



Tensor Contraction Rules for the Operational Einstein Equation: Dummy Indices, Traces, and Conserved Maps on Lead-Time Geometry

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Abstract

Einstein's field equation is already a statement about contraction. The Einstein tensor is Ricci minus a metric-weighted trace of Ricci. Ricci itself is a contraction of Riemann. The scalar curvature is a second contraction. Bianchi conservation is a further contraction of the differential Bianchi identity. None of those steps is optional bookkeeping. If a dummy index is unpaired, the object is not a tensor of the rank the equation requires. If two free indices are contracted that should have remained free, the field equation collapses to a scalar and can no longer source a metric. This paper writes the contraction rules that turn Riemann into $G_{\mu\nu}$ and that keep $\nabla^\mu G_{\mu\nu} = 0$, then reads the same rules on an operational lead-time metric whose stress-energy is inventory, congestion, charge, and blocked lots.

Keywords: Einstein summation; dummy index; metric contraction; Ricci tensor; Ricci scalar; Einstein tensor; Bianchi identity; operational metric.

1. Introduction

A field equation that cannot be contracted cannot be checked. Digital replicas that write $G_{\mu\nu}$ on a dashboard without showing the dummy pairs that produced it are displaying a symbol, not an identity. Bidirectional twins make the replica writable as policy [1]. They do not disclose which index was summed when Ricci was formed from Riemann. That disclosure is the first

task of a contraction paper.

The summation convention is older than logistics software and younger than the need to keep rank straight. A repeated index, once up and once down, is a dummy and is summed. An index that appears once is free and labels a slot the tensor still carries. Raising and lowering with the metric changes the height of a slot without changing the dummy status of a pair. Those three sentences are the whole grammar. The rest of the paper is that grammar applied to the operational Einstein equation already written for lead-time geometry.

Section 2 states related work as sources of T and gauges of spectra, not as a second pairing law. Section 3 writes the summation convention. Section 4 writes single and double contraction. Section 5 writes the metric as a pairing machine. Section 6 contracts Riemann to Ricci. Section 7 contracts Ricci to R. Section 8 builds $G_{\mu\nu}$. Section 9 treats Bianchi as a contracted identity. Section 10 concludes.

Complexity written as a weave [27] is already a warning against flattening rank. A weave that sums every index because a dashboard has one colour is how becomes a slogan. Visibility after digital transformation [13] does not license that flattening. Mixed shocks still have tensor rank [26]. $G_{\mu\nu}$

2. Related work as sources, not as pairing

Green-AI predictors populate entries that will sit in after they are converted to energy density [2]. They do not decide which index of Riemann is dummy. Append-only custody restricts which worldlines may carry T [3]. Clocked sensing reminds the reader that components are stamped in a frame [4]. Battery sizing and management cap the charge piece of T [5]. A swipe-controller ancestor is a clock, not a contraction [6]. Quantum roadmaps list engines that may later change

occupancy and therefore T [7]. Unit-load annealing realizes those occupancies only after execution [8]. Mobility surveys restrict the domain on which is defined [9]. Reserve floors from battery reviews cap dissipation [10]. Sample sets matter because two unexecuted plans share T [11]. Solar inflow changes a worldline source without changing dummy rules [12]. The solar-hybrid concept is the architectural ancestor of that source [25]. Automotive battery reviews fix the numerical knee [28]. $T_{00}g_{\mu\nu}$

Distributional FKF pairing is a different pairing: test functions against signals [14]. Sequential windows compare contracted scalars across time [15]. Residue extraction locates energy that will enter [16]. Shift-invariant lead-time maps are translations on a flat patch [17]. Spectral resilience readings apply after a scalar has been formed [18]. Differential symbols keep honest in a chosen gauge [19]. Algebraic FKF identities constrain that gauge [20]. Time-shift multipliers act on already contracted scalars [21]. Exponential modulation belongs on payloads, not on dummy pairs [22]. IoT–AI–quantum stacks populate observations that feed T [23]. Industry 5.0 requires that dummy pairs be readable on a desk [24]. None of those papers is a substitute for $T_{00} \partial_\mu g^{\{\mu\nu\}} R_{\mu\nu} = R$

