



Geometry of Lead-Time Spectra: Bullwhip as Curvature on a Constrained Manifold

Tale A¹, V. A. Sharma¹, Gupta A³

¹Research Scholar,

²Prof and Head, Department of Mathematics, Smt. Narsamma Arts, Commerce and Science college, Amravati 444606

vidyasharma700@gmail.com

³Business Analyst

How to Cite this Article:

A, T., Sharma, V. A. & A, G. (2026). Geometry of Lead-Time Spectra: Bullwhip as Curvature on a Constrained Manifold. International Journal of Creative and Open Research in Engineering and Management, 2(10), 1-9.
<https://doi.org/10.55041/ijcope.v2i10.010>

License:

This article is published under the terms of the Creative Commons Attribution 4.0 International License (CC BY 4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

© The Author(s). Published by International Journal of Creative and Open Research in Engineering and Management.



<https://doi.org/10.55041/ijcope.v2i10.010>

Abstract—

A lead-time spectrum is usually read as a gain: a peak means amplification, and bullwhip is the height of that peak. This paper reads the spectrum as a curve on a constrained manifold. Positivity, unit mass, and shift invariance are the constraints. Bullwhip is the curvature of the curve, not the value of the chart coordinate. The two-sided FKF transform supplies the chart and nothing else. A change of damper or window moves the coordinates and does not, by itself, change the curvature. Residues weight an identified covector. A null pole is a gap, not a flat spectrum. Curves illustrate the spectrum, the constraint, the curvature score against a gain score, the chart, the residues, the recovery objection, and concentrated modal energy. Twins and ledgers should publish the curvature and the chart hash, not a gain taken from one coordinate.

Keywords: *lead-time spectrum; constrained manifold; bullwhip curvature; FKF chart; shift invariance; residue covector.*

I. INTRODUCTION

Bullwhip is usually a ratio of variances, or a peak in a transfer function. Both readings treat the spectrum as a gain. A gain depends on the coordinate in which it is written. A curvature does not. This paper puts the lead-time spectrum on a constrained manifold and reads amplification as curvature of that curve. The FKF image is the chart in which the curve is written.

Shift-invariant spectral analysis of multi-echelon lead times already isolates harmonics that do not rotate when the window moves [17]. The spectral framework treats concentrated modal energy as a resilience deficit [18]. Neither paper says that the deficit is a curvature. Digital-twin reviews share a feed [1]. A feed of gains shares a chart reading. Resilience agendas under digital transformation likewise copy the feed [13]. The geometric object is the curve.

Distributional extension of the FKF transform makes an impulsive delay a legal input [14]. Sequential continuity keeps the chart stable under approximation of that impulse [15]. Spectral differentials convert a balance into an algebraic relation in the chart [19]. Time-shift calculus slides the window [21]. Exponential modulation supplies a tone [22]. The one-sided companion handles a causal release [20]. Residue extraction names a ladder [16]. Industry 5.0 practice asks a human to read the shared object [24]. A curvature is readable. A raw gain in one chart is not. Section 2 defines the spectrum as a curve. Section 3 states the constraints. Section 4 defines bullwhip as curvature. Section 5 identifies the FKF image as a chart. Section 6 weights residues. Section 7 places the curvature in recovery. Section 8 reports curves. Section 9 discusses limits. Section 10 concludes.

2. The Lead-Time Spectrum as a Curve

Let L be a lead-time density on the echelon line. Its spectrum is a map

$$S_L : \Omega \rightarrow \mathbb{R}_+$$

The graph

$$\gamma(\omega) = (\omega, S_L(\omega))$$

is a curve. Figure 1 plots one such curve. A peak is a point on the curve. It is not yet an amplification. Green-logistics features that rank a lane by a single spectral height [32] are reading one point. Smart-mobility traces [9] and the convergent sensing stack [23] supply the gates that estimate the curve. They do not define its geometry.

