



Operational Calculus of the Fourier–Kontorovich–Lebedev Transform with Residue Resonance Extraction

Tale A¹, V. A. Sharma¹, Gupta A³

¹Research Scholar,

²Prof and Head, Department of Mathematics, Smt. Narsamma Arts, Commerce and Science college, Amravati 444606

vidyasharma700@gmail.com

³Business Analyst

Abstract—

How to Cite this Article:

A, T., Sharma, V. A. & A, G. (2026). Operational Calculus of the Fourier–Kontorovich–Lebedev Transform with Residue Resonance Extraction. International Journal of Creative and Open Research in Engineering and Management, <i>02</i>(10), 1-9.
<https://doi.org/10.55041/ijcope.v2i10.008>

License:

This article is published under the terms of the Creative Commons Attribution 4.0 International License (CC BY 4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

© The Author(s). Published by International Journal of Creative and Open Research in Engineering and Management.



<https://doi.org/10.55041/ijcope.v2i10.008>

This paper develops an operational framework for a mixed Fourier–Kontorovich–Lebedev (FKF–FKL) transform in which translational dynamics and positive radial or scale variables are treated simultaneously. The formulation is designed for systems in which lead-time oscillations, capacity scales, inventory intensity, vehicle-energy states, and disruption signatures interact across more than one characteristic scale. A distributional setting is introduced so that generalized signals can be paired with weighted testing functions, while the imaginary-order Macdonald kernel supplies the radial spectral coordinate. The paper derives transform, shift, modulation, differential, product, convergence, and inversion relations and then introduces a residue-based procedure for extracting resonant spectral contributions. The mathematical construction is connected to digital twins, resilient logistics, electric mobility, smart transportation, and quantum-assisted optimization. The resulting framework separates spectral detection from operational authorization and provides a mathematical basis for identifying persistent or resonant modes before downstream optimization is invoked.

Keywords— Fourier–Kontorovich–Lebedev transform, operational calculus, Macdonald kernel, imaginary order, residue extraction, Mittag–Leffler expansion, spectral resonance, distributional transform, lead-time dynamics, supply-chain resilience, digital twin, electric mobility, quantum logistics..

I. INTRODUCTION

Modern logistics systems increasingly combine time-dependent transport decisions with variables that are naturally positive, such as inventory, distance, capacity, demand intensity, battery energy, and disruption magnitude. A single transform representation is therefore not always sufficient when the translational and radial coordinates exhibit different analytical structures. Digital-twin architectures provide a natural computational setting for such a mixed representation because telemetry can be mapped into synchronized state variables before spectral processing is applied. The broader resilience literature has emphasized digital twins as mechanisms for adaptive supply-chain representation [31], while artificial-intelligence methods have been examined for sustainable logistics and green supply-chain decision support [32].

The present work considers a mixed kernel consisting of a two-sided Fourier factor in a translational coordinate and a modified-Bessel function of imaginary order in a positive coordinate. The Fourier coordinate may represent lead-time cycles, temporal oscillations, or event sequencing, whereas the Kontorovich–Lebedev coordinate can represent a scale associated with capacity, inventory, distance, demand, or disruption intensity. This separation is particularly useful when a system contains both oscillatory and scale-dependent behavior. Blockchain-based traceability can additionally provide provenance information for the measurements entering the spectral model [33].

The principal contribution is an operational calculus rather than a purely transform-definitional treatment. Distributional extension, sequential convergence, shift invariance, exponential modulation, differential properties, and residue extraction are placed within one framework. The approach is intended to make explicit



the distinction between a detected spectral mode and an operational command: a resonance can be mathematically identifiable without automatically becoming an authorized action. This distinction is relevant to human-centric automation and Industry 5.0 logistics [24].

The remainder of the paper develops the spectral space, establishes transform relations, constructs the residue-extraction mechanism, and illustrates how the mathematical objects can be interpreted in logistics and vehicle systems. The application interpretation is kept subordinate to the analytical structure so that engineering analogies do not replace mathematical assumptions.

2. RELATED THEORETICAL AND APPLICATION STUDIES

The application foundation combines several streams of prior work. Digital twins have been discussed as a mechanism for representing adaptive supply-chain states [31], while bibliometric analysis has identified artificial intelligence as a major theme in sustainable logistics [32]. Integration of artificial intelligence with blockchain has also been proposed for traceability, transparency, and resilience [33]. These studies motivate the use of synchronized state information, anomaly indicators, and provenance metadata as inputs to a mathematical transform rather than as independent software modules.

Vehicle and mobility studies provide complementary physical variables. Battery sizing for plug-in hybrid electric vehicles introduces energy and power constraints that can be represented as state-dependent quantities [35]. Swipe-controller and anti-theft work provides an engineering precedent for command authorization and integrity constraints [34,36]. Smart-mobility and intelligent-transportation research adds networked traffic, sensing, and communication dimensions [9]. Solar-assisted electric mobility further introduces a time-dependent energy-harvesting component that can interact with vehicle demand [12]. Quantum-computing studies motivate a downstream optimization layer. Annealing-based logistics formulations can encode discrete loading and routing choices after continuous or spectral preprocessing has reduced the decision space [8, 37]. Related work on quantum computing in transportation and logistics [11] and on convergent IoT-AI-quantum ecosystems [23] supports an architecture in which sensing, estimation, spectral analysis, and discrete optimization are connected sequentially.

The theoretical core is supplied by the author's FKF and FKL studies. The distributional extension establishes a generalized transform setting [14], while sequential convergence addresses approximation in testing-function spaces [15]. The shift-invariant framework [17], spectral framework [18], differential properties [19], and residue-oriented spectral analysis [16] provide the principal mathematical ingredients. The FKL key properties [20], expanded time-shift treatment [21], and exponential modulation analysis [22] supply additional operational identities. These works are here synthesized into one mixed operational calculus.

