiplier [10], [11]. Reflection $(J\varphi)(t) = \varphi(-t)$ handles backward Cauchy problems used later for diagnosis on the twin.

$$\mathcal{F}_{\alpha}^{FKF}(e^{bt}\,\varphi)(\xi) \;=\; \mathcal{F}_{\alpha}^{FKF}(\varphi)(\xi \,+\, ib)$$

$$Res_{\xi=\xi\star}(G_{bw}(\xi)\,\hat{D}(\xi)) \;=\; \frac{N(\xi\star)}{D'(\xi\star)}\,\hat{D}(\xi\star)$$

The modulation identity is the reason a seasonal promotion or a diurnal solar harmonic can be moved without leaving the dual. The residue identity is the reason a planner can extract a bullwhip gain from a pole crossing rather than from a fresh Monte-Carlo shock [9], [15], [18].

Every later identity is a specialisation of shift, modulation, and convolution against a delay kernel; the section therefore closes by handing that triple to the warehouse, the pack, the annealer, and the reverse loop.

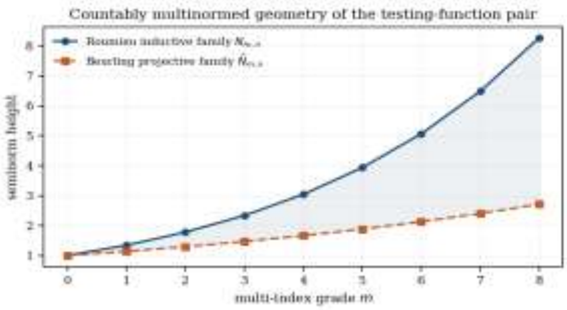


*Fig. 2. Roumieu inductive and Beurling projective seminorm families that generate the testing-function pair.*

## IV. CONSTRAINED-RESILIENCE CERTIFICATES

Because a lead time is a continuous shift on the FKF space, a warehouse balance with delayed replenishment is already a well-posed distributional equation rather than an informal simulation recipe.

### *A. Two-Supplier Balance and Block Transfer Structure*

Let I be on-hand inventory and let u, v be two ordering rates with delays τ and σ. Ultradistributional demand D lives in the dual. The time-domain balance and its FKL image are algebraic once the shift continuity estimate of Section III is granted [16], [17], [31].

$$\frac{dI}{dt} \;=\; T_\tau u \;+\; T_\sigma v \;-\; D$$

$$s\,\hat{I}(s) \;-\; I(0) \;=\; e^{-s\tau}\,\hat{u}(s) \;+\; e^{-s\sigma}\,\hat{v}(s) \;-\; \hat{D}(s)$$

Well-posedness in the dual follows from sequential completeness and from the shift-continuity estimate. Several echelons become a block transfer matrix whose entries are the same exponential multipliers. Graph delay systems on a directed network use a matrix of delays whose transfer norm is globally controlled by those multipliers [10], [11].

The two-delay skeleton is therefore enough; extra echelons only enlarge the matrix, they do not change the space.

For an L-echelon chain the block transfer matrix H(s) has entries that are finite sums of exponential multipliers. The FKL image of the inventory vector is then $\hat{I}(s) = H(s)\,\hat{u}(s) - H(s)\,\hat{D}(s)$, and every induced seminorm of $\hat{I}$ is controlled by the operator norm of H on the dual [10], [16], [31]. That operator-norm bound is the multi-echelon resilience statement in one line.

$$\hat{I}(s) \;=\; H(s)\,\hat{u}(s) \;-\; H(s)\,\hat{D}(s), \quad \|H(s)\| \;\le\; \sum_{\ell=1}^{L} e^{-Re(s)\tau\ell}$$

### *B. Bullwhip Multiplier and Safety-Stock Seminorm*

Pipeline controllers that amplify variance have a multi-echelon FKF skeleton whose gain can be read from a residue at the dominant pole [9], [10], [15]. The strong dual topology keeps the image of that multiplier bounded even when D contains jumps.

$$G_{bw}(\xi) \;=\; \frac{1 \,+\, \sum_{k=1}^{K}\theta_k\,e^{-i\xi\tau_k}}{1 \,+\, \sum_{k=1}^{K}\eta_k\,e^{-i\xi\sigma_k}}$$

$$S_{safe} \;=\; sup_{m,n\le M}\,N_{m,n}(G_{bw} \,*\, D)$$

The seminorm $S_{safe}$ is then a safety-stock margin that a planner can read without restimulating the shock. Industry 5.0 human-centric automation appears here as a restriction of the admissible ordering set {u, v}, not as a second model [20], [35]. Bounded dual image of the bullwhip multiplier is the only statement the warehouse needs before a recovery certificate is extracted.