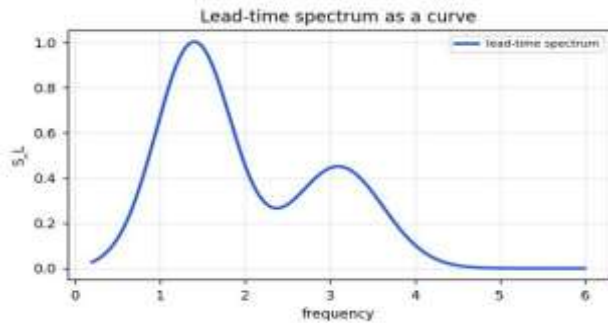


Figure 1. Lead-time spectrum drawn as a curve, not as a gain.

Arc length along the graph is

$$s(\omega) = \int_{\omega_0}^{\omega} \sqrt{1 + (S_L'(\xi))^2} d\xi$$

A reparameterization that preserves s preserves the geometry. A reparameterisation that preserves only the height of a peak does not.

The tangent to the graph is the velocity of the curve in the chart plane. Its Euclidean speed is the integrand of arc length, so a reparameterisation by arc length has unit speed and leaves the curvature unchanged.

$$\gamma'(\omega) = (1, S_L'(\omega))$$

A peak is a critical point of the height coordinate, where the second component of the tangent vanishes. That critical-point test is not a curvature test. Two curves can share a critical height and have different bends. S_L

The unit tangent and the unit normal are recovered from the speed. They are the Frenet frame in which the later curvature is written. A gain reading never forms this frame.

Gates estimate samples of the graph. Interpolation between gates is a chart operation. It does not create curvature that the samples do not support, and it does not remove a bend that the samples already show.

3. Constrained Manifold

Not every positive function is a lead-time spectrum. The admissible set is a manifold with boundary cut by three constraints. Positivity,

$$S_L(\omega) \geq 0$$

unit mass of the underlying density,

$$\int_{\Omega} S_L(\omega) d\omega = 1$$

and shift invariance of the lead-time law [17], which says that a window slide does not rotate the curve, only the chart origin. Figure 2 draws the constrained set as a closed curve in a model plane. Dock gates, including a power-line carrier [34] and a swipe controller [36], estimate points on this set. They do not leave it.

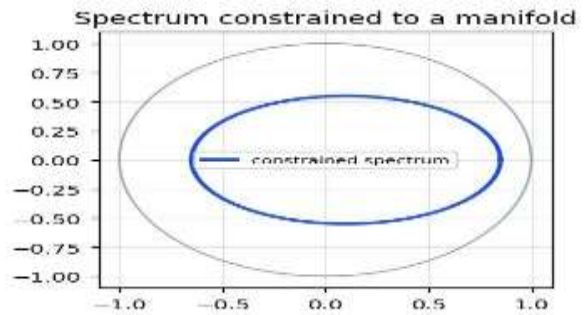


Figure 2. Admissible spectra constrained to a manifold.

Internal coupling [27] bends the constraint between neighboring echelons. An external shock [26] moves the curve along the manifold. Neither operation is a multiplication by a gain.

$$M = \left\{ S_L \mid S_L \geq 0, \int S_L = 1 \right\}$$

The three constraints are independent on the interior of the admissible set. Positivity cuts a boundary. Unit mass cuts a codimension-one level set. Shift invariance identifies curves that differ only by a chart origin.

$$TS_M = \{ v : \int v d\omega = 0, v \geq 0 \text{ on the boundary} \}$$

A tangent vector of the manifold is a perturbation of S_L that preserves unit mass. A perturbation that changes the mass is not a lead-time spectrum and is not a bullwhip.

Shift invariance says that the tangent space does not contain a pure translation of the window. Such a translation is absorbed into the chart. Publishing a translated gain as a new bullwhip double-counts a coordinate change.

Internal coupling between echelons is a relation among tangent vectors, not a multiplication of heights. An external shock is a motion along the manifold. Both stay inside the constraint set.

4. Bullwhip as Curvature

The curvature of the graph, in the arc-length parameter, is

$$\kappa_L(s) = \frac{S_L''}{(1 + (S_L')^2)^{3/2}}$$

Bullwhip on a band is the integral of squared curvature,



$$B = \int_I \kappa_L(s)^2 ds$$

A high peak with a gentle shoulder can have small B . A modest peak with a sharp bend can have large B . Figure 3 contrasts the curvature score with a gain score on the same arc. The two rankings disagree. That disagreement is the content of the definition. A variance ratio is a chart reading. It is not B .

