



The Burgers-Fisher Equation: Convection, Diffusion, and Reaction in a Single Model -- A Comprehensive Review

Dr. Weiguang Huang

DrHuang.com, Sydney, NSW 2033, Australia

ORCID: 0000-0002-5766-4516

How to Cite this Article:

Huang, W. (2026). The Burgers-Fisher Equation: Convection, Diffusion, and Reaction in a Single Model -- A Comprehensive Review. International Journal of Creative and Open Research in Engineering and Management, 2(9), 1-9. <https://doi.org/10.55041/ijcope.v2i9.252>

License:

This article is published under the terms of the Creative Commons Attribution 4.0 International License (CC BY 4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

© The Author(s). Published by International Journal of Creative and Open Research in Engineering and Management.



<https://doi.org/10.55041/ijcope.v2i9.252>

Abstract

The Burgers-Fisher equation unites the two most influential scalar models of nonlinear science: the Burgers equation, which captures the competition between convection and diffusion, and the Fisher-Kolmogorov-Petrovskii-Piskunov (Fisher-KPP) equation, which describes the propagation of reaction fronts. The resulting convection-reaction-diffusion equation is a prototypical model for travelling fronts in population genetics, combustion, plasma physics, and biology, and it is one of the few nonlinear partial differential equations for which exact travelling-wave solutions are known in closed form. This review provides a comprehensive and critical assessment of the literature on the Burgers-Fisher equation and its generalizations. We survey the analytical theory of travelling waves, including the exact solutions that make the equation a benchmark, and the rigorous results on existence, uniqueness, and stability of these waves. We examine the broad family of numerical methods that have been devised to approximate its solutions, from finite difference and finite element schemes to B-spline collocation, spectral, meshless, lattice Boltzmann, and operator-splitting methods. We discuss the fractional-order extensions of the equation, its relation to the Burgers-Huxley and FitzHugh-Nagumo equations, and the wide range of applications in biology, chemistry, physics, and engineering. Particular attention is given to recent developments, including machine-learning approaches such as physics-informed neural networks and deep learning algorithms, and to the newest results published in 2025 and 2026. We close by identifying open problems and future

directions.

1. Introduction

The Burgers-Fisher equation combines two of the most celebrated scalar models in nonlinear science into a single equation. In its generalized form it reads

$$u_t + \alpha u^\delta u_x = \nu u_{xx} + \beta u(1-u^\delta),$$

where $u(x,t)$ is a scalar field, and α , δ , ν , and β are positive parameters. When $\beta = 0$, one recovers the generalized Burgers equation, whose theory was reviewed in the companion article on the Burgers equation [1-3]. When $\alpha = 0$, one recovers the generalized Fisher-KPP equation, the celebrated model introduced by Fisher for the spread of an advantageous gene and studied in parallel by Kolmogorov, Petrovskii, and Piskunov [4]. The union of the two gives a minimal model in which convection, diffusion, and reaction interact, and it has become a standard benchmark in both analysis and scientific computing [5-6].

The interest in the Burgers-Fisher equation rests on three pillars. First, it possesses exact travelling-wave solutions expressible in closed form, which makes it an ideal test problem for numerical methods and a rare example of an



exactly solvable reaction-diffusion-convection equation [6-7]. Second, it is a prototypical model for front propagation in spatially extended systems, and its analysis illuminates the general theory of travelling waves for reaction-diffusion-convection equations [5,8]. Third, it arises, in one form or another, across an exceptionally wide range of applications, from the spread of biological populations and the propagation of nerve impulses to combustion waves, plasma dynamics, and traffic flow [4,9-11].

This breadth is not accidental. The Burgers nonlinearity $u u_x$ represents the self-steepening of waves and fluxes, while the Fisher reaction term $u(1-u)$ represents logistic growth with saturation. The equation therefore sits at the intersection of two of the most important traditions in nonlinear PDE theory: the theory of hyperbolic conservation laws and viscous shocks, and the theory of reaction-diffusion fronts. Their combination produces a model in which a travelling front is simultaneously a shock-like structure and a reaction front, and in which the balance between convection and reaction selects both the speed and the shape of the transition layer [5,8]. Understanding this balance is the central theme of the subject, and it recurs in every section of this review.

The purpose of this review is to provide a comprehensive and critical survey of the literature on the Burgers-Fisher equation. We organize the material thematically. Section 2 recalls the exact travelling-wave solutions that give the equation its special status and surveys the analytical methods used to derive them. Section 3 reviews the rigorous theory of existence, uniqueness, and stability of these waves, including the influence of delays and nonlocal effects. Section 4 surveys the extensive numerical literature, spanning finite difference, finite element, spline, spectral, meshless, lattice Boltzmann, and splitting methods. Section 5 discusses the fractional-order extensions of the equation. Section 6 examines its relation to the Burgers-Huxley and FitzHugh-Nagumo equations and to the general theory of reaction-diffusion-convection systems. Section 7 surveys applications, and Section 8 reviews the emerging machine-learning literature, including physics-informed neural networks. We close with open problems and an outlook.

2. Exact travelling-wave solutions

The defining feature of the Burgers-Fisher equation is that it admits exact travelling-wave solutions of a remarkably simple form. Substituting the ansatz $u(x,t) = v(\xi)$ with $\xi = x - ct$ reduces the PDE to a nonlinear second-order ordinary differential equation, which can be integrated for special parameter values. The result is a wavefront of the form

$$u(x,t) = [1/2 + (1/2) \tanh(A(\xi - \xi_0))]^{1/\delta},$$

with wave speed c determined explicitly by the parameters [6-7]. These solutions interpolate monotonically between the stable and unstable steady states $u = 0$ and $u = 1$ and move with a constant speed that depends on the coefficients of convection, diffusion, and reaction [6,12]. The existence of such explicit solutions is remarkable; it follows from a delicate cancellation between the convective and reactive nonlinearities that renders the resulting ODE integrable [13-14].

Several analytical techniques have been used to derive and systematize these exact solutions. The tanh method, developed by Malfliet and Hereman, expresses travelling-wave solutions as finite polynomials in the hyperbolic tangent and has become the standard tool for this class of equations [12,15]. Its extensions, including the unified method of Akcagil and Aydemir, provide a general framework that unifies the family of hyperbolic tangent methods [15-17]. The restrictive Padé approximation has been applied to the generalized Fisher and Burgers-Fisher equations, and the homotopy perturbation method provides approximate analytical solutions with high accuracy [7,18]. The exp-function method, sometimes hybridized with genetic algorithms, has also been used to construct approximate travelling waves [19]. For the generalized Fisher equation, the first-integral method and related techniques yield a family of exact solutions that includes the classical wavefronts as special cases [20-21].

A unifying perspective on exact solutions has emerged through the Simple Equations Method of Vitanov, which subsumes the tanh method, the method of the simplest equation, and the general theory of exact solutions for nonlinear differential equations [22]. Within this framework, the travelling waves of the Fisher, Burgers-Huxley, and related equations arise as compositions of solutions of simpler auxiliary equations, and the choice of the auxiliary equation encodes the balance between nonlinearity and dissipation [22]. Related expansions, such as the generalized G'/G -expansion method for equations with variable coefficients, extend the construction of closed-form solutions to time-dependent and spatially inhomogeneous settings [23]. Variational principles for generalized Burgers-type equations



with fractal derivatives provide yet another route to approximate solutions, connecting the exact-solution program to the calculus of variations [24]. The convergence and uniqueness of these solutions can also be studied directly, as demonstrated for the generalized Fisher equation through the analysis of its travelling-wave behaviour [20].

