Twin-Driven Spectral Modeling of Fractional Battery Dynamics with Dual FKF Topology

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Abstract—
This paper develops a twin-orchestrated framework for fractional battery trajectories in which delayed state evolution, fractional memory, and spectral representations are treated within a dual-topology FKF setting. The construction is motivated by the battery sizing, ageing, mobility, solar-assistance, and intelligent-transport perspectives represented in the supplied literature. A countably multinormed test-function structure is used to keep delay shifts continuous while its dual admits impulsive, incomplete, and disruption-driven observations. Fractional state-of-charge and state-of-health trajectories are then coupled to a synchronized digital-twin layer so that the physical battery state, its delayed spectral representation, and the computational replica remain mutually consistent. The resulting architecture also provides a controlled interface to resilience analytics, artificial intelligence, blockchain traceability, and hybrid quantum optimization. The emphasis is theoretical: the FKF topology supplies admissible growth, sequential completeness, shift continuity, and recovery bounds, while the twin supplies an operational mechanism for replay, prediction, and uncertainty management. The framework therefore connects battery trajectory mathematics with intelligent mobility and resilient logistics without treating any individual computational technology as a replacement for the underlying physical or functional-analytic constraints.

Keywords—Fractional battery dynamics; FKF transform; dual topology; digital twin; state of charge; state of health; delay systems; spectral analysis; smart mobility; supply chain resilience; quantum logistics; sustainable mobility.

I. INTRODUCTION

Electrified mobility converts battery state from a local engineering variable into a time-dependent trajectory that must remain interpretable under delay, ageing, sensing uncertainty, and changing operating conditions. Battery-sizing analysis establishes the energy requirement at pack level, while subsequent battery reviews show that usable performance depends on cell, module, thermal, management, and ageing effects [1]. These effects become more consequential when a controller receives delayed measurements or when a digital twin must replay a trajectory that contains abrupt changes.

The central premise of this study is that a fractional trajectory should not be handled only as a numerical curve. Fractional models represent memory, and delayed measurements introduce a second nonlocal mechanism. A useful mathematical environment must therefore control derivatives, growth, and time translation simultaneously. The FKF construction is used here as that environment, with the primal space carrying admissible battery trajectories and the dual space carrying generalized observations, impulses, missing samples, and disruption signatures.

The physical interpretation is strengthened by recent work on automotive batteries and electric-vehicle technologies. Battery architecture, materials, management systems, and sustainability considerations determine the bounds within which a trajectory can be regarded as physically meaningful [4], [5]. Smart-mobility systems add an observation layer in which vehicle and infrastructure telemetry are continuously acquired, transmitted, and interpreted [6]. Solar-assisted mobility introduces another energy contribution that can be synchronized with the battery state rather than treated as an independent subsystem [7].

The FKF viewpoint also connects naturally to multi-scale spectral analysis. Residual oscillations in hybrid-vehicle and supply-chain traces can be represented by spectral markers and residue-based resonance measures, providing a bridge between raw trajectories and topological seminorms [8]. This makes the framework suitable for trajectories in which ordinary smoothness is too restrictive but uncontrolled generalized functions would be too broad.

A second motivation is resilience. Anti-theft sensing and release-control mechanisms illustrate that an observed electrical change should not automatically be interpreted as a legitimate operational event [2], [3]. The mathematical model therefore separates observation from authorization: the twin may estimate a future state, but an operational action remains subject to its admissibility rules. This distinction is important for battery dispatch, charging, recovery, and logistics decisions.

The paper develops the construction in four stages. First, a dual FKF topology is defined for delayed fractional trajectories. Second, the fractional battery model and its spectral representation are established. Third, the digital twin is coupled to the mathematical state through synchronization and uncertainty envelopes. Finally, resilience, intelligent mobility, sustainability, traceability, and hybrid quantum optimization are treated as controlled interfaces rather than as replacements for the battery dynamics.

Problem formulation and contribution

Let x(t) denote a normalized battery state, such as state of charge, and let u(t) denote a bounded charging or discharge input. The basic fractional-delay model considered here is

$$D_t \alpha\, x(t) = -a\, x(t) - b\, x(t-\tau) + c\, u(t) + q(t)$$

where $0<\alpha\leq 1$ represents the fractional memory order, τ is an effective delay, and q(t) collects disturbances. The model is intentionally compact: detailed electrochemical submodels can be inserted later provided their trajectories satisfy the admissible FKF bounds.