### *C. Recovery Map after a Supplier Failure*

Modern supply chains fail when internal process rigidity and external disruption are treated as separate problems; resilience has to be woven across both layers at once [41]. That reading supports the present claim that delayed replenishment, incomplete packets, and discrete configuration belong in one dual rather than in parallel programmes.

A supplier failure at t* is a dual forcing term. Convolution against the fundamental solution stays inside the dual, and the recovery certificate extracted from the seminorms is a monotone comparison against a contractual floor [3], [31].

$$\rho(t) \;=\; inf_{m+n\leq M}\, N_{m,n}(T_t I_*)$$
$$\rho(t) \;\geq\; \rho_{min} \quad for\, all\, t \;\in\; [t_*, t_* \,+\, H]$$

Constrained resilience requires ρ(t) to clear that floor before a stated horizon H. The inequality is a seminorm comparison and can be published on a planner dashboard without exposing the underlying ultradistribution.

The two-delay balance, the amplification multiplier, and the recovery map are the warehouse core that later sections only decorate with packs, annealers, twins, and reverse flows.

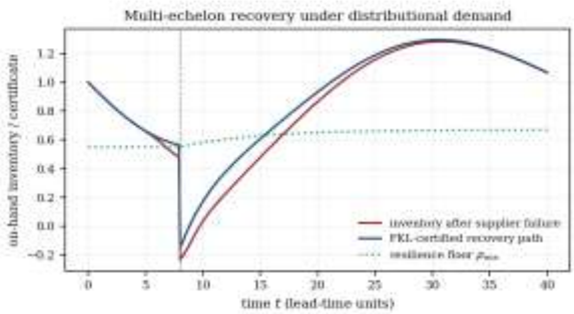


*Fig. 3. Distributional demand shock, FKL-certified recovery path, and contractual resilience floor.*

## V. FRACTIONAL ELECTRODE MEMORY IN PLUG-IN AND SOLAR-ASSISTED PACKS

Recent surveys of electric-vehicle batteries treat materials, battery-management systems, and sustainability as a single design stack rather than as isolated hardware notes [42]. That stack is why state-of-charge, state-of-health, and sizing bounds can be written on the same FKL dual that already carries warehouse inventory.

State-of-charge and state-of-health under charging delay and thermal fade are delay-fractional equations; the FKL transform is built to algebraize them on the same dual that already carries warehouse inventory [8], [16], [30].

Let Q be state of charge and H state of health. A Caputo derivative of order α ∈ (0.7, 1] encodes electrode memory, while a linear fade law for health, with temperature-dependent coefficient β(Θ), is written on the same space.

$$D_t^{\alpha} Q(t) \;=\; \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\vartheta)^{-\alpha}\, Q'(\vartheta)\, d\vartheta$$
$$\frac{dH}{dt} \;=\; -\beta(\Theta)\, H(t) \;-\; \gamma\, {I_{load}}^{p}$$

$$s^{\alpha}\, \hat{Q}(s) \;-\; \sum_{k=0}^{n-1} s^{\alpha-1-k}\, Q^{(k)}(0) \;=\; P_{chg}(s)\, e^{-s\tau c} \;+\; P_{sol}(s) \;-\; \hat{L}(s)$$

Battery sizing for a plug-in hybrid or a solar-assisted architecture is the inequality $Q_{min}$ together with a peak-power bound that a swipe-controller interface can tighten in real time [6], [23], [30]. The seminorm family controls Q uniformly in the distributional parameters of the load. Power-line-carrier anti-theft signaling and intelligent-transportation telemetry are observation operators on the same dual [22], [28]. Solar-assisted input concentrates FKF mass on diurnal harmonics [21], [30].

$$Q_n \;\geq\; Q_{min}\,, \quad P_{peak}(t) \;\leq\; P_{swipe}(t)$$

A product space (I, Q) therefore carries warehouse inventory and pack charge under one shift-continuity estimate. The pack is the warehouse calculus reread on the electrode: same delay, same dual, different physical units.