The exact solutions of the Burgers-Fisher equation play a role that goes well beyond the equation itself. They provide benchmark data against which numerical methods can be validated, since the error can be measured against the exact wavefront [6,25-26]. They also illuminate the structure of more general reaction-diffusion-convection equations, for which exact solutions are the exception rather than the rule [5]. The general theory of travelling waves in nonlinear diffusion-convection-reaction processes, developed by Gilding and Kersner, shows that the Burgers-Fisher equation sits at a special point in the parameter space where the competition between convection and reaction admits explicit waves [5,8].

3. Existence, uniqueness, and stability of travelling waves

The rigorous theory of travelling waves for the Burgers-Fisher equation and its relatives has developed along several lines. For the Fisher-KPP equation, the classical theory establishes the existence of monotone travelling waves for all speeds $c \geq c^*$, where the minimal speed $c^* = 2\sqrt{\beta\nu}$ is determined by linearizing about the unstable state [4,8]. The selection of the speed for localized initial data, and the celebrated logarithmic correction to the front position discovered by Bramson, are now well understood [27-28]. For the full Burgers-Fisher equation, the convective term modifies both the speed and the shape of the front, and the theory must account for the competition between convection and reaction [5,13].

The interplay between delay and front propagation has been a central theme in the modern theory. Reaction-diffusion equations of KPP-Fisher type with time delays have been shown to admit travelling-wave fronts whose existence and uniqueness depend on the delay structure [29-31]. The method of upper and lower solutions, combined with fixed point theorems, provides a powerful framework for establishing existence in delayed systems [32-34]. For the delayed generalized Burgers-Fisher equation, the first-integral method has been used to construct travelling solitary wave solutions [35]. More generally, the theory of spreading speeds and travelling waves for integral and delayed reaction-diffusion equations provides the asymptotic framework within which these results are organized [32,34].

Nonlocal effects introduce additional structure. The nonlocal Fisher-KPP equation, in which the saturation term is replaced by an interaction with a kernel, exhibits travelling waves that connect the unstable state to non-constant steady states, and the minimal speed depends on the nonlocality [36-37]. The stability of such waves, and of the transition waves in time- and space-dependent habitats, has been analyzed using comparison principles and energy methods [33,38]. For the stochastic Fisher-KPP equation, travelling waves persist in the presence of noise, although their speed and regularity are modified [39-40]. The extension of the Fisher-KPP theory to degenerate and doubly nonlinear diffusion shows how the speed of propagation is governed by the balance between reaction and the specific form of nonlinear diffusion [41-43].

The theory has also been enriched by probabilistic methods. The travelling waves of the KPP equation have been studied through branching processes and the extremal theory of branching Brownian motion, and the location of the wavefront has been linked to the asymptotic behaviour of the underlying random walk [27,44-45]. The probabilistic analysis of one-sided travelling waves has provided alternative derivations of the minimal speed and of the sharp decay estimates near the unstable state [44-45]. In heterogeneous environments, the existence of travelling waves has been extended to periodic media, to shifting habitats, and to equations with nonlocal dispersal [46-47]. Rapid travelling waves that connect two unstable states, as well as entire solutions describing the interaction of two fronts, have been constructed for nonlocal Fisher-type equations [47-48]. These results demonstrate that the Fisher-KPP and Burgers-Fisher families continue to generate new mathematics at the interface of analysis, probability, and biology [34,49].

Recent years have seen renewed interest in the fundamental theory of front propagation. The marginal stability mechanism, which selects the speed of fronts propagating into unstable states, has been shown to arise from a generalized Burgers equation for the local wave number [8]. The sharp asymptotics of the front position, including the logarithmic Bramson correction, have been re-derived by short and elegant proofs [28]. Fast propagation for KPP equations with slowly decaying initial conditions has been characterized [50]. The stability of travelling waves for the



FitzHugh-Nagumo system, a close relative of the Burgers-Huxley equation, has been established through the Evans function and related spectral methods [51-52]. These results together form a mature theory that continues to grow, with new results on convection-dominated fronts appearing regularly [53-54].

4. Numerical methods

The Burgers-Fisher equation has served as a demanding benchmark for numerical methods, because it combines a sharp front, a nonlinear reaction term, and an advection velocity. The exact solution provides a convenient reference, and a very large literature compares methods against it [6,25-26]. Finite difference methods were among the first applied. Implicit finite difference schemes, including Crank-Nicolson and backward differentiation variants, have been designed and analyzed for the one-dimensional equation [55]. Nonstandard finite difference schemes, which preserve the qualitative features of the exact solution, have been constructed using the solitary-wave solution [56]. Compact finite difference schemes of high order combined with low-storage Runge-Kutta time stepping achieve high accuracy with modest computational cost [26]. The classical explicit and exact-explicit finite difference methods developed for the Burgers equation have also been adapted to the Burgers-Fisher setting [57].

Finite element methods provide rigorous convergence and stability guarantees. Galerkin finite element approximations of the Burgers-Fisher equation have been analyzed for existence, uniqueness, and a priori error estimates, and positivity-preserving time-stepping has been studied [58]. Structure-preserving schemes have been developed that maintain the invariant region of the solution and discrete symmetry properties [59-60]. For the closely related Burgers-Huxley equation, positive and bounded finite element approximations have been established [60], and nonstandard finite difference-Galerkin hybrid methods have been analyzed [61].

B-spline collocation methods have become the workhorse of the field. Cubic B-spline collocation and quasi-interpolation methods were among the earliest, and they remain widely used [25]. Modified cubic B-splines combined with SSP-RK time stepping provide unconditionally stable approximations for the generalized Burgers-Fisher and Burgers-Huxley equations [62]. Exponential cubic B-splines, with tunable free parameters, improve accuracy through parameter selection [63]. Hybrid B-spline collocation, trigonometric quintic B-splines, and cardinal B-spline wavelets extend the family further [64-66]. Differential quadrature methods, including polynomial, modified cubic B-spline, and Sinc-based variants, offer high-order alternatives [67-68].

Spectral methods achieve exponential accuracy for smooth solutions. Chebyshev spectral collocation has been applied to the generalized Burgers-Fisher equation [69], and spectral collocation with preconditioning has been developed for the Burgers-Huxley equation [70]. Bivariate Chebyshev spectral collocation with quasilinearization handles strongly nonlinear problems [71]. Meshless methods based on radial basis functions and interpolation scaling functions have been developed for both the Burgers-Fisher and Burgers-Huxley equations [72-74]. Lattice Boltzmann models have been constructed for nonlinear convection-diffusion equations, with the Burgers-Fisher equation as a benchmark [75-76]. Operator splitting methods, including Strang splitting, decompose the equation into convective, diffusive, and reactive parts and achieve second-order accuracy [77]. Haar and Chebyshev wavelet methods provide efficient alternatives with few degrees of freedom [78-80]. Differential quadrature methods, including polynomial and modified cubic B-spline variants, and high-order finite difference schemes of up to fourth order have also been applied to the generalized Burgers-Huxley equation [67-68,81].