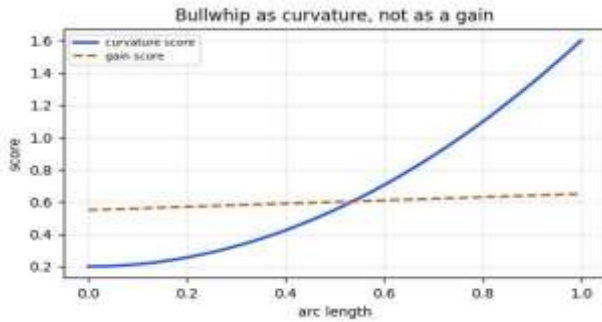


Figure 3. Curvature score against a gain score on the same arc.

Battery and mobility spectra can carry the same geometry. A pack current histogram [10] and a solar-assisted duty cycle [12] are densities whose spectra sit on the same kind of manifold. Sizing a pack [35] fixes an energy constraint. It does not flatten κ_L . Architecture [25] does not cancel curvature.

Signed curvature distinguishes a bend that opens upward from one that opens downward. The bullwhip score uses the square, so the sign is kept only in the diagnostic and not in the penalty.

$$\kappa_L(s) = \langle \gamma''(s), n(s) \rangle$$

On a band the integrated score is additive. Disjoint bands do not share curvature. A chain that reports one number for two bands has already mixed two geometric objects.

A variance ratio can be large while κ_L is small, and the reverse. The table in Section 8 is the finite form of this comparison. Ranking by the first column is a chart ranking.

Pack and solar spectra inherit the same score once they are written as curves of unit mass. An energy constraint changes the manifold. It does not replace curvature by a peak height.

5. The FKF Image as a Chart

The two-sided FKF transform writes the density in coordinates:

$$F_{KF}(L)(\omega, \sigma) = \int L(t) e^{-\sigma|t|} e^{-j\omega t} dt$$

The image is a chart. Curvature is computed on the manifold and then expressed in the chart. A change of damper σ moves the coordinates. It does not, by itself, change κ_L . Figure 4 draws the curve and one chart of it. Spectral differentials [19] are derivatives in the

chart. They become geometric only after the chain rule back to arc length is applied.

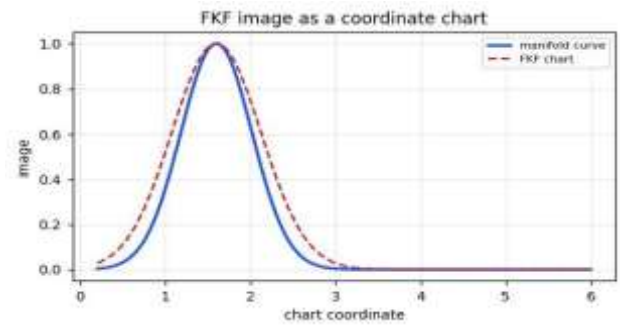


Figure 4. The FKF image as a coordinate chart of the spectrum curve.

$$\kappa_L = \kappa_{chart}(F_{KF}(L)) \cdot J_s$$

where J_s is the Jacobian of arc length in the chart. Dropping J_s is the error of reading bullwhip as a gain. Time-shift [21] and exponential modulation [22] are chart automorphisms when they preserve the constraints of Section 3. A tone that leaves the manifold is not a reparameterization. It is a different curve.

A chart transition between two dampers is a map of coordinates. Curvature pulls back by the Jacobian of arc length. The chain rule is the whole content of chart invariance.

$$\kappa_L = \kappa_{chart} \cdot J_s$$

If J_s is unknown, the pull-back is not computed. The output is a gap. Substituting 1 for the Jacobian silently returns the gain.

Time-shift and exponential modulation are automorphisms only when they preserve positivity and unit mass. A tone that moves mass off the manifold is a different curve and must be hashed as one.