The contribution is a unified mathematical interface in which fractional memory, delay shifts, generalized observations, and twin synchronization share one topology. The supplied literature supports the physical battery and mobility constraints [1], [4]–[7], the spectral interpretation [8], and the resilience and digital-transformation context [11]–[15]. Quantum-logistics studies further motivate a selective discrete optimization interface for decisions that must be coupled to continuous state propagation [9], [16], [17].

II. LITERATURE SYNTHESIS

Battery trajectory foundations

Battery sizing provides the first constraint: the trajectory cannot be interpreted independently of the available energy capacity and the power demand imposed by the vehicle. The Formula Hybrid study gives a constructive engineering basis for pack sizing [1]. Broader automotive battery research expands this view toward chemistry, thermal behavior, ageing, and battery-management functions [4]. Recent EV battery technology reviews place materials, management, and sustainability in the same design space [5]. For the present framework, these findings are translated into admissible bounds on x(t), its derivatives, and the rate of change of the trajectory.

Mobility sensing and energy augmentation

Smart mobility and intelligent transportation systems provide the telemetry layer from which the twin obtains observations [6]. The data stream may contain irregular sampling, missing intervals, abrupt transitions, and communication delays. Solar-assisted electric mobility adds an exogenous energy channel that modifies the effective input u(t), while preserving the need for a consistent state trajectory [7]. Thus, sensing and energy harvesting are represented as inputs to the same delayed fractional state rather than as disconnected modules.

Spectral and generalized-function foundations

Multi-scale spectral analysis motivates the use of frequency-like coordinates to distinguish persistent oscillations from isolated events [8]. Generalized-function theory supplies the mathematical language for impulses and singular observations. The classical distributional perspective is extended here through a countable family of seminorms so that the FKF transform can act on trajectories whose ordinary pointwise description is insufficient. Fractional transform theory and generalized integral-transform theory in the supplied references provide supporting context for nonlocal operators and transform-domain reasoning [18]–[24].

Intelligent, traceable, and resilient mobility

Artificial intelligence in sustainable logistics emphasizes forecasting, optimization, and data-driven decision support [10]. AI becomes safer when its output is constrained by physical state bounds rather than used as an unconstrained trajectory generator. Blockchain integration adds provenance and traceability, allowing the observation interval used by the twin to be audited [11]. Digital-twin research provides the synchronization architecture required to maintain a computational replica of the physical system [12]. Resilience reviews and complexity studies reinforce the need to model recovery, interdependence, and internal–external disruption jointly [13], [14], while digital transformation frames these mechanisms as part of a closed resilience loop [15].

Quantum optimization as a controlled interface

Quantum-computing studies in transportation and logistics identify combinatorial optimization as a suitable application domain, but also emphasize the importance of comparing quantum and classical methods on equivalent instances [9], [16]. ULD configuration under disruption provides a concrete example in which a discrete decision can be optimized while the battery or logistics state remains propagated classically [17]. The present architecture adopts this separation: the FKF system determines admissible continuous trajectories, while an optimizer receives only a constrained decision representation.

III. FKF DUAL BATTERY TRAJECTORIES

Admissible test-function space

Let Φ denote the battery test-function space. Its topology is generated by a countable family of seminorms that jointly control derivative order, temporal growth, and the scale variable used by the FKF kernel. A representative seminorm is written as

$$pm_k(\varphi) = sup_{\{t\in R\}}\,(1+|t|)-m^{\,|D}k^{\varphi(t)|}$$

The notation emphasizes the design principle rather than a single mandatory weight: increasing m controls temporal growth while increasing k controls derivative regularity. The resulting space is locally convex and countably multinormed. Sequential completeness is essential because a convergent sequence of approximating battery trajectories must remain within the admissible class.

Dual space and singular observations

The strong dual Φ′ contains generalized observations that need not be represented by ordinary continuous functions. A sudden current pulse, a communication interruption, or a sharp ageing transition can therefore be admitted without forcing the entire trajectory model outside the analytical framework. This is particularly useful when telemetry from intelligent transportation systems is incomplete or irregular [6].