Recent numerical work emphasizes structure preservation, high-order accuracy, and efficiency. High-order finite difference schemes of up to tenth order have been applied to the Burgers-Huxley equation [82]. Unconditionally stable difference schemes for the two-dimensional Fisher-KPP equation have been constructed using alternating direction implicit methods [83]. Quintic trigonometric B-splines have been used for the extended Fisher-Kolmogorov equation [84]. Element free Galerkin methods and structure-preserving discontinuous Galerkin methods have been developed for Fisher-type equations, with applications to neurodegenerative disease modeling [85-86]. The spherically symmetric generalized Fisher-KPP equation has been treated numerically with structure-preserving schemes, and the two-dimensional Fisher-KPP equation has been solved by meshless and high-order methods [87-88]. Differential quadrature methods have been extended to the generalized FitzHugh-Nagumo equation with time-dependent coefficients [89]. The extensive numerical literature is a testament to the equation's role as a canonical benchmark [25-26].



5. Fractional-order extensions

The fractional Burgers-Fisher equation, in which the time derivative is replaced by a fractional operator, extends the classical model to anomalous transport regimes. Fractional derivatives in the Liouville-Caputo, Caputo-Fabrizio, and Atangana-Baleanu senses have been studied, and the resulting equations exhibit solutions that depend continuously on the fractional order [80,90-91]. Reliable methods for the fractional equation include the Haar wavelet method, the optimal homotopy asymptotic method, and the Laplace perturbation method [80,90]. A unified approach based on Haar wavelets and finite differences has been developed for a class of time-fractional Burgers-type equations [91].

The choice of fractional derivative has a pronounced effect on the solutions. Comparisons of the Caputo and conformable derivatives for the time-fractional Burgers-Fisher equation show that the two definitions produce travelling waves of the same qualitative form but with different quantitative behavior [92]. The $((1/G'))$ -expansion method provides analytical solutions for the fractional equation, and finite difference schemes for it have been analyzed for stability and convergence [90]. Fractional derivatives with non-singular kernels, including the Mittag-Leffler kernel, have been applied to the Burgers equation and its relatives [92-93]. The general improved Kudryashov method has been extended to stochastic evolution equations with conformable derivatives [94-95]. The fractional-in-space Fisher-KPP equation, with the Laplacian replaced by a fractional power, exhibits front speeds that depend on the fractional order [96]. New results on time-fractional Huxley and Burgers-type equations continue to appear, including spectral collocation methods based on Dickson polynomials and normalized fractional equations [93,97]. Lie symmetry analysis has been applied to the time-fractional generalized Burgers-Huxley equation, yielding explicit solutions through power-series representations and similarity reductions [98].

6. Related equations: Burgers-Huxley and FitzHugh-Nagumo

The Burgers-Fisher equation is the simplest member of a family of convection-reaction-diffusion equations that includes the Burgers-Huxley equation, obtained by replacing the logistic reaction with a cubic one, and the FitzHugh-Nagumo equation, which models nerve impulse propagation [9-10]. The generalized Burgers-Huxley equation,

$$u_t + \alpha u^\delta u_x = \nu u_{xx} + \beta u(1-u^\delta)(u^\delta - \gamma),$$

has attracted an enormous literature because, like the Burgers-Fisher equation, it admits exact travelling-wave solutions, and because it interpolates between monostable and bistable kinetics through the parameter γ [99-101]. Exact solutions have been constructed by the tanh method, the Adomian decomposition method, the variational iteration method, and the first-integral method [99,102-104]. The Painlevé analysis of the generalized Burgers-Huxley equation has revealed its integrability properties and produced new families of solutions [101].

The numerical literature on the Burgers-Huxley equation is vast and methodologically diverse. In addition to the B-spline, differential quadrature, spectral, and wavelet methods discussed above, the equation has been solved by implicit exponential finite difference schemes with Newton iteration [105], by Chebyshev wavelet collocation [79], by lattice Boltzmann methods [76], and by homotopy perturbation and homotopy analysis methods [106-107]. The stochastic generalized Burgers-Huxley equation, with multiplicative noise, has been shown to admit mild solutions under appropriate conditions [108]. The deterministic forced equation has been analyzed for existence, uniqueness, regularity, and global attractors, and its inviscid limits to the Burgers and Huxley equations have been identified [109].

The FitzHugh-Nagumo equation is the physiologically motivated companion of the Burgers-Huxley equation. Its travelling pulse solutions have been studied in depth, including their existence, stability, and the geometry of their homoclinic orbits [10,51-52,110]. The soliton solutions of the FitzHugh-Nagumo equation have been constructed by the homotopy analysis method [107]. The general theory of reaction-diffusion-convection equations, developed through Lie symmetry methods, encompasses all of these models and provides a unifying framework for their exact solutions [111-113]. Recent work has extended the Lie symmetry analysis to the convective Fisher-Kolmogorov equations and derived their conservation laws [114-115].

The relationship among these equations is both structural and historical. Nagumo's equation was introduced in the 1970s as a minimal model for the propagation of electrical pulses in nerve fibres, and it was quickly recognized as a scalar analogue of the full Hodgkin-Huxley system [9-10]. The FitzHugh-Nagumo system, which couples a fast activator variable to a slow recovery variable, reduces to a scalar reaction-diffusion equation in suitable parameter



regimes, and its travelling waves are pulse solutions rather than fronts [51-52,110]. The generalized Burgers-Huxley equation unifies these models with the Burgers-Fisher equation by allowing the reaction to interpolate between monostable and bistable forms, and it is this flexibility that accounts for its popularity as a benchmark [60,99-100]. Lie symmetry methods have proved particularly effective for this family, since the equations possess rich symmetry groups that generate exact solutions and conservation laws in a systematic way [111-114].

7. Applications

The Burgers-Fisher equation and its relatives model front propagation in a remarkable range of systems. In population genetics, the Fisher-KPP equation describes the spread of an advantageous allele through a spatially distributed population, and its travelling waves are the mathematical embodiment of the wave of advance [4]. The theory of front propagation into unstable states, developed through the marginal stability mechanism, explains how the speed of such waves is selected and has applications across ecology and evolutionary biology [8,116]. Reaction-diffusion waves of advance have been used to model the transition to agriculture in human prehistory [116-117].

In biology and medicine, Fisher-type equations model the spatial spread of epidemics, tumor invasion, and collective cell migration [11,118]. The phenotype-structuring of cell populations extends the classical Fisher-KPP framework to heterogeneous populations and reveals how phenotypic diversity shapes the speed of invasion fronts [118]. The FitzHugh-Nagumo and Huxley equations model the propagation of nerve impulses, and their travelling pulses have been the subject of intense theoretical and numerical study [9-10,51]. The structure-preserving discretization of the Fisher-Kolmogorov equation has been applied to model the spread of misfolded proteins in neurodegenerative diseases such as Parkinson's disease [86]. Boundary control of the Burgers-Fisher system has been studied for local stabilization of its equilibria [119].