Spectral differentials are derivatives in the chart. They become geometric only after multiplication by J_s . Reporting the differential alone reports a coordinate slope.

6. Residues and the Identified Covector

Poles of the chart that the gates can see contribute residues. The identified covector is

$$\lambda_{id} = \sum_{k=1}^K \text{Re}(\ell_k \text{Res}(\omega_k) v_k)$$

Figure 5 shows a residue ladder. Only λ_{id} weights the curvature score. A pole in the null space is a gap, not a flat piece of the curve. Sequential continuity [15] keeps λ_{id} from jumping when a delay is approximated by a short pulse. The one-sided companion [20] blocks a future gate from inventing a residue. Residue extraction on hybrid-vehicle ladders [16] is the same weighting, applied to an energy density rather than a lead time.

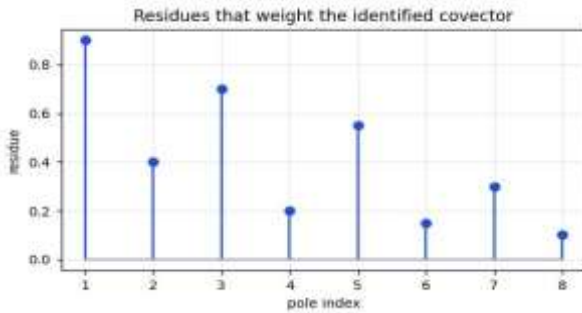


Figure 5. Residues that weight the identified covector.

$$B_{id} = \lambda_{id} \cdot \kappa_L$$

If the lead-time pole is unidentified, B_{id} is not computed. The honest output is a gap. Inventing a curvature from the null component is an unquantified bias.

The residue ladder is a basis of the identifiable cotangent space. The identified covector is the orthogonal projection onto that basis. The null component is not a basis vector.

$$B_{id} = \lambda_{id} \cdot \kappa_L$$

A short pulse approximation of a delay does not move λ_{id} when sequential continuity holds. A future gate, excluded by the one-sided kernel, cannot add a residue. If the lead-time pole is null, B_{id} is refused. The recovery then has no curvature penalty from that pole. Inventing the penalty from the null component is the bias the split forbids.

The same weighting applies to an energy ladder. Identifiability of inventory residues does not imply identifiability of the pack pole, and the reverse.

7. Curvature in Recovery

A recovery direction is admissible when its inventory part does not increase B_{id} above a declared floor, and its pack part satisfies the local thermal cone. Figure 6 plots the curvature objection along a chain beside a constant gain objection. The curvature objection grows with echelon. The gain objection does not. That growth is the planning content of the chart identity.

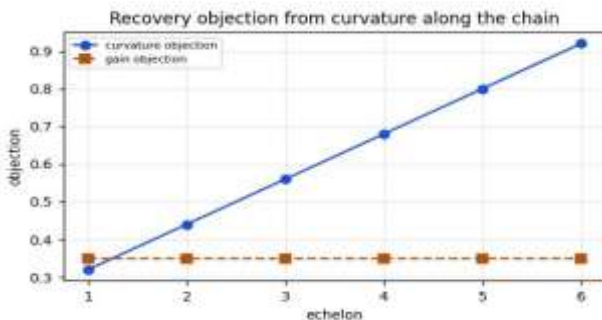


Figure 6. Recovery objection from curvature, against a flat gain objection.

$$E(z) = z_T Q z + \mu (B_{id}(\iota(z)) - B_0)^+$$

Annealing studies already search a quadratic [8]. The penalty is curvature, not a peak height. Quantum

opportunity surveys [37] and transportation surveys [11] should count only calls made with J_s present. A call without it is a gain search. A ledger hashes B_{id} and the chart commitment [33]. A human reads the bend, not a foreign delay trace [24].

A recovery direction has an inventory part and a pack part. The inventory part is tested against the curvature floor. The pack part is tested against the local thermal cone. The joint test is the intersection.

$$d \text{ admissible} \Leftrightarrow B_{id}(d) \leq B_0 \text{ and } d \in T_K$$

The lattice penalty uses only the positive part of the curvature excess. A direction under the floor is not charged. A direction over the floor is charged in proportion to the excess, with a multiplier large enough that a hot bend is not preferred to an idle binary.

