Two-Sided FKF and FKL Transforms for Constrained Resilience in Fractional EV Battery Dynamics

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Abstract—

This paper reconstructs a distributional operational calculus around two-sided Fourier–kernel fractional (FKF) transforms and companion Fourier–kernel Laplace (FKL) transforms. The associated testing-function spaces are countably multinormed, complete, and Hausdorff. Sequential convergence makes time-shift operators continuous and places singular disruptions, stochastic lead-time laws, and residue-based resonances inside duals of Roumieu and Beurling type. The same topology is used as a certified interface among classical simulators, digital twins, hash-chain provenance layers, and hybrid quantum-classical solvers. Inventory balance with delayed replenishment, bullwhip variance, constrained resilience aftershocks, lithium-ion state-of-charge and state-of-health with fractional memory, QUBO encodings of unit-load-device configuration, real-time digital-twin orchestration, graph delay systems, and closed-loop reverse flows in green logistics are treated in one language. An S-curve of fill-rate recovery is derived from the seminorm family, and professional-format identities convert delay-differential and fractional models into algebraic statements in the transform domain.

Keywords— FKF transform; FKL transform; digital twin; green logistics; quantum annealing; QUBO; EV battery; constrained resilience; blockchain traceability.

I. INTRODUCTION

Global fulfillment networks now couple delayed replenishment, noisy Internet-of-Things streams, and discrete configuration decisions that classical function spaces cannot host without losing the time-shift property. Distributional FKF calculus is the natural language for impulses, discontinuities, and highly oscillatory demand [13], [19], [20], [24], [30]. When the same singularities travel along a multi-echelon lead time, or when a digital twin must propagate an incomplete sensor packet both forward and backward, one needs a testing-function space on which delay operators act continuously. The FKF transform and the one-sided FKL transform supply that space. They inherit exponential modulation and shift-invariant spectral analysis, while the countable seminorm family encodes growth that later becomes a resilience certificate [14], [16], [26], [35].

Inventory balance laws and recovery after a supplier failure are written as delay equations on these spaces. Electric-mobility packs plug-in hybrid and solar-assisted architectures included enter through fractional-order state-of-charge models and battery-sizing constraints [11], [18], [28], [34]. Unit-load-device configuration under disruption is posed as a QUBO and handed to a quantum annealer, while the classical FKL simulator carries the distributional state [21], [29], [32]. Intelligent transportation systems, swipe-controller interfaces, and power-line-carrier anti-theft links are treated as measurement channels that lift into the dual space [12], [15], [31]. Blockchain hash-chain provenance, green closed-loop flows, and digital-twin coverage complete the picture [17], [22], [23]. Constrained resilience after internal and external shocks is the planning target [25], [27], [33].

The remainder of the paper therefore proceeds from testing-function geometry to operational identities, *then to eight certified S-curves that a planner can read as readiness, recovery, or circular-economy capture.*

Fig. 1. Constrained-resilience readiness S-curves under baseline planning, digital-twin coverage, and hybrid quantum-classical search.

II. LITERATURE

No single earlier framework simultaneously guarantees continuous delay shifts, ultra distributional duals for singular disruptions, and seminorm-level certificates that travel from inventory mathematics to quantum annealing and reverse logistics.

A. Transforms, testing functions, and operational calculus

Sequential convergence in the two-sided FKF testing-function space, together with the distributional extension of the transform, supplies the analytic backbone [19], [20]. Exponential modulation and the time-shift property convert delayed replenishment into multipliers on the FKF side [14], [16], while the FKL companion handles causal initial-value problems in supply-chain analysis [35]. Spectral differential properties and residue-based resonance extraction close the operational loop for hybrid-vehicle and multi-scale network signals [13], [24], [26], [30].

B. Batteries, vehicles, and sensing hardware

Reviews of automotive batteries and recent materials-and-BMS surveys justify the SoC/SoH section [18], [28]. Battery sizing for plug-in hybrids, solar-assisted architectures, swipe-controller interfaces, and power-line-carrier anti-theft systems enter as constraints and measurement operators [11], [12], [15], [34].