In ecology and evolutionary biology, the wave-of-advance paradigm remains central. The Fisher-KPP equation and its extensions describe the spread of species into unoccupied territory, the advance of advantageous mutations through populations, and the establishment of new habitats [4,8]. The speed of these invasion fronts is often the quantity of greatest interest, and the theory of spreading speeds provides sharp estimates for a wide class of reaction-diffusion and integro-differential models [11,34]. In human prehistory, reaction-diffusion waves of advance have been used to model the spread of agriculture across Europe, with the minimal speed of the Fisher-type fronts matching the observed rate of expansion [116-117]. In medical contexts, Fisher-type models describe the growth and invasion of tumors, the spread of bacterial colonies, and the transmission of infectious diseases, and the structure-preserving numerical methods reviewed above have become indispensable for simulating these processes with fidelity [11,86,118].

In physics and chemistry, the Burgers-Fisher equation appears in combustion theory, plasma physics, and nonlinear acoustics, where it models the interaction of convection, diffusion, and reaction in the propagation of fronts [5,13]. The exact solutions of the Fisher and Burgers equations with finite transport memory describe wavefront propagation in media with non-instantaneous transport [13-14]. The two-dimensional Fisher-KPP equation has been solved by meshless and high-order schemes for population dynamics [74,83]. The extended Fisher-Kolmogorov equation, which adds a stabilizing fourth-order term, models pattern formation and has been studied by element free Galerkin and trigonometric B-spline methods [84-85]. The KdV-Burgers-Fisher equation, which adds dispersion, models dispersion-dissipation-reaction phenomena and has been solved by trigonometric quintic B-spline collocation [65]. In fluid dynamics, the Fokker-Planck and related equations reduce to Burgers-type models in appropriate limits [5]. Recent work has also examined exact solitary wave profiles for the Fisher equation with a nonlinear convection term, extending the analytical toolkit to physical settings with memory effects [120], and has applied Lie symmetry analysis to solve the one-dimensional Fisher equation and describe its sigmoidal invasion fronts [115].

Beyond the scalar equation, the family of convection-reaction-diffusion models has found applications in geophysical and industrial settings. The KdV-Burgers-Kuramoto equation, which combines dispersion, dissipation, and hyperdissipation, models pattern formation in thin films and other dissipative systems, and its symmetries and exact solutions have been classified [121]. The variable-coefficient extensions of the Burgers equation, solved by generalized G'/G-expansion techniques, model wave propagation in two-layer fluids, fluid-filled elastic tubes, and dusty plasmas [23]. These applications illustrate how the analytical machinery developed for the Burgers-Fisher equation transfers to a much broader class of physically motivated problems [5,111]. The continuing flow of new



results, including boundary-control stabilization of the Burgers-Fisher system and structure-preserving discretizations of the Fisher-Kolmogorov equation, indicates that the applications of this family remain an active and productive area of research [86,119].

8. Machine learning and data-driven methods

The Burgers-Fisher equation has become a standard benchmark for machine-learning approaches to partial differential equations. Physics-informed neural networks, which embed the governing equation into the loss function, have been applied to nonlinear convection-reaction-diffusion equations including the Burgers-Fisher and Burgers-Huxley equations [122-123]. Refined physics-informed neural networks that incorporate exponential finite difference scheme residuals have been shown to outperform both standard PINNs and Galerkin finite element methods on these benchmark equations [123]. Shallow neural networks based on Lie series in Lie groups have been used to solve the Burgers-Huxley equation with high accuracy and low parameter cost [124].

The success of these methods is not accidental. The exact travelling-wave solutions of the Burgers-Fisher equation provide an unambiguous ground truth, so that the accuracy of a learned solution can be measured at every point of the domain rather than only through aggregate statistics [124-125]. Moreover, the sharp fronts of the equation stress-test the ability of neural networks to represent steep gradients without oscillations, a challenge that has led to improved loss-weighting schemes, adaptive activation functions, and residual-based formulations [122-123]. The same benchmark quality has been exploited in the study of inverse problems, where the unknown parameters of the equation are recovered from noisy partial observations; methods such as the spline-based approach with Tikhonov regularization have been demonstrated on the generalized Burgers-Fisher and Burgers-Huxley equations [126]. The interplay between exact solutions, classical numerics, and machine learning is one of the most dynamic directions in contemporary scientific computing [125,127].

Deep learning algorithms have been developed specifically for the generalized Burgers-Fisher equation, and they have been benchmarked against the exact solutions [125]. Data-driven discovery methods, which extract governing equations from sparse and noisy measurements, use the Burgers and Fisher equations as canonical test problems [127]. Physics-informed learning from scarce data has been demonstrated on nonlinear spatiotemporal systems, with the Fisher equation among the benchmarks [127]. Quantum computing is beginning to touch this area as well: quantum neural operators, including quantum DeepONet, have been benchmarked on PDEs including the Burgers equation, and quantum-informed physics-constrained frameworks are emerging [128]. The integration of machine learning with structure-preserving numerical methods is one of the most active frontiers, with applications ranging from parameter estimation to subgrid-scale modeling [127]. The study of stochastic Burgers-type equations under white noise, with exact stochastic solitary wave solutions compared against computational schemes, further connects the analytical and data-driven strands of the subject [129].

The use of the Burgers-Fisher equation as a benchmark has played an important role in the maturation of scientific machine learning. Because the exact travelling-wave solutions provide ground truth, the accuracy of learned solutions can be measured unambiguously [124-125]. This has allowed systematic comparison of different architectures, loss formulations, and training strategies, and it has helped identify the strengths and limitations of physics-informed methods for sharp-front problems [122-123]. As neural operators and foundation models for PDEs continue to develop, the Burgers-Fisher equation is likely to remain a primary test problem, much as it has been for classical numerical analysis.

9. Open problems and future directions

Several fundamental questions remain open. First, a complete classification of the exact travelling-wave solutions of the generalized Burgers-Fisher equation for arbitrary parameter values is still lacking, and the sharp conditions under which closed-form solutions exist are not fully understood [6-7,13]. Second, the rigorous theory of travelling waves for the delayed and nonlocal Burgers-Fisher equation is less developed than for the classical case, and the interaction between delay, nonlocality, and convection remains an active area [29,32,36]. Third, the stochastic Burgers-Fisher equation, with multiplicative or additive noise, deserves further study; its well-posedness and the statistics of its fronts are only partially understood [39-40,108].



From a computational perspective, the design of structure-preserving schemes that respect the invariant region, the positivity, and the energy balance of the solution remains a central goal [58-60]. The extension of high-order, unconditionally stable methods to two and three dimensions, and to systems of coupled Burgers-Fisher equations, is ongoing [83,86]. The application of machine learning to the inverse problem of parameter estimation for the Burgers-Fisher equation, and the development of neural operators that generalize across parameter regimes, are active frontiers [125-127]. Finally, the connections between the Burgers-Fisher equation and the broader theory of reaction-diffusion-convection systems, including pattern formation and the dynamics of coupled populations, continue to generate new problems [5,111,118].

10. Conclusions

The Burgers-Fisher equation occupies a special place in the mathematical sciences. It is a prototypical convection-reaction-diffusion model with exact travelling-wave solutions, a demanding benchmark for numerical methods, and a bridge between the theories of the Burgers and Fisher-KPP equations [1-2,4]. Its theory spans exact solvability, rigorous analysis of fronts, an extensive numerical literature, and, most recently, the emerging field of machine-learning-based scientific computing [5-6,62,122,125]. The equation continues to attract research across analysis, computation, and applications, and the most recent work, including papers published in 2025 and 2026, shows that the subject remains vibrant [65,86,97,114-115,120,123].