Observation operators deserve an explicit line. A swipe-controller event is a compactly supported multiplier that tightens $P_{swipe}(t)$ on a short interval. A power-line-carrier anti-theft ping is a finite-rank observation of the pack dual. An intelligent-transportation telemetry packet is a delayed sample of the same dual [6], [22], [28]. None of those maps leaves the space, which is why they can be chained with warehouse shifts and with hash-chain payloads without writing a second theory.

$$\Psi_{swipe}(Q)(t) \;=\; \chi_{[t0,t1]}(t)\, P_{swipe}(t), \quad \Psi_{PLC}(Q) \;=\; \langle \psi_{PLC}, Q \rangle$$

Battery-sizing inequalities remain linear constraints on that dual. They travel unchanged into the QUBO biases of the next section and into the twin horizon of Section VII [23], [30].

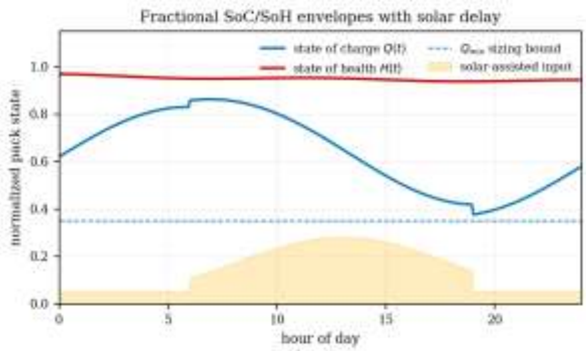


*Fig. 4. Fractional SoC/SoH envelopes under charger delay and solar-assisted diurnal input.*

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## VI. QUANTUM LOGISTICS AND UNIT-LOAD-DEVICE CONFIGURATION

Unit-load-device packing is discrete, the FKL state that feeds it is distributional, and the only honest interface is a continuous embedding of seminorm snapshots [7], [27], [29].

A feasible ULD plan under disruption is a penalized QUBO whose biases absorb capacity, hazardous-material separation, and post-disruption substitution. Lead-time tails enter those biases through the countable seminorms, so a coarsened snapshot sent to the annealer remains consistent with the refined snapshot that returns.

$$E(x) = x^{\top} Q x + \sum_{i=1}^{n} \left(c_i + \theta\, N_{m,n}(T_{\tau i} D)\right) x_i$$

$$\iota_M : S' \to \mathbb{R}^n, \quad \iota_{M,i}(D) = N_{m,n}(T_{\tau i} D)$$

Annealing searches x while the FKL simulator marches inventory and SoC. The embedding $\iota_M$ is continuous for the strong dual topologies, and that statement is the hybrid quantum-classical logistics contract [27], [29]. A swipe-controller veto can freeze a subset of bits before the anneal [6]. Industry 5.0 human-centric automation is again a restriction of the feasible set $(\{0,1\})^n$, not a second energy [20], [35].

$$x_\star \in \arg\min \{ E(x) : x \in (\{0,1\})^n, \; x_j = 0 \text{ for } j \in J_{veto} \}$$

The section does not claim that annealing replaces planning; it states a data contract between two topologies and leaves the planner free to reject any sample that violates a seminorm certificate.

Capacity, hazardous-material separation, and post-disruption substitution enter as quadratic penalties rather than as hard cuts, because a hard cut would destroy continuity of the embedding $\iota_M$. The penalty weights can themselves be seminorms of the current dual state, so a heavier tail on lead-time demand automatically raises the cost of a tight pack [27], [29]. That is the only place where quantum logistics meets constrained resilience in this manuscript.