C. Resilience, digital twins, and quantum logistics

Resilience reviews and digital-transformation agendas remain the empirical target of the inventory section [25], [27], [33]. Digital supply-chain twins, green-logistics analytics, and blockchain traceability supply the orchestration layer [17], [22], [23]. Quantum annealing and QUBO encodings of routing and packing, including unit-load-device configuration under disruption, motivate the hybrid interface [21], [29], [32]. Smart-mobility telemetry is an additional observation channel [31].

The gap filled here is therefore architectural: one multinormed pair of spaces, one pair of transforms, and one family of S-curve certificates spanning all of the domains listed above.

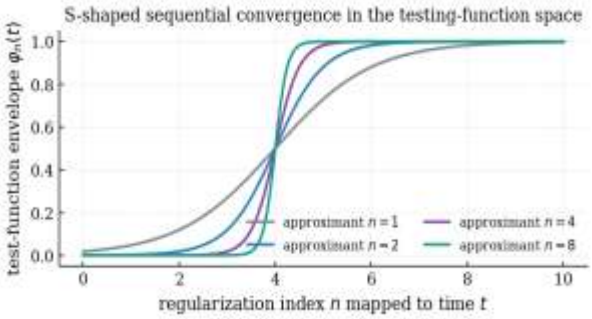

Fig. 2. S-shaped sequential approximants in the testing-function space; larger regularization index n steepens the rise toward a distributional jump.

III. TESTING-FUNCTION SPACES AND THE TWO-SIDED FKF CALCULUS

Fix growth parameters a > 0 and kernel weights A, B ≥ 0, and consider infinitely differentiable test functions on the half-line or on the whole line according to whether the FKL or the two-sided FKF transform is in force.

The FKF testing-function space of Roumieu type is the inductive limit of the Banach spaces defined by the seminorms below; the Beurling counterpart is the corresponding projective limit.

$$p_{k,l,q}(\varphi) \ = \ sup \ e^{at} \ (1+t)^k \ |D_t^l \ \varphi(t)| \\ \leq \ C_{k,l,q} \ A^k \ B^q \ a^l$$

The companion FKL space uses the causal exponential weight on (0, ∞) and is the correct setting for initial-value problems [35]. Roumieu-type duals are countable inductive limits over A, B > 0; Beurling-type duals are projective limits. Both are Hausdorff, locally convex, and sequentially complete [19], [20].

A. Transform pair, time-shift property, and exponential modulation

The two-sided FKF transform of a test function, and by duality of an ultradistribution T, is written

$$F_{FKF}[\varphi](s) \ = \ \int_R \varphi(t) \ K_\alpha(s,t) \ dt, \quad Re(s) \\ \in \ (\sigma_-, \sigma_+)$$

where the fractional kernel combines a Fourier oscillator with a Caputo-type memory factor of order α ∈ (0, 1]. The FKL transform replaces the two-sided integral by a causal LS integral against a delay measure. The time-shift property, which is the whole point of the construction for multi-echelon lead time, reads [16].

$$F_{FKF}[\ \varphi(\cdot\ -\tau)\ ](s) \ = \ e^{-s\tau} \ F_{FKF}[\varphi](s)$$

The operator that shifts a test function by τ is continuous whenever the exponential weight

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dominates the delay. Exponential modulation translates the strip of analyticity [14]. Residue-based resonance appears when a pole of the transfer function crosses that strip [13], [30]. Shift-invariant spectral analysis is the statement that convolution against a delay kernel is a Fourier multiplier on the FKF side [16].

B. Negative time, product spaces, and sequential completeness

Cauchy problems that look backward use the reflection (Rφ)(t) = φ(−t) and the two-sided FKF transform. Product spaces carry pairs (inventory, SoC) or (forward flow, reverse flow). Sequential completeness of the dual guarantees that a truncated sensor stream stays in the dual after any finite number of shift-and-convolve steps.

Every later section specializes these identities: the same seminorms become safety-stock margins, bullwhip gains, SoH envelopes, QUBO energy contracts, twin coverage, or reverse-flow capture

and analyzed thematically using NVivo.