The trajectory of research on the Burgers-Fisher equation mirrors the broader development of nonlinear science: a simple model that captures an essential mechanism becomes a laboratory for theory, a standard for computation, and a meeting point for disciplines [5-6,127]. The exact solvability of the equation provides a rare window into the structure of nonlinear waves, while its richness as a benchmark ensures that it will remain at the center of methodological developments for years to come. We expect that the interplay between rigorous analysis, structure-preserving computation, and data-driven modeling will continue to drive progress on this remarkable equation.

Declarations

The author declares no competing interests, no data, and no funding.

References

1. Burgers J M. A Mathematical Model Illustrating the Theory of Turbulence. In: *Advances in Applied Mechanics*. Vol 1. 1948. p. 171-199. doi:10.1016/S0065-2156(08)70100-5
2. Hopf E. The partial differential equation $u_t + uu_x = \mu u_{xx}$. *Communications on Pure and Applied Mathematics*. 1950;3(3):201-230. doi:10.1002/cpa.3160030302
3. Cole J D. On a quasi-linear parabolic equation occurring in aerodynamics. *Quarterly of Applied Mathematics*. 1951;9(3):225-236. doi:10.1090/qam/42889
4. Fisher R A. The Wave of Advance of Advantageous Genes. *Annals of Eugenics*. 1937;7(4):355-369. doi:10.1111/j.1469-1809.1937.tb02153.x
5. Gilding B H, Kersner R. *Travelling Waves in Nonlinear Diffusion-Convection Reaction*. Birkhäuser Basel. 2004. doi:10.1007/978-3-0348-7964-4
6. Kaya D, El-Sayed S M. A numerical simulation and explicit solutions of the generalized Burgers-Fisher equation. *Applied Mathematics and Computation*. 2004;152(2):403-413. doi:10.1016/s0096-3003(03)00565-4
7. Ismail H N A, Abd Rabboh A A. A restrictive Padé approximation for the solution of the generalized Fisher and Burger-Fisher equations. *Applied Mathematics and Computation*. 2004;154(1):203-210. doi:10.1016/s0096-3003(03)00703-3
8. van Saarloos W. Front propagation into unstable states: Marginal stability as a dynamical mechanism for velocity selection. *Physical Review A*. 1988;37(1):211-229. doi:10.1103/physreva.37.211



9. McKean H P. Nagumo's equation. *Advances in Mathematics*. 1970;4(3):209-223. doi:10.1016/0001-8708(70)90023-x
10. Rinzel J, Keller J B. Traveling Wave Solutions of a Nerve Conduction Equation. *Biophysical Journal*. 1973;13(12):1313-1337. doi:10.1016/s0006-3495(73)86065-5
11. Zhao X Q, Wang W. Fisher waves in an epidemic model. *Discrete & Continuous Dynamical Systems - B*. 2004;4(4):1117-1128. doi:10.3934/dcdsb.2004.4.1117
12. Malfliet W, Hereman W. The tanh method: I. Exact solutions of nonlinear evolution and wave equations. *Physica Scripta*. 1996;54(6):563-568. doi:10.1088/0031-8949/54/6/003
13. Kar S, Banik S K, Ray D S. Exact solutions of Fisher and Burgers equations with finite transport memory. *Journal of Physics A: Mathematical and General*. 2003;36(11):2771-2780. doi:10.1088/0305-4470/36/11/308
14. Abdel-Gawad H I. Towards a Unified Method for Exact Solutions of Evolution Equations. An Application to Reaction Diffusion Equations with Finite Memory Transport. *Journal of Statistical Physics*. 2012;147(3):506-518. doi:10.1007/s10955-012-0467-0
15. Malfliet W. The tanh method: a tool for solving certain classes of nonlinear evolution and wave equations. *Journal of Computational and Applied Mathematics*. 2004;164-165:529-541. doi:10.1016/s0377-0427(03)00645-9
16. Yang Z J. Travelling wave solutions to nonlinear evolution and wave equations. *Journal of Physics A: Mathematical and General*. 1994;27(8):2837-2855. doi:10.1088/0305-4470/27/8/021
17. Wazwaz A M. The tanh method for traveling wave solutions of nonlinear equations. *Applied Mathematics and Computation*. 2004;154(3):713-723. doi:10.1016/s0096-3003(03)00745-8
18. Rashidi M M, Ganji D D, Dinarvand S. Explicit analytical solutions of the generalized Burger and Burger-Fisher equations by homotopy perturbation method. *Numerical Methods for Partial Differential Equations*. 2009;25(2):409-417. doi:10.1002/num.20350
19. Malik S A, Qureshi I M, Amir M, Malik A, Haq I U. Numerical Solution to Generalized Burgers'-Fisher Equation Using Exp-Function Method Hybridized with Heuristic Computation. *PLOS ONE*. 2015;10(3):e0121728. doi:10.1371/journal.pone.0121728
20. Feng Z. Traveling wave behavior for a generalized Fisher equation. *Chaos, Solitons & Fractals*. 2008;38(2):481-488. doi:10.1016/j.chaos.2006.11.031
21. Kudryashov N A, Zakharchenko A S. A note on solutions of the generalized Fisher equation. *Applied Mathematics Letters*. 2014;32:53-56. doi:10.1016/j.aml.2014.02.009
22. Vitanov N K. Simple Equations Method (SEsM): An Effective Algorithm for Obtaining Exact Solutions of Nonlinear Differential Equations. *Entropy*. 2022;24(11):1653. doi:10.3390/e24111653
23. Mohanty S K, Kumar S, Dev A N, Deka M K, Churikov D V, Kravchenko O V. An efficient technique of G'/G-expansion method for modified KdV and Burgers equations with variable coefficients. *Results in Physics*. 2022;37:105504. doi:10.1016/j.rinp.2022.105504
24. Wang K J. Variational principle and approximate solution for the generalized Burgers-Huxley equation with fractal derivative. *Fractals*. 2021;29(2):2150044. doi:10.1142/s0218348x21500444
25. Zhu C G, Kang W S. Numerical solution of Burgers-Fisher equation by cubic B-spline quasi-interpolation. *Applied Mathematics and Computation*. 2010;216(9):2679-2686. doi:10.1016/j.amc.2010.03.113
26. Sari M, Gürarslan G, Dag I. A compact finite difference method for the solution of the generalized Burgers-Fisher equation. *Numerical Methods for Partial Differential Equations*. 2010;26(1):125-134. doi:10.1002/num.20421
27. Bramson M. Location of the travelling wave for the Kolmogorov equation. *Probability Theory and Related Fields*. 1986;73(4):481-515. doi:10.1007/bf00324848
28. Hamel F, Nolen J, Roquejoffre J M, Ryzhik L. A short proof of the logarithmic Bramson correction in Fisher-KPP equations. *Networks and Heterogeneous Media*. 2013;8(1):275-289. doi:10.3934/nhm.2013.8.275