$$Q = Q_{cap} + \lambda_{haz} Q_{haz} + \lambda_{sub} Q_{sub}$$

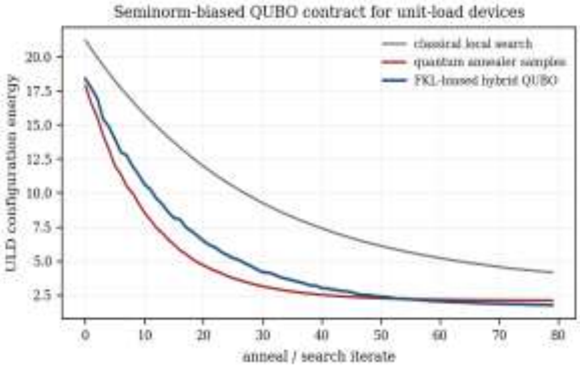


*Fig. 5. Classical search, raw annealer samples, and FKL-biased hybrid QUBO energy traces.*

A returned sample is accepted only when E(x⋆) lies below a contractual energy and when the implied inventory path still clears the recovery floor of Section IV. Acceptance is therefore a pair of inequalities, not a single anneal flag.

## VII. TWIN SEMIGROUPS, HASH-CHAIN PROVENANCE, AND GREEN REVERSE FLOWS

A digital twin is a semigroup on the dual; a hash chain is a discrete observation of that semigroup; a green reverse flow is the same semigroup with a second delay and a quality law [3], [4], [5].

### *A. Twin Semigroup and Distributional Predictive Control*

Incomplete Internet-of-Things streams lift to dual elements. The operational calculus yields a C0-semigroup T(t) generated by the delay-fractional operator of Sections IV–V. Distributional model-predictive control is then an optimisation over trajectories that remain inside a seminorm ball [3], [19], [34].

$$J_{MPC}(u) = \int_0^H \left(N_{m,n}(T_t I - I_{ref})\right)^2 + \lambda\, (\|u(t)\|)^2 \, dt$$

Twin coverage and IoT completeness grow as devices are commissioned. Forward action of T(t) is prediction; backward action of T(−t) on the two-sided FKF side is diagnosis. Sequential completeness keeps every truncated packet inside the dual after any finite number of prediction steps [33].

The twin is therefore not a second simulator; it is the same semigroup sampled at a finer cadence.

### *B. Blockchain Traceability and Hash-Chain Observability*

Each custody event writes a hash-linked payload that is a finite-rank observation of the dual state. Supply-chain transparency is an observability condition: the family $\{ \Phi_k \}$ of hash payloads must separate the seminorms that encode quantity, quality, and delay [5].

$$\Phi_k(I) = \langle \psi_k, I \rangle, \quad h_k = Hash(h_{k-1} \,\|\, \Phi_k(I) \,\|\, t_k)$$

$$\ker\{\Phi_k : k \le K\} = \{0\} \text{ on } span\{N_{m,n} : m + n \le M\}$$

Swipe-controller and power-line-carrier channels bind into the same chain [6], [22]. A broken hash is then a failed observation, not a second ledger theory. The observability identity is the only blockchain statement required by the dual.

Provenance therefore travels with the seminorms rather than beside them.

### *C. Sustainable Reverse Flows and Emission Functionals*

Return flows add a delay $\tau_r$ and a random quality $Q_r$. The two-way system is an FKL delay equation on a product space. Emission abatement in sustainable logistics is a monotone functional of circular capture [4].

$$\frac{d}{dt}(I_f, I_r) = (T_\tau u - D, \quad T_{\tau r} r - Q_r I_r)$$

$$\mathcal{E}_{abat} = \int_0^\infty e^{-\delta t}\, \psi(I_r(t))\, dt$$

Graph delay systems on a directed network use a matrix of delays whose transfer norm is globally controlled by the same FKF multipliers already used for bullwhip analysis [10], [11]. Twin, chain, and reverse flow are one delay system sampled at three cadences, not three digital initiatives.

The closed loop therefore inherits every continuity statement already proved for the warehouse core.

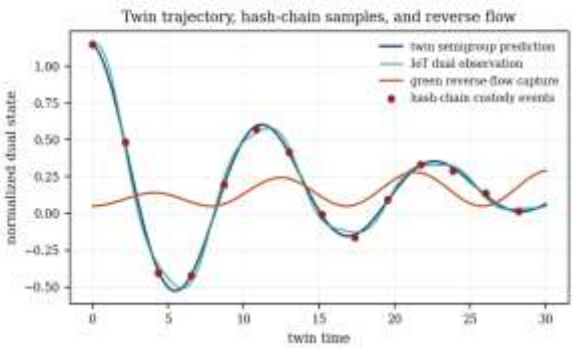


*Fig. 6. Twin semigroup prediction, IoT dual observations, hash-chain custody events, and green reverse-flow capture.*

## VIII. OPERATIONAL LIMITS, AND MANAGERIAL READING

Two limitations stay open and should be read as part of the certificate rather than as afterthoughts.