FKF representation and dual topology

For a time variable t and a positive scale variable r, the FKF representation may be expressed schematically as

$$FFKF_{\{f\}}\,(\omega,\nu) = \int f(t)e^{-i\omega t}$$

The exact kernel can be specialized to the Fourier and Kontorovich–Lebedev factors required by the selected FKF convention. The important structural property is that temporal translation acts predictably in the spectral coordinate while the scale coordinate retains the information needed for multi-scale analysis [8].

Continuous delay shifts

For the delay operator Tτφ(t)=φ(t−τ), the topology is selected so that Tτ maps Φ continuously into itself for every admissible τ. The corresponding transform-domain relation is represented by

$$FFKF_{\{T\tau\, f\}=e}-i\omega\tau\; FFKF_{\{f\}}$$

This identity is the mathematical reason the delay is treated as an operator rather than as an informal index shift. It also provides a direct bridge between lead-time analysis and battery telemetry latency. In a fractional model, the delay and memory terms can therefore be studied in the same transform-domain representation.

Fractional memory and regularity

For 0<α<1, the fractional derivative retains dependence on the trajectory history. A Caputo-type interpretation is useful when the initial state is physically prescribed. The transform framework does not require one unique fractional derivative definition; instead, it requires that the chosen operator map the admissible space continuously. This allows the battery model to be refined as electrochemical evidence becomes available.

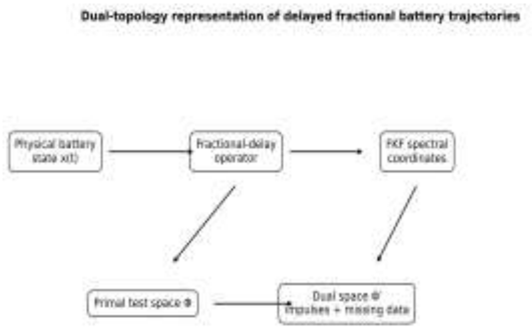

Figure 1. Dual-topology representation of delayed fractional battery trajectories.

IV. FRACTIONAL BATTERY TRAJECTORY MODELING

State-of-charge evolution

A normalized state-of-charge trajectory s(t) can be related to battery current through a fractional balance law. A convenient form is

$$D_t\alpha\, s(t) = -\eta i(t)/Qeff$$

with η representing an effective charge/discharge efficiency and Q the usable capacity. In a delayed setting, the current or its measured value may be replaced by i(t−τ). The capacity term is constrained by battery-sizing considerations [1], while the efficiency and degradation envelope can be refined using automotive battery reviews [4], [5].

State-of-health trajectory

State of health can be represented by a slowly varying fractional state h(t), with degradation driven by temperature, cycling, depth of discharge, and calendar effects. A generic memory-aware relation is

$$D_t\beta\, h(t) = -\lambda g(T, i, N),\ \ 0 < \beta \le 1$$

The function g aggregates the selected ageing drivers. The formulation is deliberately modular because the supplied battery literature discusses ageing at different levels of granularity [4], [5]. The FKF topology provides the mathematical container; it does not replace electrochemical identification.

Coupled SoC–SoH trajectory

The joint battery state can be represented as $z(t)=[s(t),h(t)]^T$. A coupled model becomes

$$D\alpha^t z(t) = A\, z(t) + B\, u(t-\tau) + w(t)$$

where A contains effective storage and degradation couplings, B maps control input to state variation, and w(t) collects disturbances. This representation is compatible with a digital twin because the twin can update the state estimate while the FKF layer evaluates admissibility and stability.

Spectral trajectory and resonance

A battery trajectory may contain charging cycles, regenerative-braking pulses, thermal oscillations, and control-induced transients. Spectral decomposition separates these scales and can expose residue peaks associated with repeated modes [8]. A compact modal representation is

$$\sum_{k=0}^{\infty} c_k \psi_k(t)$$

The coefficients c_k provide a bridge between time-domain state estimation and spectral diagnostics. A large coefficient at a persistent mode can be monitored by the twin, while an isolated generalized component can be handled in the dual space rather than being incorrectly smoothed away.