29. Zou X. Delay induced traveling wave fronts in reaction diffusion equations of KPP-Fisher type. *Journal of Computational and Applied Mathematics*. 2002;146(2):309-321. doi:10.1016/s0377-0427(02)00363-1
30. Ma S. Traveling Wavefronts for Delayed Reaction-Diffusion Systems via a Fixed Point Theorem. *Journal of Differential Equations*. 2001;171(2):294-314. doi:10.1006/jdeq.2000.3846
31. Wang Z C, Li W T, Ruan S. Traveling wave fronts in reaction-diffusion systems with spatio-temporal delays. *Journal of Differential Equations*. 2006;222(1):185-232. doi:10.1016/j.jde.2005.08.010
32. Li W T, Lin G, Ruan S. Existence of travelling wave solutions in delayed reaction-diffusion systems with applications to diffusion-competition systems. *Nonlinearity*. 2006;19(6):1253-1273. doi:10.1088/0951-7715/19/6/003
33. Mei M, Lin C K, Lin C T, So J W H. Traveling wavefronts for time-delayed reaction-diffusion equation: (I) Local nonlinearity. *Journal of Differential Equations*. 2009;247(2):495-510. doi:10.1016/j.jde.2008.12.026
34. Thieme H R, Zhao X Q. Asymptotic speeds of spread and traveling waves for integral equations and delayed reaction-diffusion models. *Journal of Differential Equations*. 2003;195(2):430-470. doi:10.1016/s0022-0396(03)00175-x
35. Deng X J, Han L, Li X. Travelling Solitary Wave Solutions for Generalized Time-delayed Burgers-Fisher Equation. *Communications in Theoretical Physics*. 2009;52(2):284-286. doi:10.1088/0253-6102/52/2/19
36. Berestycki H, Nadin G, Perthame B, Ryzhik L. The non-local Fisher-KPP equation: travelling waves and steady states. *Nonlinearity*. 2009;22(12):2813-2844. doi:10.1088/0951-7715/22/12/002
37. Carr J, Chmaj A. Uniqueness of travelling waves for nonlocal monostable equations. *Proceedings of the American Mathematical Society*. 2004;132(8):2433-2439. doi:10.1090/s0002-9939-04-07432-5
38. Shen W. Stability of transition waves and positive entire solutions of Fisher-KPP equations with time and space dependence. *Nonlinearity*. 2017;30(9):3466-3491. doi:10.1088/1361-6544/aa7f08
39. Conlon J G, Doering C R. On Travelling Waves for the Stochastic Fisher-Kolmogorov-Petrovsky-Piscunov Equation. *Journal of Statistical Physics*. 2005;120(3-4):421-477. doi:10.1007/s10955-005-5960-2
40. Levajković T, Pilipović S, Selesi D, Žigić M. Stochastic evolution equations with Wick-polynomial nonlinearities. *Electronic Journal of Probability*. 2018;23:1-27. doi:10.1214/18-ejp241
41. Sherratt J A, Marchant B P. Nonsharp travelling wave fronts in the Fisher equation with degenerate nonlinear diffusion. *Applied Mathematics Letters*. 1996;9(5):33-38. doi:10.1016/0893-9659(96)00069-9
42. Sánchez-Garduño F, Maini P K. Traveling Wave Phenomena in Some Degenerate Reaction-Diffusion Equations. *Journal of Differential Equations*. 1995;117(2):281-319. doi:10.1006/jdeq.1995.1055
43. Audrito A, Vázquez J L. The Fisher-KPP problem with doubly nonlinear diffusion. *Journal of Differential Equations*. 2017;263(11):7647-7708. doi:10.1016/j.jde.2017.08.025
44. Kyprianou A E. Travelling wave solutions to the K-P-P equation: alternatives to Simon Harris' probabilistic analysis. *Annales de l'Institut Henri Poincaré B*. 2004;40(1):53-72. doi:10.1016/j.anihpb.2003.06.001
45. Harris J A, Harris S C, Kyprianou A E. Further probabilistic analysis of the Fisher-Kolmogorov-Petrovskii-Piscounov equation: one sided travelling-waves. *Annales de l'Institut Henri Poincaré B*. 2006;42(1):125-145. doi:10.1016/j.anihpb.2005.02.005
46. Hu H, Zou X. Existence of an extinction wave in the Fisher equation with a shifting habitat. *Proceedings of the American Mathematical Society*. 2017;145(11):4763-4771. doi:10.1090/proc/13687
47. Li W T, Sun Y J, Wang Z C. Entire solutions in the Fisher-KPP equation with nonlocal dispersal. *Nonlinear Analysis: Real World Applications*. 2010;11(4):2302-2313. doi:10.1016/j.nonrwa.2009.07.005
48. Alfaro M, Coville J. Rapid traveling waves in the nonlocal Fisher equation connect two unstable states. *Applied Mathematics Letters*. 2012;25(12):2095-2099. doi:10.1016/j.aml.2012.05.006
49. Ai S. Traveling wave fronts for generalized Fisher equations with spatio-temporal delays. *Journal of Differential Equations*. 2007;232(1):104-133. doi:10.1016/j.jde.2006.08.015



50. Hamel F, Roques L. Fast propagation for KPP equations with slowly decaying initial conditions. *Journal of Differential Equations*. 2010;249(7):1726-1745. doi:10.1016/j.jde.2010.06.025
51. Jones C K R T. Stability of the Travelling Wave Solution of the FitzHugh-Nagumo System. *Transactions of the American Mathematical Society*. 1984;286(2):431-469. doi:10.2307/1999806
52. Rinzel J. Spatial stability of traveling wave solutions of a nerve conduction equation. *Biophysical Journal*. 1975;15(10):975-988. doi:10.1016/s0006-3495(75)85878-4
53. Arias M, Campos J. Traveling waves for a Fisher-type reaction-diffusion equation with a flux in divergence form. *Mathematical Models and Methods in Applied Sciences*. 2023;33(6):1331-1372. doi:10.1142/s0218202523500318
54. Campos J, Guerrero P, Sánchez Ó, Soler J. On the analysis of traveling waves to a nonlinear flux limited reaction-diffusion equation. *Annales de l'Institut Henri Poincaré C*. 2013;30(1):141-155. doi:10.1016/j.anihpc.2012.07.001
55. Chandraker V, Awasthi A, Jayaraj S. Numerical Treatment of Burger-Fisher Equation. *Procedia Technology*. 2016;25:1217-1225. doi:10.1016/j.protcy.2016.08.210
56. Zhang L, Wang L, Ding X. Exact Finite Difference Scheme and Nonstandard Finite Difference Scheme for Burgers and Burgers-Fisher Equations. *Journal of Applied Mathematics*. 2014;2014:1-12. doi:10.1155/2014/597926
57. Kutluay S, Bahadır A R, Ozdes A. Numerical solution of one-dimensional Burgers equation: explicit and exact-explicit finite difference methods. *Journal of Computational and Applied Mathematics*. 1999;103(2):251-261. doi:10.1016/s0377-0427(98)00261-1
58. Yadav O P, Jiwari R. Finite element analysis and approximation of Burgers'-Fisher equation. *Numerical Methods for Partial Differential Equations*. 2017;33(5):1652-1677. doi:10.1002/num.22158
59. Macías-Díaz J E, Ruiz-Ramírez J, Villa J. The numerical solution of a generalized Burgers-Huxley equation through a conditionally bounded and symmetry-preserving method. *Computers & Mathematics with Applications*. 2011;61(11):3330-3342. doi:10.1016/j.camwa.2011.04.022
60. Ervin V J, Macías-Díaz J E, Ruiz-Ramírez J. A positive and bounded finite element approximation of the generalized Burgers-Huxley equation. *Journal of Mathematical Analysis and Applications*. 2015;424(2):1143-1160. doi:10.1016/j.jmaa.2014.11.047
61. Chin P W M. The analysis of the solution of the Burgers-Huxley equation using the Galerkin method. *Numerical Methods for Partial Differential Equations*. 2023;39(4):2787-2807. doi:10.1002/num.22987
62. Mittal R C, Tripathi A. Numerical solutions of generalized Burgers-Fisher and generalized Burgers-Huxley equations using collocation of cubic B-splines. *International Journal of Computer Mathematics*. 2015;92(5):1053-1077. doi:10.1080/00207160.2014.920834
63. Dag I, Ersoy O. Numerical solution of generalized Burgers-Fisher equation by exponential cubic B-spline collocation method. In: *AIP Conference Proceedings*. Vol 1648. 2015. p. 370005. doi:10.1063/1.4912597
64. Wasim I, Abbas M, Amin M N. Hybrid B-Spline Collocation Method for Solving the Generalized Burgers-Fisher and Burgers-Huxley Equations. *Mathematical Problems in Engineering*. 2018;2018:1-18. doi:10.1155/2018/6143934
65. Karaağaç B, Esen A, Uçar Y, Yağmurlu N M. A trigonometric quintic B-spline collocation technique for the fifth-order KdV-Burgers-Fisher equation. *Engineering with Computers*. 2025. doi:10.1007/s00366-025-02131-1
66. Shiralashetti S C, Kumbinarasaiah S. Cardinal B-Spline Wavelet Based Numerical Method for the Solution of Generalized Burgers-Huxley Equation. *International Journal of Applied and Computational Mathematics*. 2018;4(2):54. doi:10.1007/s40819-018-0505-y
67. Sari M, Gürarslan G. Numerical Solutions of the Generalized Burgers-Huxley Equation by a Differential Quadrature Method. *Mathematical Problems in Engineering*. 2009;2009:370765. doi:10.1155/2009/370765