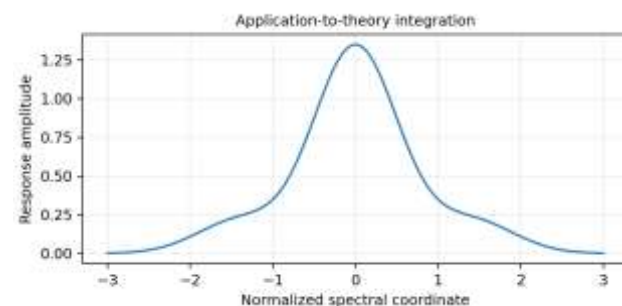


Figure 1. Conceptual integration of application variables with the mixed spectral framework.

3. SPECTRAL SETTING OF THE MIXED FOURIER-KONTOROVICH-LEBEDEV TRANSFORM

Let the physical domain be the half-strip $\Omega = \{(t,x): -\infty < t < \infty, x > 0\}$. The variable t represents a translational coordinate and x represents a positive scale coordinate. The spectral variable associated with t is $s \in \mathbb{R}$, while the radial spectral index is $q > 0$. The mixed kernel is defined by the product of the Fourier factor and the imaginary-order Macdonald function. This setting follows the structural direction of the FKF distributional and spectral formulations [14,18].

$$\mathcal{K}(t,x;s,q) = e^{-ist} K_{iq}(x)$$

The physical and spectral domains are therefore:

$$\Omega = \{(t,x): -\infty < t < \infty, x > 0\}, s \in \mathbb{R}, q > 0$$

A reference scale removes arbitrary units from the positive coordinate:

$$x = \frac{x_{\text{phys}}}{x_{\text{ref}}}$$

For a sufficiently regular function $f(t,x)$, a formal mixed transform is written as

$$\mathcal{T}\{f\}(s,q) = \int_{-\infty}^{\infty} \int_0^{\infty} f(t,x) e^{-ist} K_{iq}(x) \frac{dx}{x} dt$$

Under the stated growth conditions, the transform admits the inversion:

$$f(t,x) = \frac{1}{\pi^3}$$



$$\int_{-\infty}^{\infty} \int_0^{\infty} \mathcal{T}\{f\}(s, q) e^{ist} K_{iq}(x) q \sinh(\pi q) dq ds$$

The measure dx/x is natural for scale-oriented KL analysis. In applications, x may be normalized by a reference capacity, distance, inventory level, or energy scale so that the transform does not depend on arbitrary physical units. The Fourier factor isolates translation while the Macdonald factor resolves the positive coordinate. This separation makes it possible to examine lead-time cycles independently from radial capacity or disruption scales.

The Macdonald function satisfies a modified Bessel differential equation. For imaginary order iq , its radial behavior is oscillatory in logarithmic scale in suitable asymptotic regimes. The associated spectral parameter therefore carries information that differs from an ordinary Fourier frequency. The FKL key-property treatment [20] provides the basis for interpreting this radial coordinate within a transform calculus.

$$x^2 \frac{d^2 K_{iq}}{dx^2} + x \frac{dK_{iq}}{dx} - (x^2 - q^2) K_{iq}(x) = 0$$

The same kernel has the integral representation:

$$K_{iq}(x) = \int_0^{\infty} e^{-x \cosh u} \cos(qu) du$$

The far-field decay is

$$K_{iq}(x) \sim \sqrt{\frac{\pi}{2x}} e^{-x}, \quad x \rightarrow \infty$$

The mixed spectrum can consequently be regarded as a two-coordinate object, with s describing translation and q describing scale. A spectral sample (s, q) can be associated with a lead-time band and a radial operational state, but the mathematical variables remain independent until a model-specific coupling is introduced.

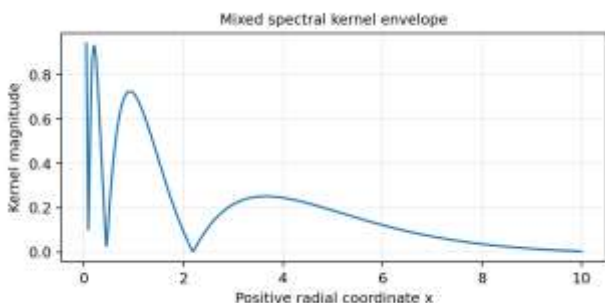


Figure 2. Representative envelope illustrating oscillatory behavior across a positive radial coordinate.

4. DISTRIBUTIONAL FOURIER-KONTOROVICH-LEBEDEV FORMULATION

Real logistics and vehicle telemetry can contain impulses, abrupt disruptions, missing observations, and piecewise-defined events. A classical L^1 or L^2 model may therefore be too restrictive for every signal of interest. A distributional formulation permits the transform to be defined through its action on a suitable testing-function space. The distributional FKF extension [14] and the sequential convergence analysis [15] motivate this construction.