3. Einstein summation and index pairing

Figure 1. Einstein summation convention and tensor index pairing

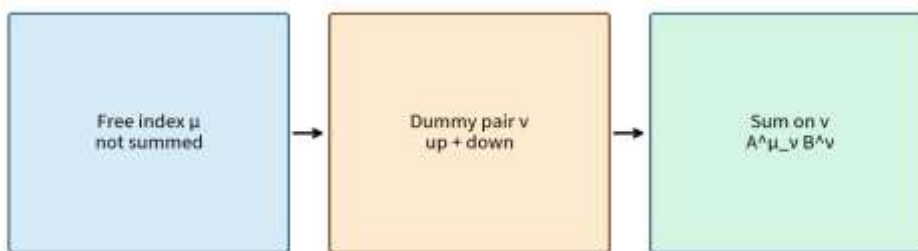


Figure 1. Einstein summation convention and tensor index pairing.

Let a tensor component carry indices written up or down. The summation convention is

$$A^\mu_\nu B^\nu \equiv \sum_\nu A^\mu_\nu B^\nu \quad (3.1)$$

The index ν is dummy. It may be renamed λ without changing the value. The index μ is free. The result is a vector, not a matrix. Writing both indices of A at the same height and then “summing” them is not a contraction unless the metric is inserted. That insertion is Section 5, not a silent habit.

$$A^\mu_\mu \text{ is legal; } A^{\mu\mu} \text{ is not a contraction} \quad (3.2)$$

A free index must appear in every term of an equation at the same height. The operational field equation therefore cannot add a scalar delay D to without a metric factor. That is already a pairing rule, not a modelling preference. $G_{\mu\nu}$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \quad (3.3)$$

Every term in (3.3) is a symmetric (0,2) tensor. Dummy indices inside $G_{\mu\nu}$ are hidden in its definition. Making them explicit is the work of Sections 6–8.

3.1 What may be renamed

Dummy names are arbitrary. Free names are not. A twin that stores components under a private dummy label must still export free indices in a disclosed frame. Relabelling μ as x on a dashboard that already used x for the lane coordinate is how pairing errors enter software.

Range of a dummy is the dimension of the operational plane. On the two-dimensional lead-time plane a dummy run over $\{0,1\}$. Adding a charge coordinate enlarges the range. Contracting as if the range were four when the metric is two-dimensional invents two silent zeros. Publishing dimension next to κ is part of the pairing discipline.

4. Single and double contraction

Figure 2. Single and double contraction of higher-rank tensors

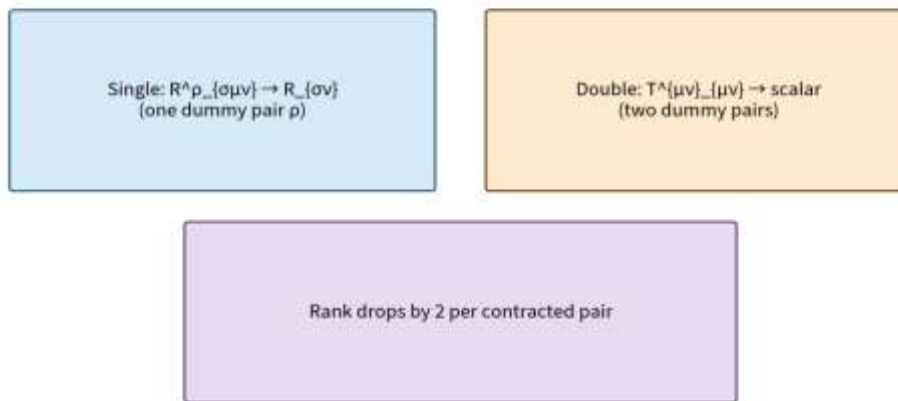


Figure 2. Single and double contraction of higher-rank tensors.