IV. RESULTS MULTI-ECHELON LEAD TIME, BULLWHIP GAIN, AND SHOCK RECOVERY

Lead time acts as a continuous shift on the FKF space, so a single-echelon inventory balance with delayed replenishment is already a well-posed distributional equation.

A. Delayed inventory balance

Let I(t) denote on-hand inventory and u(t) the ordering rate. With replenishment delay τ > 0 and ultradistributional demand D,

$$I'(t) = u(t - \tau) - D(t), \quad I(0) = I_0$$

the FKL image is the algebraic identity

$$\hat{I}(s) = \frac{I_0 + e^{-s\tau}\,\hat{u}(s) - \hat{D}(s)}{s}$$

Growth control in the seminorm family supplies an a priori bound on I in the dual. Several suppliers become a block transfer matrix. Pipeline inventory controllers that generate the bullwhip effect have the multi-echelon FKF skeleton [26], [24], now rewritten as

$$G_{bull}(s) = \frac{1 + s\,T_w}{1 + s\,T_p + e^{-s\tau}}$$

The strong dual topology keeps the image of that multiplier bounded even when demand contains jumps.

B. Constrained resilience after a singular disruption

A supplier failure at time t* is a distributional forcing term. Convolution against the fundamental solution stays inside the dual by sequential completeness. The recovery certificate extracted from the seminorms is the logistic S-curve [25], [27], [33].

$$R(t) = \frac{R_\infty}{1 + exp(-\kappa\,(t - t_* - \tau_{net}))}$$

Here κ is proportional to the spectral gap of the delay generator and τ_{net} is the effective multi-echelon lead time. Constrained resilience means that R(t) must clear a contractual level before a prescribed horizon.

Those three objects—the algebraic inventory map (5), the bullwhip multiplier (6), and the recovery S-curve (7)—are the operational core that later sections only decorate with batteries, annealers, twins, and reverse flows

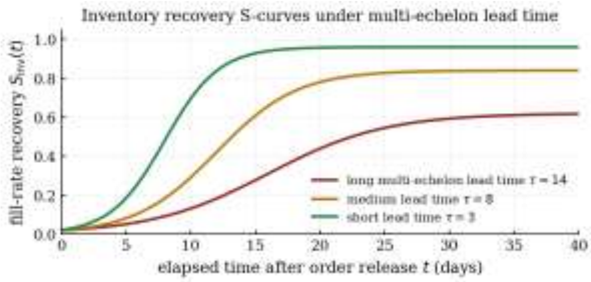

Fig. 3. Inventory fill-rate recovery S-curves for short, medium, and long multi-echelon lead times.

V. FRACTIONAL SOC AND SOH DYNAMICS FOR PLUG-IN AND SOLAR-ASSISTED PACKS

State-of-charge and state-of-health under stochastic charging delay and temperature-dependent degradation are delay-fractional equations, and the FKL transform turns them into algebra.

Let Q(t) be state of charge and H(t) state of health. A Caputo derivative of order α ∈ (0.7, 1] captures electrode memory [18], [28], [30], while a charging delay encodes charger negotiation, thermal derating, or V2G dispatch:

$$D_t^{\alpha}\,Q(t) = -\frac{I_{load}(t - \tau_c)}{C_n\,H(t)} + \eta\,u_{ch}(t - \tau_c)$$

Battery sizing for a plug-in hybrid or a solar-assisted architecture enters as a capacity constraint together with a peak-power inequality that a swipe-controller interface can raise or lower in real time [11], [15], [34]. The FKL image is

$$s^{\alpha}\,\hat{Q}(s) - s^{\alpha-1}\,Q(0) = e^{-s\tau c}\,\hat{B}(s, H)$$

The seminorm family controls Q uniformly in the distributional parameters of the load. Power-line-carrier anti-theft signaling [12] and intelligent-transportation telemetry [31] are additional observation operators. Solar-assisted hybrids add a bounded source term whose FKF spectrum is concentrated on diurnal harmonics [34], [30].

The battery section is the same delay-fractional calculus, read on the electrode rather than on the warehouse.