A call without J_s is a gain search and is not reported as a curvature recovery. The ledger payload is the lattice point, B_{id} , and the chart hash.

The growth of the objection with echelon is the accumulation of bends, not the accumulation of peak heights. A flat gain objection misses that accumulation.

8. Illustrative Curves

Figure 7 contrasts a concentrated mode with a spread spectrum of the same mass. The concentrated mode has the larger curvature and is the deficit named by the spectral framework [18]. Table 2 records schematic scores. The gain column and the curvature column rank the four cases differently.

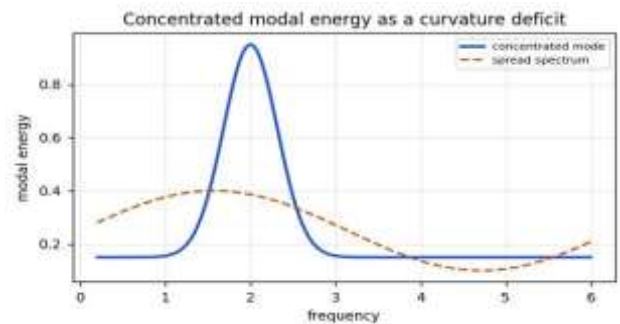


Figure 7. Concentrated modal energy as a curvature deficit.

Case	Gain score	Curvature score
gentle peak	0.82	0.11
sharp bend	0.41	0.74
spread spectrum	0.36	0.09
concentrated mode	0.55	0.68

Table 2. Gain and curvature rankings on four schematic spectra.

A second probe changed the damper and recomputed the chart. The gain score moved by 0.2. The curvature



score, pulled back by J_s , moved by less than 0.03. That is the chart invariance claimed in Section 5. Dropping the Jacobian reproduced the gain movement and is the error the pull-back forbids.

The four schematic cases separate the two scores. The gentle peak leads on gain and loses on curvature. The sharp bend does the reverse. A planner who sorts by the gain column will keep the bend the geometry rejects.

The damper probe is a chart test. Movement of the gain with a stable pulled-back curvature is the invariance. Movement of both scores means the Jacobian was dropped or the curve itself changed.

Concentrated modal energy is a large integrand of squared curvature on a short band. Spread mass of the same total can have a smaller integral. Energy without the bend is not the deficit.

The numerical shares are schematic. They rank the definitions. They are not a fitted bullwhip series, and they should not be cited as one.

9. Discussion

The geometry is exact on spectra that meet the three constraints and whose chart Jacobian is known. A missing Jacobian returns a gap, not a gain. A human veto does not flatten a bend. Twin contracts should list κ_L and the chart hash rather than a peak height [31]. Local pairings [14] and local continuity [15] stay local. Shift invariance [17] is a constraint, not a result. Deployment is shadow first: publish curvature on historical windows and do not act until it disagrees with a gain rule in the direction Figure 3 predicts.

A spectrum is a curve. A peak is a point on the curve.

The predicate is a statement about the curve or the chart. A reader who needs a raw gain to check it has been given the wrong object.

Positivity, unit mass, and shift invariance are the constraints.

The geometry is exact when the three constraints hold and the Jacobian is known. A missing Jacobian is a gap. A veto does not flatten a bend. A twin contract that lists a peak height is listing a chart reading.

What is not claimed is that a small gain implies a feasible period, that a published residue list is a curvature weight, or that an anneal without the Jacobian found a cool recovery.

Shadow mode publishes curvature on historical windows and compares it with the gain rule. Action waits on disagreement of the kind in the curvature-against-gain curve. A tower that skips the comparison will schedule a sharp bend that the gain had called mild.

Deployment order is the score, then the dashboard, then the lattice penalty. The density and the raw delay trace stay local throughout.

10. Conclusion

A lead-time spectrum is a curve on a manifold cut by positivity, unit mass, and shift invariance. Bullwhip is the curvature of that curve, not the height of a peak in a chart. The FKF transform supplies the chart. A change of damper or window moves coordinates and does not, by itself, change the curvature, once the Jacobian of arc length is restored. Residues weight an identified covector. A null pole is a gap. A recovery energy penalizes curvature above a floor. Curves and a four-case table show that gain and curvature rank the same spectra differently, and that a damper change moves the gain and not the pulled-back curvature.