A single contraction of a rank-(k+2) tensor on one up–down pair yields rank k. A double contraction yields rank k–2 relative to the original if two pairs are summed. Ricci is a single contraction of Riemann. The Ricci scalar is a second contraction. The Einstein tensor keeps rank 2; it is Ricci minus a double-contracted piece put back with the metric.

$$C_{\sigma\nu} = R^{\rho}_{\sigma\rho\nu} \quad (4.1)$$

$$S = T^{\mu\nu}_{\mu\nu} \quad (4.2)$$

Equation (4.1) is the pattern of Ricci. Equation (4.2) is a complete trace of a (2,2) tensor. One may not mix them. Tracing $T_{\mu\nu}$ with $g_{\mu\nu}$ is (4.2) only after T is written as $T^{\mu\nu}_{\mu\nu}$ or after the metric is inserted twice. The operational T built from inventory and charge is (0,2) in the planner frame, so its scalar is

$$T = g^{\mu\nu} T_{\mu\nu} \quad (4.3)$$

4.1 Rank audit

Before a line is accepted into the twin, count free indices on each term. If they disagree, the line is not an equation. If they agree but a dummy appears three times, the line is not a tensor expression. Those two audits catch most dashboard algebra.

A residue scalar has rank 0 and may multiply . It may not be added to . That is why mix error and curvature scalars enter sources only after they are converted to energy density of the correct rank. $g_{\mu\nu} T_{\mu\nu}$

5. Metric as a pairing machine

Figure 3. Metric tensor and index raising / lowering

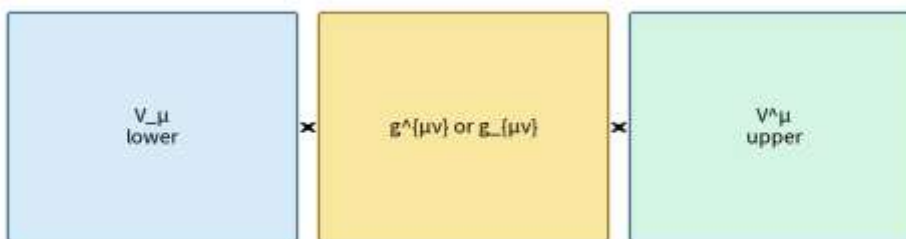


Figure 3. Metric tensor and index raising and lowering operations.

The metric and its inverse satisfy

$$g_{\mu\lambda} g^{\lambda\nu} = \delta_{\mu}^{\nu} \quad (5.1)$$



Raising and lowering are contractions with that pair

$$V^\mu = g^{\mu\nu} V_\nu, \quad V_\mu = g_{\mu\nu} V^\nu \quad (5.2)$$

$$T^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} T_{\alpha\beta} \quad (5.3)$$

On a flat patch and raising only changes signs according to signature. On a curved operational patch the metric itself carries the pile-up. Lowering an index of T with a curved g mixes energy density into flux components. That mixing is physical in the operational sense: a well in changes how a planner reads $g_{\mu\nu} = \eta_{\mu\nu} g_{tt} T^{\{0x\}}$

5.1 Compatibility

Levi-Civita compatibility means raising and lowering pass through covariant derivatives. Without it, contracting ∇T after raising an index would not match contracting first. The field equation inherits that compatibility from the connection used to build Riemann. A twin that differentiates components with ordinary ∂ and then raises with a position-dependent g has already broken pairing. $\nabla_\lambda g_{\mu\nu} = 0$