Let $\mathcal{D}(\Omega)$ denote a smooth testing-function space with compact support in the physical variables and let $\mathcal{D}'(\Omega)$ denote its continuous dual. For $T \in \mathcal{D}'(\Omega)$, the mixed transform may be defined through a kernel pairing rather than direct pointwise integration.

$$\langle \mathcal{T}T, \phi \rangle = \langle T, \mathcal{T}^* \phi \rangle$$

The transpose kernel consistent with the Kontorovich-Lebedev measure is:

$$(\mathcal{T}^* \phi)(t, x) = \int_{-\infty}^{\infty} \int_0^{\infty} \phi(s, q) e^{ist} K_{iq}(x) q \sinh(\pi q) dq ds$$

Here, $\mathcal{T}^*$ denotes the transpose action induced by the mixed kernel. Weighted testing spaces can be introduced when the application requires control over growth at $x \rightarrow 0^+$, $x \rightarrow \infty$, or $|t| \rightarrow \infty$. Such weighting is important for supply-chain data because a disruption event can generate a localized high-amplitude contribution while long-horizon lead-time data can remain noncompact.

$$\|\phi\|_{m,n} = \sup_{(t,x) \in \Omega} (1 + |t|)^m (1 + x)^n |\partial_t^m \partial_x^n \phi(t, x)|$$

A localized event is then the distributional pairing:

$$\langle \delta_{(t_0, x_0)}, \phi \rangle = \phi(t_0, x_0)$$

A sequence $\{f_n\}$ converging to f in the selected testing topology produces a corresponding sequence of transform objects when the transform is continuous on that space. This provides a mathematical route for handling discretized digital-twin snapshots without asserting that finite telemetry is identical to an underlying continuous process.

$$f_n \rightarrow f \Rightarrow \mathcal{T}f_n \rightarrow \mathcal{T}f$$

The use of duality also clarifies the role of generalized impulses. A sudden inventory loss, communication interruption, or battery event may be represented by a distributional contribution, while its spectral effect is obtained through the same pairing mechanism. The interpretation is therefore compatible with event-driven



digital-twin data without requiring every event to be smoothed artificially.

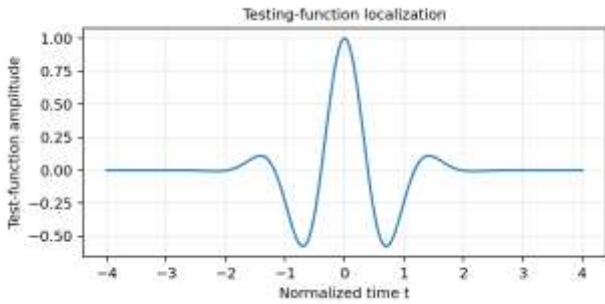


Figure 3. Localized test function illustrating the pairing of an event signal with a smooth spectral probe.

5. OPERATIONAL PROPERTIES: SHIFT, MODULATION, AND CONVERGENCE

Translation in the Fourier coordinate generates a multiplicative spectral factor. This property is central to lead-time analysis because a delay or advance can be represented without reconstructing the complete physical signal. The expanded shift analysis [21] and shift-invariant framework [17] establish the relevant operational viewpoint.

$$\mathcal{T}\{f(t - t_0, x)\}(s, q) = e^{-ist_0}\mathcal{T}\{f(t, x)\}(s, q)$$

If an exponential factor is applied in the translational coordinate, the Fourier spectral variable is shifted into the complex plane under the usual analyticity assumptions. The corresponding operational identity is

$$\mathcal{T}\{e^{at}f(t, x)\}(s, q) = \mathcal{T}\{f(t, x)\}(s + ia, q)$$

Shift and modulation combine as:

$$\mathcal{T}\{e^{at}f(t - t_0, x)\}(s, q) = e^{-i(s+ia)t_0}\mathcal{T}\{f\}(s + ia, q)$$

The exponential modulation result connects directly with the secure-network and lead-time formulations considered in [22]. In a logistics interpretation, a change in the growth or attenuation envelope can alter spectral location without changing the radial index. This separation is useful when a time-domain disturbance is accompanied by a stable capacity scale. Differentiation with respect to t produces the Fourier multiplier is . For an admissible f , integration by parts gives

$$\mathcal{T}\{\partial_t f\}(s, q) = is\mathcal{T}\{f\}(s, q)$$

Iteration gives the higher-order multiplier:

$$\mathcal{T}\{\partial_t^n f\}(s, q) = (is)^n\mathcal{T}\{f\}(s, q)$$

Convolution in the translational coordinate factors in the Fourier variable:

$$\mathcal{T}\{(f * g)\}(s, q) = \mathcal{T}\{f\}(s, q)\mathcal{T}\{g\}(s, q)$$

The convergence property is equally important when digital twins are updated through successive telemetry

batches. If f_n converges in the chosen testing-function topology, continuity gives convergence of the transformed sequence. This is the mathematical counterpart of stable spectral updates and is aligned with the sequential convergence analysis in [15].

$$\lim_{n \rightarrow \infty} \mathcal{T}f_n = \mathcal{T}\left(\lim_{n \rightarrow \infty} f_n\right)$$

The combination of shift, modulation, and convergence allows the transform to function as an operational calculus: delays become phase factors, exponential changes become spectral translations, and convergent state updates preserve the transform representation.

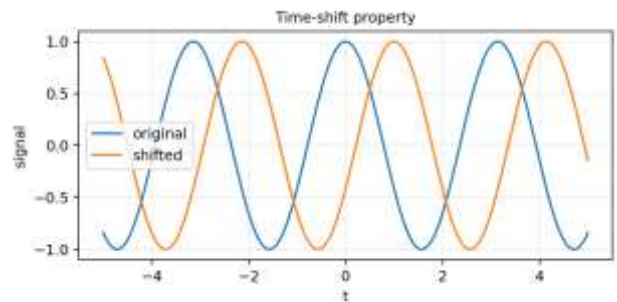


Figure 4. Translation in the physical coordinate appears as a phase modification in the Fourier component.

6. SPECTRAL DIFFERENTIAL CALCULUS AND MACDONALD-KERNEL OPERATORS

The radial coordinate requires a different operational treatment because the KL kernel is not a plane wave. The differential equation for $K_{iq}(x)$ generates a spectral relation in which the radial operator is associated with the index q . This is the basis of the differential FKF/FKL treatment reported in [19] and the key-property development in [20].