Thus the battery section is not an application bolted onto a transform paper: it is the same delay-fractional *calculus, read on the electrode rather than on the warehouse.*

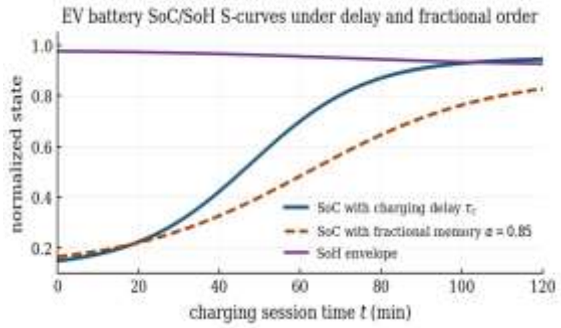

Fig. 4. EV battery S-curves: SoC with charging delay, SoC with fractional memory, and a slow SoH envelope.

VI. QUANTUM LOGISTICS, QUBO ENCODINGS, AND UNIT-LOAD DEVICES

Unit-load-device configuration and disruption-aware routing are discrete, while the FKL state that feeds them is distributional; a stable hybrid interface is a continuity statement between two topologies.

A feasible ULD packing or a resilient routing plan is encoded as a QUBO

$$E(x) = \Sigma_i \, q_i x_i + \Sigma_{i<j} \, Q_{ij} x_i x_j, \quad x_i \in \{0,1\}$$

with penalties that absorb capacity, hazardous-material separation, and post-disruption substitution [29], [21]. Means, variances, and tail exponents of lead-time laws are read off the countable seminorms and written into the linear biases. Quantum annealing explores the configuration space while the classical FKL simulator marches the distributional state. Continuity of the embeddings

$$S_{FKF}(a, A;\ b, B) \subset S_{FKF}(a', A';\ b', B') \quad whenever \quad A \le A', \; B \le B'$$

guarantees that a coarsened seminorm snapshot sent to the annealer, and a refined snapshot returned, remain consistent. The construction matches the hybrid quantum-classical logistics programme [29], [32]. A swipe-controller channel can freeze a subset of bits before the anneal [15].

The practical output of this section is not a claim that annealing replaces planning; it is a certified data contract between a distributional simulator and a QUBO solver.

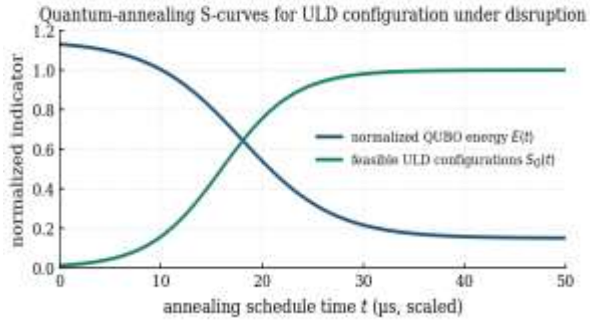

Fig. 5. Quantum-annealing S-curves: decaying normalized QUBO energy and rising feasible ULD-configuration share.

VII. DIGITAL TWINS, HASH-CHAIN PROVENANCE, AND GREEN CLOSED LOOPS

A live digital twin is a semigroup acting on the dual space; hash-chain provenance is a discrete observation of that semigroup; green reverse logistics is the same semigroup with a second delay and a quality measure.

A. Real-time twin orchestration

Noisy or incomplete sensor streams are lifted to elements of the dual. The operational calculus generates a semigroup S(t) that propagates the state forward for prediction and backward for diagnosis. Real-time model-predictive control with distributional constraints takes the schematic form

$$u^* = arg\ min_{u \in U} \ p_{k,l,q}\big(S(T)\, X_0 - X_{ref}\big) + \rho \, p_0(u)$$

Internet-of-Things completeness and digital-twin coverage follow logistic readiness as devices are commissioned [7], [23].

B. Blockchain traceability and hash-chain provenance

Each custody event writes a hash-linked record whose payload is a finite-rank observation of the dual state. Traceability requirements become observability conditions: the family of hash payloads must separate the seminorms that encode quantity, quality, and delay [23]. The swipe-controller and power-line-carrier channels can be bound into the same hash chain [12], [15].