$$\nabla_\lambda g_{\mu\nu} = 0 \quad (5.4)$$

6. Riemann to Ricci

Figure 4. Contraction of the Riemann tensor to the Ricci tensor

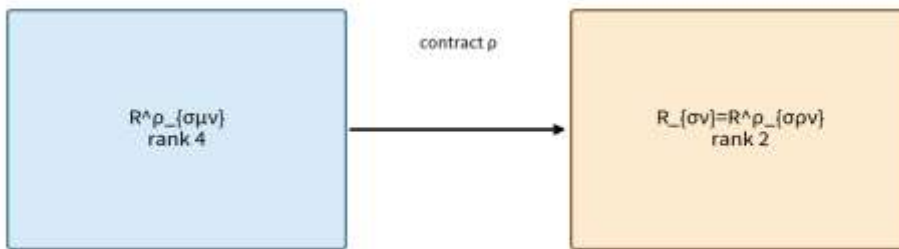


Figure 4. Contraction of the Riemann tensor to the Ricci tensor.

The Riemann tensor of the operational metric is the usual (1,3) object built from Γ . Ricci is the contraction on the first and third slots in the common convention

$$R_{\sigma\nu} = R^{\rho}_{\sigma\rho\nu} \quad (6.1)$$

Other slot choices differ by signs from Riemann symmetries. The paper freezes (6.1). A software library that contracts a different pair must disclose the pair. Silent disagreement is how two twins report opposite Ricci signs on the same hub.

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma_{\nu\sigma}^\rho - \partial_\nu \Gamma_{\mu\sigma}^\rho + \Gamma\Gamma - \Gamma\Gamma \quad (6.2)$$

Geodesic deviation uses the uncontracted Riemann. Ricci is not a substitute for that. A desk that shows only $R_{\mu\nu}$ cannot display peeling of nearby worldlines except in averaged form. Both objects are required: Riemann for (3.7) of the field-equation paper, Ricci for $G_{\mu\nu}$.

6.1 Operational reading

Ricci measures, in a contracted sense, how a small ball of nearby shipments changes volume as they pass a hub. Volume loss is a pile-up source. Volume gain is a rarefaction. That reading is why couples most strongly to in the weak field. It is not a reason to ignore off-diagonal Ricci when flux is large. $T_{00} R_{00}$

7. Ricci to Ricci scalar

Figure 5. Contraction of the Ricci tensor to the Ricci scalar

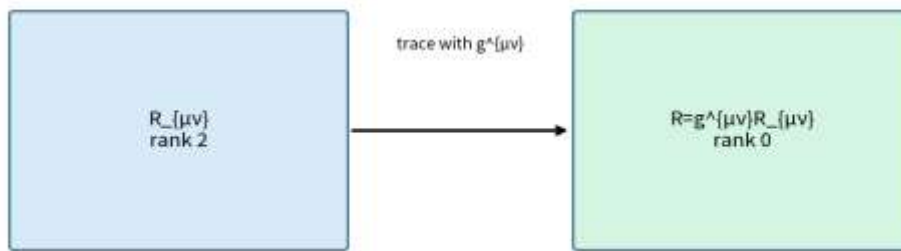


Figure 5. Contraction of the Ricci tensor to the Ricci scalar.

The scalar is the metric trace

$$R = g^{\mu\nu} R_{\mu\nu} \quad (7.1)$$

Equivalently after raising one index. On the two-dimensional operational plane the Einstein tensor is completely determined by R , because in two dimensions Ricci is proportional to the metric times R . That is a contraction identity special to dimension two and must be stated when the lead-time plane is used without a charge axis. $R = R^\mu_\mu$

$$R_{\mu\nu} = (R/2) g_{\mu\nu} \quad (dim = 2) \quad (7.2)$$

If a third coordinate is added, (7.2) fails and $G_{\mu\nu}$ carries independent information. Teams that quote (7.2) on a three-dimensional operational spacetime are using a false contraction. Publishing dimension next to (7.1) prevents that false step.