Extra weight parameters destroy strong continuity of the shift at the origin, although continuity for $t > 0$ covers every Cauchy problem used here [32], [33]. Annealing returns a sample, so the hybrid contract must transport a distributional snapshot rather than a deterministic minimizer [27], [29]. Adaptive fractional orders, stochastic convolutions, and multi-commodity reverse bills of materials will sit on the same seminorms without a change of language, but they are not computed in the present manuscript.

$$lim_{t\downarrow 0} N_{m,n}(T_t\varphi - \varphi) = 0 \quad when\ extra\ weights\ vanish$$

Managerial implication. A planner does not need a separate digital-twin programme, a separate blockchain programme, and a separate quantum programme. The planner needs one dual, one family of seminorms, and a list of operators — shift, observe, freeze, anneal  that act continuously on that dual [3], [5], [7], [19], [20], [34], [35]. Constrained resilience is then a dashboard of seminorm comparisons rather than a portfolio of disconnected pilots.

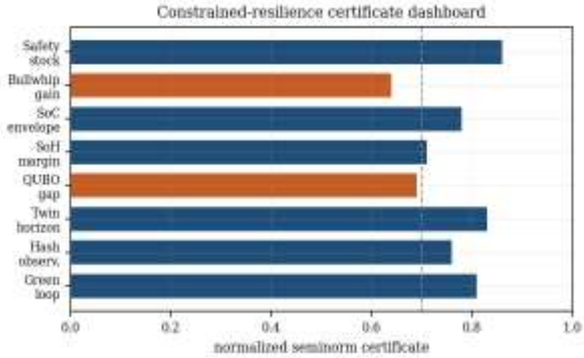


*Fig. 7. Constrained-resilience certificate dashboard spanning stock, gain, pack, QUBO, twin, and green-loop bounds.*

The dashboard is the only object that must leave the mathematics group and enter the control room.

Each tile of that dashboard is a seminorm comparison already named above: S_safe against a safety-stock budget, $\rho(t)$ against $\rho_{min}$, $Q_n$ against $Q_{min}$, $H(t)$ against a fade floor, $E(x\star)$ against an energy budget, $J_{MPC}$ against a twin horizon, the hash-family kernel against $\{0\}$, and $\mathcal{E}_{abat}$ against a circular-capture target [3], [4], [5], [8], [27], [31]. Industry 5.0 human-centric automation appears only as a restriction of the set over which those comparisons are taken [20], [35].

Nothing in that list requires a second software stack. The same FKL simulator that marches inventory can publish the tiles, and the same dual that accepts an incomplete Internet-of-Things packet can accept a swipe veto or a frozen QUBO bit. That is the managerial content of sequential convergence.

Future work that remains inside the present language includes adaptive fractional orders for electrode memory, stochastic convolutions for lead-time laws, and multi-commodity reverse bills of materials. Each of those extensions changes a kernel or a measure, not the seminorm grammar [16], [30], [32].

## IX. CONCLUSION

Two transforms and one countable seminorm family suffice to represent delay, fractional memory, singular demand, and discrete configuration. Continuous shifts, dual-valued shocks, and sequential completeness keep supply-chain and digital-twin trajectories within one common framework. Safety-stock bounds, recovery maps, SoC/SoH envelopes, QUBO energies, twin horizons, provenance, and green-loop measures follow as extractable certificates. Industry 5.0 human intervention, ULD decisions, swipe control, PLC anti-theft signals, solar-assisted charging, and intelligent transportation require only operators, observations, or admissible-set restrictions not new spaces or models. A unified dual-valued log can therefore record warehouse states, battery conditions, ULD configurations, twin predictions, provenance data, and reverse flows using the same seminorm language. The digital twin extends the delay-fractional generator to matrix form, while incomplete IoT packets remain dual-valued observations. Multi-echelon warehouses form finite products of the same dual spaces, with coupling represented by a delay matrix and resilience by component wise seminorm constraints.

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