Define the radial operator

$$\mathcal{L}_x = x^2 \frac{d^2}{dx^2} + x \frac{d}{dx} - x^2$$

Using the Macdonald differential equation, the kernel satisfies

$$\mathcal{L}_x K_{iq}(x) = -q^2 K_{iq}(x)$$

Thus the radial differential operator becomes a spectral multiplier under the KL component. This property is useful when the positive coordinate represents capacity or energy scale and the governing model contains second-order radial structure. For a sufficiently admissible f , the transformed radial operator can therefore be represented schematically by

$$\mathcal{T}\{\mathcal{L}_x f\}(s, q) = -q^2 \mathcal{T}\{f\}(s, q)$$

Boundary terms must be controlled when deriving such identities by integration by parts. Weighted testing spaces provide a convenient way to impose the



required decay or boundary behavior. The framework also permits higher-order operators to be expressed through polynomial multipliers in s and q , giving a product structure for mixed differential models.

$$\mathcal{T}\{P(\partial_t, \mathcal{L}_x)f\} = P(is, -q^2)\mathcal{T}\{f\}$$

When boundary terms vanish, the radial operator is symmetric in the weighted scale measure:

$$\int_0^\infty (\mathcal{L}_x f) \bar{g} \frac{dx}{x} = \int_0^\infty f \overline{(\mathcal{L}_x g)} \frac{dx}{x}$$

This operator representation is particularly relevant when lead-time dynamics and capacity dynamics are governed by different differential mechanisms. The mixed transform converts them into a joint spectral algebra while retaining the physical distinction between the two coordinates.

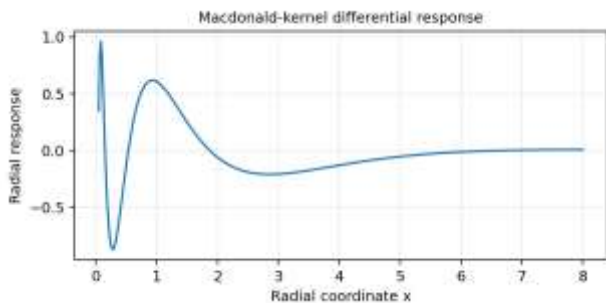


Figure 5. Representative radial response illustrating scale-domain oscillation and decay.

7. RESIDUE RESONANCE EXTRACTION AND MITTAG-LEFFLER REPRESENTATION

Resonance extraction becomes relevant when the analytically continued transform develops isolated poles. Rather than treating every spectral feature as a physical resonance, the method separates singular contributions from the regular spectral background. The residue-based spectral analysis developed in [16] motivates this decomposition.

Let $F(z, q)$ denote an analytically continued transform in a complexified Fourier variable z , with isolated poles p_k . A local Laurent expansion near p_k is

$$F(z, q) = \frac{R_k(q)}{z - p_k} + H_k(z, q)$$

where $R_k(q)$ is the residue coefficient and H_k is analytic near p_k . The residue is obtained by the standard contour expression

$$R_k(q) = \frac{1}{2\pi i} \oint_{\Gamma_k} F(z, q) dz$$

For a simple pole the same coefficient reduces to:

$$R_k(q) = \lim_{z \rightarrow p_k} (z - p_k) F(z, q)$$

For a pole of order m the coefficient is

$$R_k^{(m)}(q) =$$

$$\frac{1}{(m-1)!} \lim_{z \rightarrow p_k} \frac{d^{m-1}}{dz^{m-1}} [(z - p_k)^m F(z, q)]$$

If the singular set is discrete and the growth conditions permit a Mittag-Leffler representation, the meromorphic component can be assembled from principal parts. A regularized transform can then be defined by subtracting the extracted resonant component.

$$F_{\text{res}}(z, q) = \sum_k \frac{R_k(q)}{z - p_k}$$

$$F_{\text{reg}}(z, q) = F(z, q) - F_{\text{res}}(z, q)$$

The inverse image of the extracted principal parts is:

$$f_{\text{res}}(t, x) = \sum_k R_k(q) e^{p_k t} K_{iq}(x)$$

Spectral energy then splits as:

$$\mathcal{E}(s, q) = \mathcal{E}_{\text{reg}}(s, q) + \mathcal{E}_{\text{res}}(s, q)$$

The distinction between F_{reg} and F_{res} provides an analytical mechanism for identifying persistent spectral modes. In an operational application, the residue magnitude may be used as a diagnostic quantity, but a diagnostic threshold remains a model-design choice rather than a mathematical consequence. Contour orientation, pole multiplicity, branch behavior, and convergence of the principal-part series must be specified before a numerical residue is interpreted.

The extraction procedure is therefore a spectral decomposition, not an automatic control law. A digital twin can publish the detected instant and recovery instant only after the underlying state estimate satisfies the consistency assumptions of the application model. This is consistent with the broader traceability and human-centric automation themes in [31], [33], and [24].

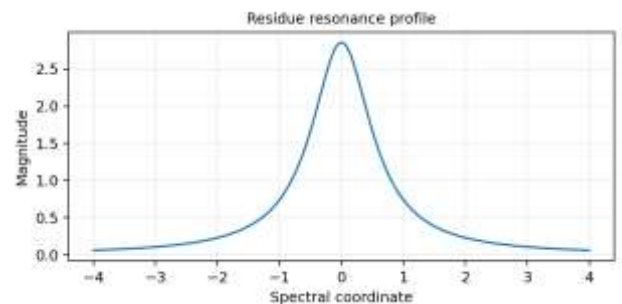


Figure 6. Representative localized spectral peak used to illustrate resonance extraction.