7.1 Trace reversal

The field equation's Ricci form is the trace-reversed partner of the Einstein form

$$R_{\mu\nu} = \kappa (T_{\mu\nu} - (1/2) T g_{\mu\nu}) + \Lambda g_{\mu\nu} \quad (7.3)$$

Trace reversal itself is a contraction: take (3.3), contract with $g^{\mu\nu}$, solve for R , and substitute back. Missing that step and writing is the Newtonian habit in tensor clothing. $g^{\mu\nu} R_{\mu\nu} = \kappa T_{\mu\nu}$

$$R - 2\Lambda = -\kappa T \quad (\text{mostly} - \text{minus or mostly} - \text{plus changes sign of } \Lambda T \text{ bookkeeping}) \quad (7.4)$$

8. Contraction structure of the Einstein tensor

Figure 6. Tensor contraction structure of the Einstein tensor

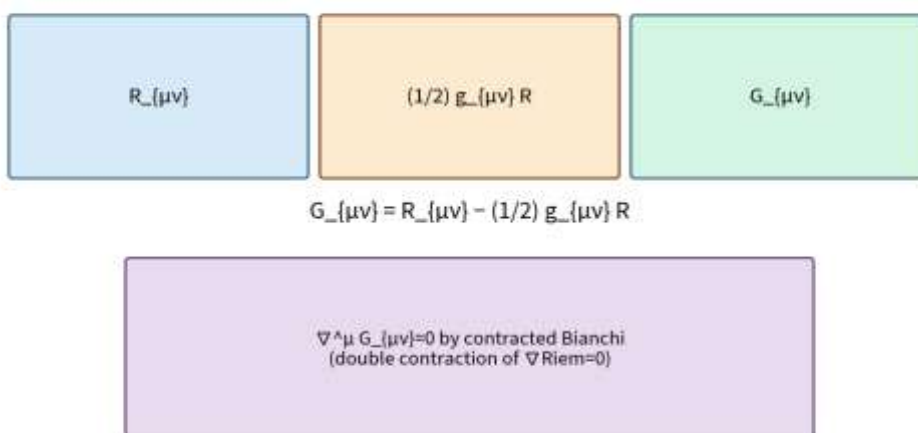


Figure 6. Tensor contraction structure of the Einstein tensor.



By definition

$$G_{\mu\nu} = R_{\mu\nu} - (1/2) g_{\mu\nu} R \quad (8.1)$$

The second term is Ricci contracted to a scalar and then spread back with the metric. That is why G is divergenceless when Ricci comes from a Levi-Civita Riemann. Removing the second term to “simplify the dashboard” destroys conservation. The desk that shows should show both pieces, or it cannot be audited. $G_{\mu\nu}$

$$G_{\mu}^{\mu} = -R \quad (\text{trace of the Einstein tensor}) \quad (8.2)$$

The trace of G is $-R$ in four dimensions with the usual factor; in two dimensions G vanishes identically by (7.2), which is why a pure 2-D operational Einstein equation needs care: $G_{\mu\nu} = 0$ forces $T_{\mu\nu} = -(\Lambda/\kappa) g_{\mu\nu}$. Sources then look cosmological. That is a contraction warning, not a reason to abandon the 2-D plane. It is a reason to add a third coordinate or to treat Λ as the standing lag that balances background T.

8.1 Field equation assembled

$$R_{\mu\nu} - (1/2) g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \quad (8.3)$$

Every symbol in (8.3) has now been named as a contraction of something prior, except Λ and κ , which are policy, and T, which is assembled from named densities. That is the whole content of “genuinely related to tensor contraction rules of the field equations.”

3.2 Explicit two-dimensional sums

On the lead-time plane the dummy range is $\{0,1\}$. The pairing (3.1) expands as

$$A_{\nu}^{\mu} B^{\nu} = A_0^{\mu} B^0 + A_1^{\mu} B^1 \quad (3.4)$$

No third term appears. Writing a sum through $\nu = 3$ as if the plane were spacetime is a four-dimensional habit and invents vanishing components. Every operational implementation of (3.3) should print the range next to the dummy.