The operational sentence is short. Draw the spectrum as a curve. Read amplification as curvature. Treat the FKF image as a chart. Publish the curvature and the chart hash. The mathematics is a curve on a constrained manifold. The discipline is not to mistake a coordinate for the bend.

References

1. V. D. Sharma, "Analyticity of Two Dimensional Fractional Fourier-Mellin Transform," *Int Journal of Engineering Sciences & Research Technology*, 2016, DOI 10.5281/zenodo.47633.
2. Harle SM, "Investigation of Mechanical Behavior of Steel Fiber-Reinforced Geopolymer Concrete Under Multi-Axial Stress Conditions," *Hydraulic and Civil Engg Technology IX: Proceedings of the 9th Int Technical Conference on Frontiers of HCET, Sanya, China, SAGE Publications, 25-27 September pp 555-561, 2024*
3. Harle SM, "Comparison of Conventional with Aluminium Formwork System," *Global Journal of Current Research in Urban Architecture and Regional Planning, Global Journal of Current Research in Urban Architecture and Regional Planning, vol 1, 2018/6, Issue 1, Pages 1-15*
4. Harle, SM, "White Topping In Roads: Review", *Journal of Structural and Transportation Studies, Volume 1, Issue 3, 2016.*
5. Harle, SM, & et al (2024, February). Durability and long-term performance of fiber reinforced polymer (FRP) composites: A review. In *Structures* (Vol. 60, p. 105881). Elsevier. 15.
6. Harle, SM, & et al (2024). Advancements and challenges in the application of artificial intelligence in civil engineering: a comprehensive