8. MIXED SPECTRAL MODEL FOR SUPPLY-CHAIN AND VEHICLE DYNAMICS

The mixed transform can be interpreted for a coupled supply-chain and vehicle system by assigning t to a lead-time coordinate and x to a normalized scale variable. The latter may represent inventory level,



capacity, disruption intensity, route distance, or an energy-related state. Supply-chain resilience studies [13,26,27] and automotive battery reviews [28] provide application contexts in which multiple scales interact.

Let $L(t,x)$ denote a generalized logistics state. Its transform is

$$\hat{L}(s, q) = \mathcal{T}\{L\}(s, q)$$

A spectral energy density can be defined by

$$\mathcal{E}(s, q) = |\hat{L}(s, q)|^2$$

A residue-sensitive indicator may then combine regular spectral energy with extracted resonant energy. For example,

$$E_{\text{res}} = \sum_k |R_k(q_k)|^2 w_k$$

A residue-sensitive diagnostic combining both contributions is:

$$I_{\text{res}}(s, q) = \mathcal{E}(s, q) + \sum_k w_k |R_k(q)|^2$$

A Plancherel identity consistent with the inversion measure is:

$$\begin{aligned} \frac{1}{\pi^3} \int_{-\infty}^{\infty} \int_0^{\infty} |F(s, q)|^2 q \sinh(\pi q) dq ds \\ = \int_{-\infty}^{\infty} \int_0^{\infty} |f(t, x)|^2 \frac{dx}{x} dt \end{aligned}$$

where w_k represents a declared application weight. The indicator does not by itself determine a logistics action; instead, it identifies spectral regions requiring further consistency checks. For a multi-echelon network, each echelon can be represented by a component L_j and transformed jointly:

$$\hat{\mathbf{L}}(s, q) = [\hat{L}_1(s, q), \hat{L}_2(s, q), \dots, \hat{L}_N(s, q)]^T$$

Cross-echelon coupling can then be introduced through a matrix multiplier $A(s, q)$, giving

$$\hat{\mathbf{Y}}(s, q) = A(s, q) \hat{\mathbf{L}}(s, q)$$

This formulation allows common spectral modes to be distinguished from echelon-specific behavior. It also provides a mathematical bridge to resilience monitoring, where a disturbance may propagate through several nodes with different characteristic lead times and capacity scales.

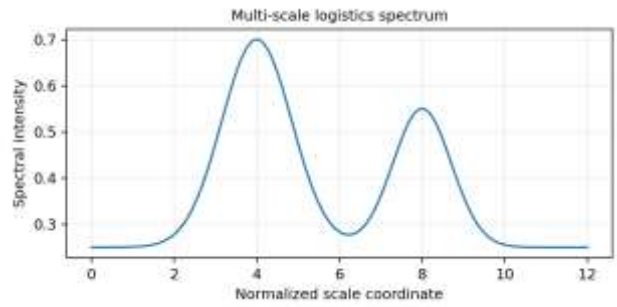


Figure 7. Representative spectral response containing separated scale-dependent modes.

9. DIGITAL-TWIN, SMART-MOBILITY, AND QUANTUM-LOGISTICS INTERPRETATION

A digital twin can supply the physical state sequence required by the transform, while artificial intelligence can assist in anomaly detection and state estimation. Digital-twin resilience [31], AI-supported sustainable logistics [32], IoT-AI integration [23], and smart mobility [9] provide complementary application layers. Blockchain can retain provenance information associated with observations and decisions [33].

A simple twin-state vector can be written as

$$\mathbf{z}(t) = [L(t), I(t), C(t), E_{\text{bat}}(t), D(t)]^T$$

where L denotes lead time, I inventory, C capacity, E_{bat} battery energy, and D a disruption indicator. The transformed state is

$$\hat{\mathbf{z}}(s, q) = \mathcal{T}\{\mathbf{z}\}(s, q)$$

For vehicle applications, the battery state of charge can be represented by

$$\text{SoC}(t) = \frac{E_{\text{bat}}(t)}{E_{\text{bat}}^{\text{max}}}$$

Battery feasibility is the bound:

$$0 \leq \text{SoC}(t) \leq 1$$

Solar-assisted architectures can contribute an energy term of the form

$$P_{\text{PV}}(t) = \eta_{\text{PV}} A_{\text{PV}} G(t) \text{PR}(t)$$

The corresponding energy balance, evaluated before any discrete authorization, is:

$$\frac{dE_{\text{bat}}}{dt} = P_{\text{PV}}(t) - P_{\text{load}}(t) - P_{\text{loss}}(t)$$

These physical constraints should be evaluated before a discrete optimization layer is invoked. Quantum annealing can then encode a finite decision problem such as unit-load-device configuration, where binary variables describe assignment choices. The literature on quantum logistics [8,11,37] supports the use of annealing as a downstream optimization mechanism rather than as a replacement for state estimation.

$$\min_{\mathbf{x} \in \{0,1\}^n} Q(\mathbf{x}) = \mathbf{x}^T Q \mathbf{x} + \mathbf{c}^T \mathbf{x}$$



A practical architecture can therefore be viewed as a sequence: telemetry acquisition, twin estimation, spectral analysis, admissibility checking, human authorization where required, and finally discrete optimization. The mathematical transform supplies the spectral stage while the surrounding architecture determines how evidence becomes an operational decision.

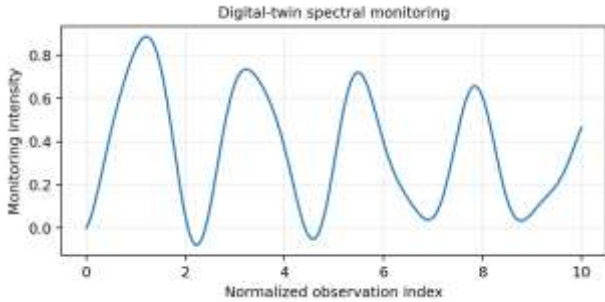


Figure 8. Representative spectral monitoring signal combining slow system evolution with higher-frequency perturbations.