Trajectory constraints and admissibility

The physically meaningful trajectory must satisfy bounds such as $0 \le s(t) \le 1$ and $h(t) \le 1$, together with current, power, thermal, and charging limits. These are not optional post-processing checks. They define the feasible subset on which AI predictions, twin updates, and optimization decisions are allowed to operate. Sustainable logistics studies similarly treat energy and carbon constraints as part of the feasible decision space [10].

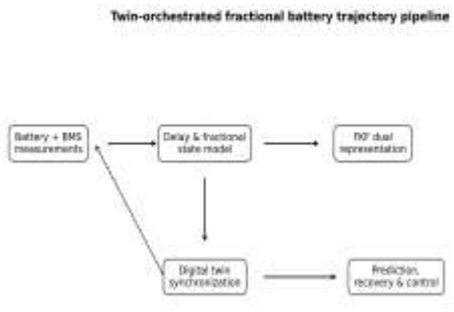

Figure 2. Twin-orchestrated pipeline for fractional battery trajectory estimation and control.

V. DIGITAL-TWIN ORCHESTRATION AND RESILIENCE

Twin synchronization

A digital twin is treated as a synchronized computational replica rather than a static visualization. Let $z_p(t)$ denote the physical state and $z_d(t)$ the twin state. The synchronization error is

$$e(t) = z_p(t) - z_d(t)$$

The twin update is admissible when the error remains inside an uncertainty envelope determined by sensor quality, delay, and model mismatch. Digital-twin literature provides the conceptual basis for synchronization and adaptive supply-chain replicas [12]. Here, the FKF topology gives the analytical condition under which delayed or generalized observations can still be propagated.

Uncertainty and incomplete telemetry

When a measurement stream is incomplete, replacing missing samples by arbitrary interpolation may create a false smooth trajectory. The dual-space formulation instead permits the observation to be represented as a generalized functional until sufficient information becomes available. This is particularly relevant to intelligent transportation systems in which communication interruptions and heterogeneous sensors are intrinsic to the data environment [6].

Resilience and recovery

Resilience is formulated as controlled recovery of the trajectory after a disruption. A supplier event, charging interruption, sensor fault, or mobility shock enters the model as a forcing term r(t). The recovered state is represented by a convolution with the system response:

$$z(t) \ = \frac{1}{2\pi} \int G(\omega)\, R(\omega)\, e^{i\omega t}\, d\omega$$

The exact recovery kernel depends on the chosen FKF convention. The important requirement is that the forcing and recovered state remain in the same dual class. Resilience reviews emphasize recovery capability and adaptive response, while digital transformation studies position these functions inside an integrated resilience architecture [13], [15]. Internal and external complexity further indicate that disruption should be evaluated across interacting layers rather than at a single node [14].

Traceability and authorization

Battery observations used in a recovery decision should be traceable to their acquisition interval. AI–blockchain integration provides a mechanism for binding data lineage and event provenance to decision records [11]. Anti-theft sensing and swipe-based release control provide a complementary principle: detection and prediction should not be confused with authorization [2], [3]. The twin can therefore publish a predicted trajectory while the operational layer retains a separate release decision.

Sustainable mobility constraints

Energy recovered through regenerative braking or solar assistance can improve the effective energy balance, but it cannot invalidate battery operating limits. Solar-assisted architectures motivate the inclusion of an energy-harvesting input, while green-logistics research emphasizes energy and carbon constraints within the feasible set [7], [10]. The twin should therefore expose both state trajectory and sustainability indicators.

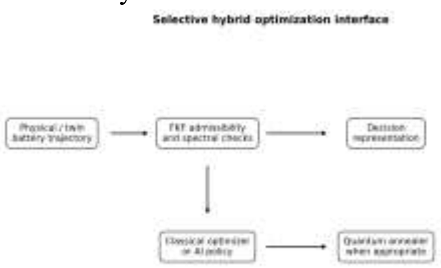

Figure 3. Selective hybrid interface between FKF-constrained trajectories and decision optimization.

VI. HYBRID OPTIMIZATION AND INTELLIGENT LOGISTICS INTERFACE

Constrained decision layer

Once the continuous battery trajectory has been certified as admissible, a discrete optimization layer can select charging schedules, route alternatives, ULD assignments, or recovery actions. The separation is important: the optimizer should not be allowed to generate a state trajectory that violates the physical or topological constraints. Quantum-computing studies in logistics motivate this selective architecture and recommend instance-level comparison with classical methods [9], [16].