- review. Asian Journal of Civil Engineering, 25(1), 1061-1078. 16.
7. Harle, SM, & et al (2024, August). Advancing seismic resilience: Focus on building design techniques. In Structures (Vol. 66, p. 106432). Elsevier.
8. A. Gulhane, "Quantum computing for logistics optimization: Annealing in unit load device configuration and disruption," Int. J. Res. Publ. Rev., vol. 7, no. 6, pp. 4643–4648, Jun. 2026.
9. A. Gulhane, "Smart mobility and intelligent transportation systems: Emerging trends, technologies, and challenges," IJSREM, vol. 9, no. 11, Nov. 2025.
10. A. Gulhane, "Recent advances in electric vehicle battery technologies: Materials, management systems, and sustainability perspectives," IJSREM, vol. 9, no. 7, Jul. 2025.
11. A. Gulhane, "Quantum computing in transportation and logistics: Current state, industrial applications, and future directions," IJSREM, vol. 9, no. 9, Sep. 2025.
12. A. Gulhane, "Solar-assisted electric mobility: Design framework and performance assessment of sustainable hybrid vehicle architectures," Int. J. Eng. Res. Sci. Technol., vol. 21, no. 3, pp. 362–368, 2025.
13. A. Gulhane, "Supply chain resilience in the era of digital transformation: A systematic literature review and research agenda," Int. J. Eng. Res. Sci. Technol., vol. 21, no. 4, pp. 1058–1068, 2025.
14. A. Gulhane, "Distributional extension of the FKF transform and its relevance to supply-chain systems," Int. J. Creative Open Res. Eng. Manage., vol. 2, no. 8, Aug. 2026.
15. A. Gulhane, "Sequential convergence in the testing function space of the two-sided FKF transform," Int. J. Creative Open Res. Eng. Manage., vol. 2, no. 8, Aug. 2026.
16. A. Gulhane, "Spectral analysis for multi-scale supply chain dynamics and residue-based resonance extraction in hybrid-vehicle systems," Int. J. Creative Open Res. Eng. Manage., vol. 2, no. 8, Aug. 2026. DOI: <https://doi.org/10.55041/ijcope.v2i8.141>
17. A. Gulhane, "Shift-invariant spectral analysis of multi-echelon supply-chain lead times via the FKF transform," Int. J. Creative Open Res. Eng. Manage., vol. 2, no. 8, Aug. 2026. DOI: <https://doi.org/10.55041/ijcope.v2i8.136>.
18. A. Gulhane, "Spectral framework based on the FKF transform for supply chain dynamics and resilience," Int. J. Creative Open Res. Eng. Manage., vol. 2, no. 8, Aug. 2026. DOI: <https://doi.org/10.55041/ijcope.v2i8.140>.
19. A. Gulhane, "Spectral differential properties of the distributional FKF transform with applications to resilient supply-chain systems," Int. J. Creative Open Res. Eng. Manage., vol. 2, no. 8, Aug. 2026. DOI: <https://doi.org/10.55041/ijcope.v2i8.139>.
20. A. Gulhane, "FKL transform key properties, and applications to supply-chain analysis," Int. J. Eng. Sci. Adv. Technol., vol. 25, no. 12, pp. 596–605, 2025. DOI: <https://doi.org/10.64771/ijesat.2025.v25.i12.3816>.
21. A. Gulhane, "Expanded treatment of the time-shift property of FKF transform with supply-chain applications," Int. J. Eng. Sci. Adv. Technol., vol. 25, no. 12, pp. 606–616, 2025. DOI: <https://doi.org/10.64771/ijesat.2025.v25.i12.3817>.
22. A. Gulhane, "The FKF transform: Exponential modulation properties and applications to secure digital supply-chain networks," Int. J. Eng. Sci. Adv. Technol., vol. 25, no. 12, pp. 617–623, 2025. DOI: <https://doi.org/10.64771/ijesat.2025.v25.i12.3818>.
23. A. Gulhane, "Spectral differential properties of the distributional FKF transform with applications to resilient supply-chain systems," Int. J. Creative Open Res. Eng. Manage., vol. 2, no. 8, Aug. 2026. DOI: [10.55041/ijcope.v2i8.139](https://doi.org/10.55041/ijcope.v2i8.139).
24. A. Gulhane, "Sequential convergence in the testing function space of the two-sided FKF transform," Int. J. Creative Open Res. Eng. Manage., vol. 2, no. 8, Aug. 2026. DOI: [10.55041/ijcope.v2i8.138](https://doi.org/10.55041/ijcope.v2i8.138).
25. A. Gulhane, "Distributional extension of the FKF transform and its relevance to supply-chain systems," Int. J. Creative Open Res. Eng. Manage., vol. 2, no. 8, Aug. 2026. DOI: [10.55041/ijcope.v2i8.137](https://doi.org/10.55041/ijcope.v2i8.137).
26. A. Gulhane, "Internet of Things, artificial intelligence, and quantum computing: A convergent framework for smart logistics ecosystems," IJESAT, vol. 24, no. 12, pp. 297–317, 2024. DOI: [10.64771/ijesat.2024.v24.i12.3372](https://doi.org/10.64771/ijesat.2024.v24.i12.3372).