Free-index matching on (3.3) is a three-term check: $G_{\mu\nu}$ has two free slots, $\Lambda g_{\mu\nu}$ has two, $\kappa T_{\mu\nu}$ has two. Adding a residue scalar $R(t)$ without $g_{\mu\nu}$ fails the check. Multiplying that scalar by $g_{\mu\nu}$ restores rank and is the only legal insertion of a rank-0 diagnostic into the field equation.

4.2 Component form of a single contraction

For Riemann on a 2-D plane the only independent component may be taken as R^0_{01}. Ricci then has

$$R_{00} = R^{\rho}_{0\rho} = R^0_{000} + R^1_{010} \quad (4.4)$$

$$R_{11} = R^{\rho}_{1\rho}, \quad R_{01} = R^{\rho}_{0\rho 1} \quad (4.5)$$

Terms with three identical indices vanish by antisymmetry of Riemann in the last pair. The surviving pieces are the ones a twin must store. Storing all 16 naive slots of a 2-D (1,3) tensor without symmetries wastes space and invites dummy errors.

5.2 Raised field equation

Raising one index on (3.3) produces a mixed tensor equation used when comparing to $\nabla^{\mu} G_{\mu\nu}$

$$G_{\nu}^{\mu} + \Lambda \delta_{\nu}^{\mu} = \kappa T_{\nu}^{\mu} \quad (5.5)$$

The Kronecker delta is the contraction (5.1). Mixed indices make conservation look like an ordinary divergence in a coordinate basis only after connection terms are included. Dropping Γ terms while using (5.5) is another pairing error: height was changed but the derivative was not.

6.2 Symmetries that survive contraction

Ricci is symmetric. That is a contraction identity, not an extra assumption. A twin that stores independent R_{01} and R_{10} and lets them drift has broken the pairing that produced Ricci. The audit is

$$R_{\mu\nu} - R_{\nu\mu} = 0 \quad (6.3)$$

Einstein G inherits the symmetry. T on the right-hand side must therefore be symmetrized if a flux construction produces an antisymmetric remainder. That remainder is not sourced curvature; it is a modelling error in T.

8.3 Weak-field traces

In linearized theory the trace of h is $h = \eta^{\{\mu\nu\}} h_{\mu\nu}$, another contraction. The trace-reversed perturbation is

$$h_{\mu\nu} - (1/2) \eta_{\mu\nu} h \quad (8.4)$$

That combination is G linearized, up to constants and gauge. Teaching contraction on the exact G and then again on h keeps the same dummy habit in the weak-field paper's Poisson map. Skipping the trace piece of (8.4) is the same crime as dropping $(1/2)gR$ in (8.1).



A compact hub with only active still produces a scalar plus corrections. Using without the metric factor mis-sets the trace-reversed source in (7.3). The factor is a contraction. It is not optional rounding. $T_{00}T = g^{\{00\}} T_{00} T = T_{00}$

9. Contracted Bianchi identity

The differential Bianchi identity on Riemann is a rank-5 statement. One contraction produces a relation among ∇Ricci and ∇R . A second contraction produces

$$\nabla^\mu G_{\mu\nu} = 0 \quad (9.1)$$

hence from the field equation when κ and Λ are constant. Dummy indices in (9.1) are μ . The free index ν labels the conserved current. Losing track of which is dummy is how a twin conserves the wrong component. $\nabla^\mu T_{\mu\nu} = 0$

$$\nabla^\mu T_{\mu\nu} = 0 \quad (9.2)$$

9.1 Worked two-dimensional count

On a 2-D plane Riemann has one independent component, Ricci has three components subject to (7.2), and G vanishes. Conservation (9.2) remains and is the useful identity. The Einstein tensor is the wrong poster in that dimension; the contracted Bianchi on Ricci is not. Publishing that count keeps a 2-D deployment from hunting a nonzero that algebra forbids. G_{00}

9.2 What a trial must freeze

A trial of conservation should freeze dummy conventions, metric signature, and dimension. Changing the contracted pair in (6.1) mid-trial changes the sign of Ricci and is not a test of T . Changing dimension mid-trial changes whether G vanishes. Those are algebra changes, not operational findings.