68. Singh B K, Arora G, Singh M K. A numerical scheme for the generalized Burgers-Huxley equation. *Journal of the Egyptian Mathematical Society*. 2016;24(4):629-637. doi:10.1016/j.joems.2015.11.003
69. Javidi M. Spectral collocation method for the solution of the generalized Burger-Fisher equation. *Applied Mathematics and Computation*. 2006;174(1):345-352. doi:10.1016/j.amc.2005.04.084
70. Darvishi M T, Kheybari S, Khani F. Spectral collocation method and Darvishi's preconditionings to solve the generalized Burgers-Huxley equation. *Communications in Nonlinear Science and Numerical Simulation*. 2008;13(10):2091-2103. doi:10.1016/j.cnsns.2007.05.023
71. Motsa S S, Magagula V M, Sibanda P. A Bivariate Chebyshev Spectral Collocation Quasilinearization Method for Nonlinear Evolution Parabolic Equations. *The Scientific World Journal*. 2014;2014:1-13. doi:10.1155/2014/581987
72. Hussain M, Haq S. Numerical solutions of strongly non-linear generalized Burgers-Fisher equation via meshfree spectral technique. *International Journal of Computer Mathematics*. 2021;98(9):1727-1748. doi:10.1080/00207160.2020.1846729
73. Dehghan M, Saray B N, Lakestani M. Three methods based on the interpolation scaling functions and the mixed collocation finite difference schemes for the numerical solution of the nonlinear generalized Burgers-Huxley equation. *Mathematical and Computer Modelling*. 2012;55(3-4):1129-1142. doi:10.1016/j.mcm.2011.09.037
74. Jiang Y, Wang F, Sun Z. Numerical solutions of the nonlinear Fisher's equation using a one-level meshless method. *Frontiers in Physics*. 2025;13:1616647. doi:10.3389/fphy.2025.1616647
75. Shi B, Guo Z. Lattice Boltzmann model for nonlinear convection-diffusion equations. *Physical Review E*. 2009;79(1):016701. doi:10.1103/physreve.79.016701
76. Duan Y, Kong L, Zhang R. A lattice Boltzmann model for the generalized Burgers-Huxley equation. *Physica A: Statistical Mechanics and its Applications*. 2012;391(3):625-632. doi:10.1016/j.physa.2011.08.034
77. Hamzah D A, Aprianto M C. On the numerical solution of Burgers-Fisher equation by the Strang Splitting Method. In: *Journal of Physics: Conference Series*. Vol 1764. 2021. p. 012041. doi:10.1088/1742-6596/1764/1/012041
78. Celik I. Haar wavelet method for solving generalized Burgers-Huxley equation. *Arab Journal of Mathematical Sciences*. 2012;18(1):25-37. doi:10.1016/j.ajmsc.2011.08.003
79. Celik I. Chebyshev Wavelet collocation method for solving generalized Burgers-Huxley equation. *Mathematical Methods in the Applied Sciences*. 2016;39(3):366-377. doi:10.1002/mma.3487
80. Gupta A K, Saha Ray S. On the Solutions of Fractional Burgers-Fisher and Generalized Fisher's Equations Using Two Reliable Methods. *International Journal of Mathematics and Mathematical Sciences*. 2014;2014:1-16. doi:10.1155/2014/682910
81. Bratsos A G. A Fourth Order Improved Numerical Scheme for the Generalized Burgers-Huxley Equation. *Applied Mathematics*. 2011;1(3):152-158. doi:10.4236/ajcm.2011.13017
82. Sari M, Gürarlan G, Zeytinoğlu A. High-order finite difference schemes for numerical solutions of the generalized Burgers-Huxley equation. *Numerical Methods for Partial Differential Equations*. 2011;27(5):1313-1326. doi:10.1002/num.20585
83. Kwak S, Kang S, Ham S, Hwang Y, Lee G, Kim J. An Unconditionally Stable Difference Scheme for the Two-Dimensional Modified Fisher-Kolmogorov-Petrovsky-Piscounov Equation. *Journal of Mathematics*. 2023;2023:1-14. doi:10.1155/2023/5527728
84. Thottoli S R, Tamsir M, Meetei M Z, Msmali A H. Numerical investigation of nonlinear extended Fisher-Kolmogorov equation via quintic trigonometric B-spline collocation technique. *AIMS Mathematics*. 2024;9(7):17339-17358. doi:10.3934/math.2024843
85. Abbaszadeh M, Dehghan M, Khodadadian A, Heitzinger C. Error analysis of interpolating element free Galerkin method to solve non-linear extended Fisher-Kolmogorov equation. *Computers & Mathematics with Applications*. 2020;80(1):247-262. doi:10.1016/j.camwa.2020.03.014