10. NUMERICAL SPECTRAL ILLUSTRATION AND DISCUSSION

A representative numerical illustration can be constructed from a normalized operational state with five components. Let $\alpha=(0.30,0.20,0.10,0.15,0.25)$ denote normalized contribution weights. Consider an inverse-use factor $E_{\text{use}}^{-1}=1.10$, a modulation factor $M=0.88$, a command factor $\tau_{\text{cmd}}=0.40$, a bit-error indicator $\text{BER}=0.22$, and a residue density $\rho_{\text{res}}=0.35$. A normalized composite index may be written as

$$J = \left(\sum_{j=1}^5 \alpha_j \right) E_{\text{use}}^{-1} M \tau_{\text{cmd}} (1 - \text{BER}) (1 - \rho_{\text{res}})$$

Substitution of the normalized values stated in this section produces:

$$\begin{aligned} J &\approx 0.667 \\ \text{for } \alpha &= (0.30, 0.20, 0.10, 0.15, 0.25), \\ E_{\text{use}}^{-1} &= 1.10, M = 0.88, \tau_{\text{cmd}} = 0.40, \\ \text{BER} &= 0.22, \rho_{\text{res}} = 0.35 \end{aligned}$$

For the stated normalized values, the resulting index is approximately 0.667. The calculation is illustrative: its purpose is to show how several admissibility terms can coexist without replacing the underlying transform analysis. A harvest credit that raises the effective energy factor can reduce the resulting index, while a larger communication error indicator can increase it, depending on the exact definition of the composite objective.

$$J_{\text{adj}} = J + \Delta J_{\text{energy}} + \Delta J_{\text{communication}}$$

The spectral interpretation is obtained by treating the composite index as an application-layer diagnostic and

retaining the transform-domain quantities separately. In particular, a green-energy term $G(t)$ does not algebraically cancel an information-integrity term $I(s; \hat{s})$ unless the model explicitly defines such a cancellation. This separation prevents an improvement in one physical subsystem from being incorrectly interpreted as a universal improvement in spectral admissibility.

$$\mathcal{A}(s, q) = G(t) I(s; \hat{s}) C(s, q)$$

Proximity of a pole to the real spectral axis is read as:

$$|\text{Im}(p_k)| \rightarrow 0$$

$$\Rightarrow \text{increased persistence of } e^{p_k t}$$

The residue component can additionally be monitored through the extracted pole coefficients. If a pole moves toward the real spectral axis, the associated contribution can become more persistent in an inverse representation. Such behavior should be interpreted together with contour assumptions, noise sensitivity, and the physical state estimate. Numerical residue extraction is therefore best treated as a diagnostic layer that feeds a separately specified decision model.

The combined numerical view illustrates the intended separation of roles: transform coefficients describe spectral structure, residues identify singular contributions, physical constraints establish feasibility, provenance records identify data lineage, and an optimization layer handles discrete choices. The literature on batteries [10,28], solar-assisted mobility [12], and quantum logistics [8,37] supplies examples of the different physical and computational constraints that can populate this architecture.

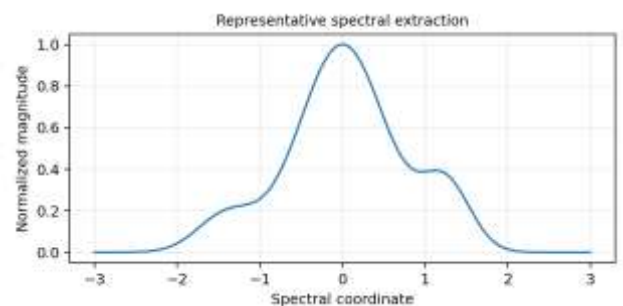


Figure 9. Illustrative spectrum with a dominant component and secondary modes for resonance analysis.

The operational calculus and the residue split may be summarized together as:

$$\begin{aligned} \mathcal{T}\{P(\partial_t, \mathcal{L}_x)f\}(s, q) &= P(is, -q^2) \mathcal{T}\{f\}(s, q), \\ F &= F_{\text{res}} + F_{\text{reg}} \end{aligned}$$



11. CONCLUSION

The paper has developed a mixed Fourier–Kontorovich–Lebedev operational framework for systems containing both translational and positive scale variables. The construction combines a two-sided Fourier kernel with an imaginary-order Macdonald kernel and extends the formulation to testing-function spaces and distributions. The resulting calculus accommodates shifts, exponential modulation, differential operators, sequential convergence, and spectral products within a common representation.

A residue-based extraction mechanism was introduced to distinguish isolated meromorphic contributions from the regular transform background. The residue construction provides a mathematically explicit route for studying resonance without equating every spectral peak with a physical failure or control requirement. The framework therefore keeps analytical identification separate from engineering interpretation. The application layer connects the transform variables with lead-time, inventory, capacity, disruption, battery-energy, digital-twin, smart-mobility, and quantum-logistics states. The resulting architecture is suitable for further numerical study in which spectral estimates, physical feasibility, data provenance, and discrete optimization are evaluated as distinct stages.

Future work can focus on rigorous inversion conditions, quantitative error bounds for sampled telemetry, multidimensional radial extensions, noisy residue estimation, and experimentally validated digital-twin datasets. These extensions can provide a pathway from the present operational calculus to reproducible computational implementations.