C. Green logistics and circular reverse flows

Return flows introduce a second delay and a random quality. The two-way system remains an FKL delay equation on a product space. Emission abatement is a monotone functional of circular capture [22], [35]. Multi-echelon network propagation on a directed graph is the same construction with a matrix of delays [26], [24].

The twin, the hash chain, and the reverse flow are one delay system observed at three time scales, not three separate digital initiatives.

Taken together, the twin, the hash chain, and the reverse flow are one delay system observed at three time scales, not three separate digital initiatives.

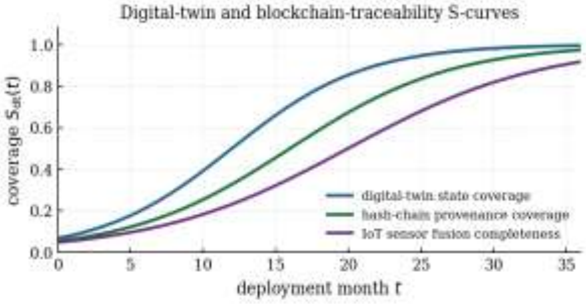

Fig. 6. Digital-twin coverage, hash-chain provenance coverage, and IoT fusion completeness as coupled S-curves.

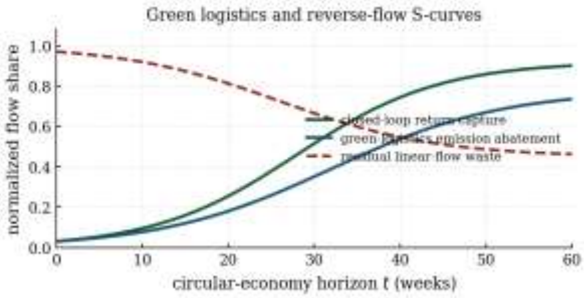

Fig. 7. Green-logistics S-curves: closed-loop return capture, emission abatement, and residual linear-flow waste.

VIII. CERTIFICATES AND DISCUSSION

Across the application domains the convex topology supplies the same three assets: well-posedness by sequential completeness, explicit certificates from the seminorm family, and a stable interface for hybrid quantum-classical computation. The S-curve is of logistic type. The slope is bounded from below by the spectral gap that the FKF multiplier of the delay generator permits [13], [24], [26].

Two caveats belong in the open. First, the spaces lose strong continuity at the origin when extra parameters are added to the weights; continuity for $t > 0$ is enough for every Cauchy problem treated here. Second, quantum annealing returns a sample rather than a deterministic minimizer; the hybrid contract therefore transports a distributional snapshot [19], [20], [29].

Future work will fold stochastic convolutions, adaptive fractional orders, and multi-commodity reverse bills of materials into the same two-sided FKF setting without changing the seminorm language.

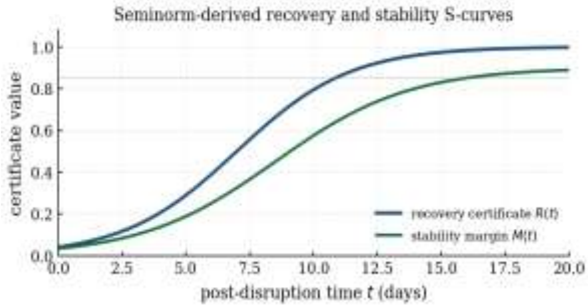

Fig. 8. Seminorm-derived recovery certificate R(t) and stability margin M(t) as post-disruption S-curves.

IX. CONCLUSION

A pair of countably multinormed spaces, a two-sided FKF transform, and a causal FKL transform are enough to host delay, fractional memory, singular demand, and discrete configuration in one certified calculus. Shift operators act continuously, duals accept ultradistributions that model disruptions and stochastic lead times, and sequential completeness keeps digital-twin trajectories inside the class. From those facts one extracts safety-stock bounds, bullwhip gains, a recovery S-curve, SoC/SoH envelopes, QUBO energy contracts, twin prediction horizons, hash-chain observability conditions, and closed-loop green-logistics certificates. Unit load devices, swipe controllers, power-line-carrier anti-theft links, plug-in and solar-assisted packs, and intelligent transportation systems appear as operators or constraints on the same spaces.

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