Predictors, annealers, and solar certificates may still vary as sources. They vary T . They must not vary the pairing law. That separation is how this paper stays distinct from the source papers.

10. Discussion and conclusion

An index that appears twice at the same height is a programming smell. Either the metric was forgotten or the component array was flattened. Code review of a twin should search for that smell before it searches for a new kernel.

Renaming a dummy after a partial contraction is legal and often clarifying. Renaming a free index to the name of a dummy already in scope is not. Scope is the term. Languages that reuse μ for every slot invite the bug the convention was invented to prevent.

Parentheses do not contract. Writing $(R_{\mu\nu})(g^{\{\mu\nu\}})$ without summation intent is juxtaposition. The convention supplies the sum only when the same letter is used as a pair. Two different letters that happen to be printed in similar fonts are not a pair.

The operational plane's volume form is not needed to contract tensors of rank two, but it is needed if one later dualizes. Dualization is not in this paper. Mentioning it only marks a boundary: contraction and Hodge dual are different pairings. When charge is added as the dummy range becomes $\{0,1,2\}$. Every identity from (3.4) onward gains one term. Ricci is no longer forced to become a genuine three-component story in 3-D plus symmetries. That enlargement should be a versioned change of the twin, not a silent extra loop in a summer intern's script. $x^2(R/2)g_{\mu\nu}$

10.1 Discussion

Contraction is not a visual style. It is the only way Riemann becomes a source-ready tensor of rank 2 and the only way that tensor stays conserved. The operational reading does not change the algebra. It changes what is allowed to sit in T and what a desk must show. Companion papers keep their lanes: cones, sourced G , residue wakes, desks, multi-commodity budgets. This paper is the dummy-index manual those lanes were using whenever they wrote or $R_{\mu\nu}$

Limits remain. Dimension of the operational manifold is a choice. Slot convention for Ricci is a choice. Both must be published. Linearized traces ignore back-reaction. FKF windows act on scalars after contraction and cannot repair a wrong dummy pair.

10.2 Conclusion

The Einstein equation is a chain of contractions: Riemann to Ricci, Ricci to R , Ricci and R to G , Bianchi to $\nabla \cdot G = 0$, and the field equation to $\nabla \cdot T = 0$. Dummy indices are paired once up and once down. The metric is the machine that changes height so pairing is possible. On an operational lead-time metric those rules decide whether a dashboard symbol



is $G_{\mu\nu}$ or a flattened delay colour. Implementation is an audit of rank, of dummy range, and of the two traces that produce R and G. Measurement that keeps those audits fixed and varies only T is a test of the sourced geometry. Measurement that relabels dummies until a colour looks right is not.

The short test is the same as the algebra. If a line adds tensors of unequal free rank, it is not the field equation. If G is shown without the $(1/2)gR$ piece, conservation is unearned. If R is shown without $g^{\{\mu\nu\}}$, the scalar is unearned. Earning those three pieces is the work contraction names.

Summation	one dummy pair	rank – 2
Riemann $\rightarrow$ Ricci	$R^\rho_{\{\sigma\rho\nu\}}$	$4 \rightarrow 2$
Ricci $\rightarrow$ R	$g^{\{\mu\}} R_{\{\mu\nu\}}$	$2 \rightarrow 0$
Ricci, R $\rightarrow$ G	$R_{\{\mu\nu\}} - \left(\frac{1}{2}\right)g_{\{\mu\nu\}}R$	2
Bianchi	$\nabla^\mu G_{\{\mu\nu\}}$	1

Table 1. Contraction chain from Riemann to conservation.

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