QUBO representation

For binary decision variables y_i, a generic constrained objective may be written as

$$min\, H(y) \ = \sum_{i}^{n} c_i y_i \ + \sum_{i<j}^{n} q_{ij} y_i y_j \ + \rho\, P(y)$$

Here P(y) represents penalties for violated capacity, routing, timing, or energy constraints. ULD configuration under disruption is a concrete logistics application in the supplied literature [17]. In the present framework, the penalty coefficients are conditioned on the battery trajectory and resilience state before the binary problem is submitted to an annealer.

Classical–quantum boundary

The classical FKF simulator propagates the fractional state and computes admissibility measures. The optimizer receives a finite decision representation derived from that state. A quantum annealer is used only when the resulting discrete problem is appropriate and when equivalent classical baselines are available [9], [16]. This avoids interpreting quantum optimization as a universal substitute for continuous trajectory analysis.

AI-assisted orchestration

AI models can estimate demand, forecast state trajectories, detect anomalies, or rank recovery actions. Their outputs are treated as candidate controls rather than unconstrained truths. The admissibility layer evaluates each candidate against the fractional battery model, the FKF seminorms, and the twin uncertainty envelope. This constrained use of AI is consistent with the role of artificial intelligence in sustainable logistics and green supply-chain decision support [10].

VII. THEORETICAL AND ENGINEERING IMPLICATIONS

Well-posedness and continuity

The principal theoretical implication is that delay, fractional memory, and generalized observations can be studied without abandoning a common locally convex structure. If the input sequence converges in the selected seminorm family and the delay operator remains continuous, the corresponding transformed trajectories inherit controlled convergence. This is the basis for stable numerical approximation and repeated twin updates.

Spectral diagnostics

The FKF domain provides a compact location for persistent modes, transient components, and residue signatures. Multi-scale spectral analysis can therefore be used as a diagnostic layer between physical sensing and resilience decisions [8]. Rather than treating every high-frequency event as a fault, the twin can compare the observed spectral pattern with the admissible trajectory envelope.

Resilience certification

A recovery certificate can be expressed as a bound on the deviation between the nominal and disrupted trajectories. For example, with a seminorm p, one may monitor

$$p(z_r - z_0) \leq C\, p(r)$$

where C depends on the delay, model operator, and chosen topology. Such a bound is a mathematical certificate, not a claim of industrial performance. Its practical value is that it gives the twin a quantitative criterion for deciding whether a predicted recovery remains within the admissible state family.

Implications for smart and sustainable mobility

The combined framework links battery energy, sensing, digital replication, traceability, resilience, and optimization. Battery technology defines the physical envelope [4], [5]; mobility systems provide observations [6]; solar assistance modifies available energy [7]; spectral analysis extracts multi-scale signatures [8]; AI and blockchain support decision intelligence and provenance [10], [11]; digital twins provide synchronization [12]; resilience and complexity studies provide system-level interpretation [13]–[15]; and quantum-logistics studies motivate the discrete optimization interface [9], [16], [17].

VIII. CONCLUSION

A twin-orchestrated framework for fractional battery trajectories under an FKF dual topology has been formulated. The approach treats battery state evolution, delay, fractional memory, generalized observations, and spectral structure within a common analytical setting. The resulting architecture separates the physical trajectory from its computational replica while retaining a mathematically controlled interface between them.

The framework also establishes a disciplined boundary for intelligent decision support. AI predictions, blockchain records, digital-twin updates, and quantum optimization are connected to the battery state only after physical and topological admissibility has been considered. This separation supports resilient mobility and logistics applications in which prediction, diagnosis, optimization, and authorization must remain distinguishable.

Future development can specialize the fractional operator for experimentally identified battery ageing laws, introduce temperature-dependent kernels, incorporate richer cell-to-pack coupling, and validate the FKF spectral indicators against measured telemetry. The same architecture can then be extended to multi-vehicle fleets, energy-aware logistics, and closed-loop mobility systems while preserving the underlying continuity and recovery principles.

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