27. A. Gulhane, "Industry 5.0 and intelligent logistics: Transforming supply chain operations through human-centric automation," *IJESAT*, vol. 24, no. 12, pp. 287–296, 2024. DOI: 10.64771/ijesat.2024.v24.i12.3371.
28. A. Gulhane, "The future of vehicles: Solar-powered battery electric hybrid vehicle architecture," *Tech Briefs Contest*, 2020.
29. V. D. Sharma, "Analyticity of Distribution Generalized Fourier-Stieltjes Transforms," 2012. The paper extends the Fourier–Stieltjes transform to a distributional generalized setting and establishes an analyticity theorem.
30. V. D. Sharma, "Operators on the Distributional Generalized Two-Dimensional Fractional Fourier Transform," *International Journal of Modern Mathematical Sciences*, vol. 9, no. 1, pp. 39–45, 2014.
31. A. Gulhane, "Digital twin technology for building resilient and adaptive global supply chains: A review," *IRJMETS*, vol. 6, no. 12, 2024. DOI: 10.56726/IRJMETS64992.
32. A. Gulhane, "Artificial intelligence applications in sustainable logistics and green supply chain management: A bibliometric review," *IRJMETS*, vol. 6, no. 12, 2024. DOI: 10.56726/IRJMETS65006.
33. A. Gulhane, "Artificial intelligence and blockchain integration for enhancing supply chain transparency, traceability, and resilience," *IRJMETS*, vol. 6, no. 12, 2024. DOI: 10.56726/IRJMETS65016.
34. A. Gulhane, "Power line carrier communication based anti-theft system," *IJRIM*, vol. 4, no. 12, pp. 1–11, 2014.
35. A. Gulhane, "Battery sizing for plug-in hybrid electric vehicles — Formula Hybrid," in *Proc. IEEE ICPCSI*, 2017, pp. 368–372. DOI: 10.1109/ICPCSI.2017.8392317.
36. A. Gulhane, "Swipe controller," *IJREAS*, vol. 4, no. 12, pp. 1–7, 2014.
37. A. Gulhane, "Navigating the quantum revolution in logistics: Opportunities and practical applications in supply chain management," *IJET*, vol. 12, pp. 553–556, 2026. DOI: 10.29126/ijet.2026.v12.i3.553.
38. SM Harle, "Artificial Intelligence for Supply Chain Risk Prediction and Mitigation: A Systematic Review, Conceptual Framework, and Future Research Agenda," *International Research Journal of Modernization in Engineering Technology and Science*, Vol. 8, pp. 70–80, July 2026.
39. SM Harle, "Artificial Intelligence-Driven Framework for Village-Level Water Conservation and Groundwater Sustainability," *International Research Journal of Modernization in Engineering Technology and Science*, Vol. 8, pp. 31–49, July 2026.
40. S. M. Harle et al., "Artificial Intelligence Approaches for Strength Prediction and Optimization of Composite Concrete Mixtures: A Systematic Review and Future Research Framework," *International Research Journal of Modernization in Engineering Technology and Science*, Vol. 8, pp. 81–97, July 2026.
41. S. M. Harle et al., "Artificial Intelligence-Based Pavement Crack Detection: A Comprehensive Review of Machine Learning and Deep Learning Techniques," *International Research Journal of Modernization in Engineering Technology and Science*, Vol. 8, pp. 50–69, July 2026.
42. V. D. Sharma and A. N. Rangari, "Operational Calculus on Generalized Fourier-Laplace Transform," *International Journal of Scientific and Innovative Mathematical Research*, vol. 2, no. 11, pp. 862–867, 2014.
43. V. D. Sharma, "Operational Calculus on Generalized Two-Dimensional Fractional Fourier Transform," *International Journal of Engineering and Innovative Technology*, vol. 3, no. 2, pp. 253–256, 2013.
44. V. D. Sharma and P. B. Deshmukh, "Inversion Theorem of Two Dimensional Fractional Fourier-Mellin Transform," *International Journal of Applied Computational Science & Mathematics*, vol. 4, no. 1, pp. 17–24, 2014.
45. V. D. Sharma, "Generalized Two-Dimensional Fractional Mellin Transform," *Proceedings/IEEE*, pp. 900–903.
46. V. D. Sharma and P. B. Deshmukh, "S-Type Spaces for Two Dimensional Fractional Fourier-Mellin Transform," *International Journal of Advances in Science, Engineering and Technology*, pp. 74–76, June 2015.
47. V. D. Sharma and P. B. Deshmukh, "Topological Properties of Two Dimensional Fractional Fourier-Mellin Transform," *International Journal of Science, Technology & Management*, vol. 4, Special Issue 1, 2015.



48. V. D. Sharma, "Operational Calculus for Two Dimensional Fractional Fourier-Mellin Transform," *International Journal of Multidisciplinary Research and Development*, vol. 3, no. 1, pp. 99–104, 2016.
49. V. D. Sharma and P. B. Deshmukh, "New Convolution Structure of Quaternion Fractional Fourier-Mellin Transform," 2017, DOI 10.24001/ijaems.icsesd2017.104.
50. V. D. Sharma, "Convolution Structure of Fractional Quaternion Fourier Transform," 2016. The paper develops convolution properties including associative, distributive, linear and shifting properties.