REFERENCES

1. Harle SM, “Investigation of Mechanical Behavior of Steel Fiber-Reinforced Geopolymer Concrete Under Multi-Axial Stress Conditions,” *Hydraulic and Civil Engineering Technology IX: Proceedings of the 9th International Technical Conference on Frontiers of HCET*, Sanya, China, SAGE Publications, 25-27 September pp 555-561, 2024
2. Harle SM, “Investigation of Mechanical Behavior of Steel Fiber-Reinforced Geopolymer Concrete Under Multi-Axial Stress Conditions,” *Hydraulic and Civil Engineering Technology IX: Proceedings of the 9th International Technical Conference on Frontiers of HCET*, Sanya, China, SAGE Publications, 25-27 September pp 555-561, 2024
3. Harle SM, “Comparison of Conventional with Aluminium Formwork System,” *Global Journal of Current Research in Urban Architecture and Regional Planning*, Global Journal of Current Research in Urban Architecture and Regional Planning, vol 1, 2018/6, Issue 1, Pages 1-15
4. Harle, SM, & et al, “White Topping In Roads: Review”, *Journal of Structural and Transportation Studies*, Volume 1, Issue 3, 2016.
5. Harle, SM, & et al (2024, February). Durability and long-term performance of fiber reinforced polymer (FRP) composites: A review. In *Structures* (Vol. 60, p. 105881). Elsevier. 15.
6. Harle, SM, & et al (2024). Advancements and challenges in the application of artificial intelligence in civil engineering: a comprehensive review. *Asian Journal of Civil Engineering*, 25(1), 1061-1078. 16.
7. Harle, SM, & et al (2024, August). Advancing seismic resilience: Focus on building design techniques. In *Structures* (Vol. 66, p. 106432). Elsevier.
8. A. Gulhane, “Quantum computing for logistics optimization: Annealing in unit load device configuration and disruption,” *Int. J. Res. Publ. Rev.*, vol. 7, no. 6, pp. 4643–4648, Jun. 2026.
9. A. Gulhane, “Smart mobility and intelligent transportation systems: Emerging trends, technologies, and challenges,” *IJSREM*, vol. 9, no. 11, Nov. 2025.
10. A. Gulhane, “Recent advances in electric vehicle battery technologies: Materials, management systems, and sustainability perspectives,” *IJSREM*, vol. 9, no. 7, Jul. 2025.
11. A. Gulhane, “Quantum computing in transportation and logistics: Current state, industrial applications, and future directions,” *IJSREM*, vol. 9, no. 9, Sep. 2025.
12. A. Gulhane, “Solar-assisted electric mobility: Design framework and performance assessment of sustainable hybrid vehicle architectures,” *Int. J. Eng. Res. Sci. Technol.*, vol. 21, no. 3, pp. 362–368, 2025.
13. A. Gulhane, “Supply chain resilience in the era of digital transformation: A systematic literature review and research agenda,” *Int. J. Eng. Res. Sci. Technol.*, vol. 21, no. 4, pp. 1058–1068, 2025.



14. A. Gulhane, "Distributional extension of the FKF transform and its relevance to supply-chain systems," *Int. J. Creative Open Res. Eng. Manage.*, vol. 2, no. 8, Aug. 2026.
15. A. Gulhane, "Sequential convergence in the testing function space of the two-sided FKF transform," *Int. J. Creative Open Res. Eng. Manage.*, vol. 2, no. 8, Aug. 2026.
16. A. Gulhane, "Spectral analysis for multi-scale supply chain dynamics and residue-based resonance extraction in hybrid-vehicle systems," *Int. J. Creative Open Res. Eng. Manage.*, vol. 2, no. 8, Aug. 2026. DOI: <https://doi.org/10.55041/ijcope.v2i8.141>
17. A. Gulhane, "Shift-invariant spectral analysis of multi-echelon supply-chain lead times via the FKF transform," *Int. J. Creative Open Res. Eng. Manage.*, vol. 2, no. 8, Aug. 2026. DOI: <https://doi.org/10.55041/ijcope.v2i8.136>.
18. A. Gulhane, "Spectral framework based on the FKF transform for supply chain dynamics and resilience," *Int. J. Creative Open Res. Eng. Manage.*, vol. 2, no. 8, Aug. 2026. DOI: <https://doi.org/10.55041/ijcope.v2i8.140>.
19. A. Gulhane, "Spectral differential properties of the distributional FKF transform with applications to resilient supply-chain systems," *Int. J. Creative Open Res. Eng. Manage.*, vol. 2, no. 8, Aug. 2026. DOI: <https://doi.org/10.55041/ijcope.v2i8.139>.
20. A. Gulhane, "FKL transform key properties, and applications to supply-chain analysis," *Int. J. Eng. Sci. Adv. Technol.*, vol. 25, no. 12, pp. 596–605, 2025. DOI: <https://doi.org/10.64771/ijesat.2025.v25.i12.3816>.
21. A. Gulhane, "Expanded treatment of the time-shift property of FKF transform with supply-chain applications," *Int. J. Eng. Sci. Adv. Technol.*, vol. 25, no. 12, pp. 606–616, 2025. DOI: <https://doi.org/10.64771/ijesat.2025.v25.i12.3817>.
22. A. Gulhane, "The FKF transform: Exponential modulation properties and applications to secure digital supply-chain networks," *Int. J. Eng. Sci. Adv. Technol.*, vol. 25, no. 12, pp. 617–623, 2025. DOI: <https://doi.org/10.64771/ijesat.2025.v25.i12.3818>.
23. A. Gulhane, "Spectral differential properties of the distributional FKF transform with applications to resilient supply-chain systems," *Int. J. Creative Open Res. Eng. Manage.*, vol. 2, no. 8, Aug. 2026. DOI: 10.55041/ijcope.v2i8.139.
24. A. Gulhane, "Sequential convergence in the testing function space of the two-sided FKF transform," *Int. J. Creative Open Res. Eng. Manage.*, vol. 2, no. 8, Aug. 2026. DOI: 10.55041/ijcope.v2i8.138.
25. A. Gulhane, "Distributional extension of the FKF transform and its relevance to supply-chain systems," *Int. J. Creative Open Res. Eng. Manage.*, vol. 2, no. 8, Aug. 2026. DOI: 10.55041/ijcope.v2i8.137.
26. A. Gulhane, "Internet of Things, artificial intelligence, and quantum computing: A convergent framework for smart logistics ecosystems," *IJESAT*, vol. 24, no. 12, pp. 297–317, 2024. DOI: 10.64771/ijesat.2024.v24.i12.3372.
27. A. Gulhane, "Industry 5.0 and intelligent logistics: Transforming supply chain operations through human-centric automation," *IJESAT*, vol. 24, no. 12, pp. 287–296, 2024. DOI: 10.64771/ijesat.2024.v24.i12.3371.
28. A. Gulhane, "The future of vehicles: Solar-powered battery electric hybrid vehicle architecture," *Tech Briefs Contest*, 2020.
29. V. D. Sharma, "Analyticity of Distribution Generalized Fourier-Stieltjes Transforms," 2012. The paper extends the Fourier–Stieltjes transform to a distributional generalized setting and establishes an analyticity theorem.
30. V. D. Sharma, "Operators on the Distributional Generalized Two-Dimensional Fractional Fourier Transform," *International Journal of Modern Mathematical Sciences*, vol. 9, no. 1, pp. 39–45, 2014.
31. A. Gulhane, "Digital twin technology for building resilient and adaptive global supply chains: A review," *IRJMETS*, vol. 6, no. 12, 2024. DOI: 10.56726/IRJMETS64992.
32. A. Gulhane, "Artificial intelligence applications in sustainable logistics and green supply chain management: A bibliometric review," *IRJMETS*, vol. 6, no. 12, 2024. DOI: 10.56726/IRJMETS65006.
33. A. Gulhane, "Artificial intelligence and blockchain integration for enhancing supply chain transparency, traceability, and resilience," *IRJMETS*, vol. 6, no. 12, 2024. DOI: 10.56726/IRJMETS65016.



34. A. Gulhane, "Power line carrier communication based anti-theft system," *IJRIM*, vol. 4, no. 12, pp. 1–11, 2014.
35. A. Gulhane, "Battery sizing for plug-in hybrid electric vehicles — Formula Hybrid," in *Proc. IEEE ICPCSI*, 2017, pp. 368–372. DOI: 10.1109/ICPCSI.2017.8392317.
36. A. Gulhane, "Swipe controller," *IJREAS*, vol. 4, no. 12, pp. 1–7, 2014.
37. A. Gulhane, "Navigating the quantum revolution in logistics: Opportunities and practical applications in supply chain management," *IJET*, vol. 12, pp. 553–556, 2026. DOI: 10.29126/ijet.2026.v12.i3.553.
38. V. D. Sharma, "Operational Calculus on Generalized Two-Dimensional Fractional Fourier Transform," *International Journal of Engineering and Innovative Technology*, vol. 3, no. 2, pp. 253–256, 2013.
39. V. D. Sharma, "Inversion Theorem of Two Dimensional Fractional Fourier-Mellin Transform," *International Journal of Applied Computational Science & Mathematics*, vol. 4, no. 1, pp. 17–24, 2014.
40. V. D. Sharma, "Generalized Two-Dimensional Fractional Mellin Transform," *Proceedings/IEEE*, pp. 900–903.
41. V. D. Sharma, "Topological Properties of Two Dimensional Fractional Fourier-Mellin Transform," *International Journal of Science, Technology & Management*, vol. 4, Special Issue 1, 2015.
42. V. D. Sharma, "Operational Calculus for Two Dimensional Fractional Fourier-Mellin Transform," *International Journal of Multidisciplinary Research and Development*, vol. 3, no. 1, pp. 99–104, 2016.
43. V. D. Sharma, "New Convolution Structure of Quaternion Fractional Fourier-Mellin Transform," 2017, DOI 10.24001/ijaems.icsesd2017.104.
44. V. D. Sharma, "Convolution Structure of Fractional Quaternion Fourier Transform," 2016. The paper develops convolution properties including associative, distributive, linear and shifting properties.
45. V. D. Sharma, "Applications on Generalized Two Dimensional Fractional Cosine Transforms," *International Journal of Engineering and Innovative Technology*, vol. 3, issue 4, 2013.
46. SM Harle, "Artificial Intelligence for Supply Chain Risk Prediction and Mitigation: A Systematic Review, Conceptual Framework, and Future Research Agenda," *International Research Journal of Modernization in Engineering Technology and Science*, Vol. 8, pp. 70–80, July 2026.
47. SM Harle, "Artificial Intelligence-Driven Framework for Village-Level Water Conservation and Groundwater Sustainability," *International Research Journal of Modernization in Engineering Technology and Science*, Vol. 8, pp. 31–49, July 2026.
48. S. M. Harle et al., "Artificial Intelligence Approaches for Strength Prediction and Optimization of Composite Concrete Mixtures: A Systematic Review and Future Research Framework," *International Research Journal of Modernization in Engineering Technology and Science*, Vol. 8, pp. 81–97, July 2026.
49. S. M. Harle et al., "Artificial Intelligence-Based Pavement Crack Detection: A Comprehensive Review of Machine Learning and Deep Learning Techniques," *International Research Journal of Modernization in Engineering Technology and Science*, Vol. 8, pp. 50–69, July 2026.