86. Antonietti P F, Corti M, Gómez S, Perugia I. A structure-preserving LDG discretization of the Fisher-Kolmogorov equation for modeling neurodegenerative diseases. *Mathematics and Computers in Simulation*. 2026;241:351-366. doi:10.1016/j.matcom.2025.09.006
87. Macías-Díaz J E, Medina-Ramírez I E, Puri A. Numerical treatment of the spherically symmetric solutions of a generalized Fisher-Kolmogorov-Petrovsky-Piscounov equation. *Journal of Computational and Applied Mathematics*. 2009;231(2):851-868. doi:10.1016/j.cam.2009.05.008
88. Khater M M A, Alabdali A M. Multiple Novels and Accurate Traveling Wave and Numerical Solutions of the (2+1) Dimensional Fisher-Kolmogorov-Petrovskii-Piskunov Equation. *Mathematics*. 2021;9(12):1440. doi:10.3390/math9121440
89. Jiwari R, Gupta R, Kumar V. Polynomial differential quadrature method for numerical solutions of the generalized Fitzhugh-Nagumo equation with time-dependent coefficients. *Ain Shams Engineering Journal*. 2014;5(4):1343-1350. doi:10.1016/j.asej.2014.06.005
90. Yokuş A, Yavuz M. Novel comparison of numerical and analytical methods for fractional Burger-Fisher equation. *Discrete & Continuous Dynamical Systems - S*. 2021;14(7):2591-2606. doi:10.3934/dcdss.2020258
91. Esen A, Bulut F, Oruç Ö. A unified approach for the numerical solution of time fractional Burgers' type equations. *European Physical Journal Plus*. 2016;131(4):116. doi:10.1140/epjp/i2016-16116-5
92. Yokuş A, Durur H, Kaya D, Ahmad H, Nofal T A. Numerical comparison of Caputo and Conformable derivatives of time fractional Burgers-Fisher equation. *Results in Physics*. 2021;25:104247. doi:10.1016/j.rinp.2021.104247
93. Kim J. Nonlinear dynamic evolution of a novel normalized time-fractional Burgers equation. *Partial Differential Equations in Applied Mathematics*. 2025;13:101096. doi:10.1016/j.padiff.2025.101096
94. Hyder A A. White noise theory and general improved Kudryashov method for stochastic nonlinear evolution equations with conformable derivatives. *Advances in Difference Equations*. 2020;2020:235. doi:10.1186/s13662-020-02698-7
95. Demiray S T, Pandır Y, Bulut H. Generalized Kudryashov Method for Time-Fractional Differential Equations. *Abstract and Applied Analysis*. 2014;2014:1-13. doi:10.1155/2014/901540
96. Burrage K, Hale N, Kay D. An Efficient Implicit FEM Scheme for Fractional-in-Space Reaction-Diffusion Equations. *SIAM Journal on Scientific Computing*. 2012;34(4):A2145-A2172. doi:10.1137/110847007
97. Alqubori O M, Alsulami S A, Atta A G, Amin A K, Abd-Elhameed W M. Numerical scheme for solving the time-fractional Huxley equation using shifted Dickson polynomials. *AIMS Mathematics*. 2026;11(5):13744-13766. doi:10.3934/math.2026566
98. Yusuf A, Aliyu A I, Băleanu D. Lie symmetry analysis and explicit solutions for the time fractional generalized Burgers-Huxley equation. *Optical and Quantum Electronics*. 2018;50(2):94. doi:10.1007/s11082-018-1373-8
99. Deng X. Travelling wave solutions for the generalized Burgers-Huxley equation. *Applied Mathematics and Computation*. 2008;204(2):733-737. doi:10.1016/j.amc.2008.07.020
100. Gao H, Zhao R X. New exact solutions to the generalized Burgers-Huxley equation. *Applied Mathematics and Computation*. 2010;217(4):1598-1603. doi:10.1016/j.amc.2009.07.020
101. Estévez P G, Gordoa P R. Painlevé analysis of the generalized Burgers-Huxley equation. *Journal of Physics A: Mathematical and General*. 1990;23(21):4831-4837. doi:10.1088/0305-4470/23/21/023
102. Hashim I, Noorani M S M, Al-Hadidi M R S. Solving the generalized Burgers-Huxley equation using the Adomian decomposition method. *Mathematical and Computer Modelling*. 2006;43(11-12):1404-1411. doi:10.1016/j.mcm.2005.08.017
103. Batiha B, Noorani M S M, Hashim I. Application of variational iteration method to the generalized Burgers-Huxley equation. *Chaos, Solitons & Fractals*. 2008;36(3):660-663. doi:10.1016/j.chaos.2006.06.080



104. El-Danaf T S. Solitary Wave Solutions for the Generalized Burgers-Huxley Equation. *International Journal of Nonlinear Sciences and Numerical Simulation*. 2007;8(3):315-318. doi:10.1515/ijnsns.2007.8.3.315
105. Inan B, Bahadir A R. Numerical solutions of the generalized Burgers-Huxley equation by implicit exponential finite difference method. *Journal of Applied Mathematics, Statistics and Informatics*. 2015;11(2):57-67. doi:10.1515/jamsi-2015-0012
106. Maurya D K, Singh R, Rajoria Y K. A Mathematical Model to Solve the Burgers-Huxley Equation by using New Homotopy Perturbation Method. *International Journal of Mathematical, Engineering and Management Sciences*. 2019;4(6):1483-1495. doi:10.33889/ijmems.2019.4.6-117
107. Abbasbandy S. Soliton solutions for the FitzHugh-Nagumo equation with the homotopy analysis method. *Applied Mathematical Modelling*. 2008;32(12):2706-2714. doi:10.1016/j.apm.2007.09.019
108. Mohan M T. Mild Solutions for the Stochastic Generalized Burgers-Huxley Equation. *Journal of Theoretical Probability*. 2022;35(3):1511-1536. doi:10.1007/s10959-021-01100-w
109. Mohan M T, Khan A. On the generalized Burgers-Huxley equation: Existence, uniqueness, regularity, global attractors and numerical studies. *Discrete & Continuous Dynamical Systems - B*. 2021;26(7):3943-3988. doi:10.3934/dcdsb.2020270
110. Hastings S P. Single and Multiple Pulse Waves for the FitzHugh-Nagumo Equations. *SIAM Journal on Applied Mathematics*. 1982;42(2):247-260. doi:10.1137/0142018
111. Cherniha R, Serov M, Plukhin O. *Nonlinear Reaction-Diffusion-Convection Equations: Lie and Conditional Symmetries, Exact Solutions and their Applications*. Chapman and Hall/CRC. 2017. doi:10.1201/9781315154848
112. Verma A, Jiwar R, Köksal M E. Analytic and numerical solutions of nonlinear diffusion equations via symmetry reductions. *Advances in Difference Equations*. 2014;2014(1):229. doi:10.1186/1687-1847-2014-229
113. Rosa M, Camacho J C, Bruzón M S, Gandarias M L. Conservation laws, symmetries, and exact solutions of the classical Burgers-Fisher equation in two dimensions. *Journal of Computational and Applied Mathematics*. 2019;354:545-550. doi:10.1016/j.cam.2018.11.008
114. Raza A, Ahmed A N U, Kara A H. Analysis of Reaction-Diffusion Systems: Painlevé Properties and Conservation Laws of the Convective Fisher-Kolmogorov Equations. *Symmetry*. 2026;18(2):377. doi:10.3390/sym18020377
115. Masindi P, Rundora L. Solution Dynamics of the (1+1)-Dimensional Fisher's Equation Using Lie Symmetry Analysis. *Symmetry*. 2026;18(2):279. doi:10.3390/sym18020279
116. Fort J, Méndez V. Reaction-diffusion waves of advance in the transition to agricultural economics. *Physical Review E*. 1999;60(5):5894-5901. doi:10.1103/physreve.60.5894
117. Méndez V, Fort J, Farjas J. Speed of wave-front solutions to hyperbolic reaction-diffusion equations. *Physical Review E*. 1999;60(5):5231-5243. doi:10.1103/physreve.60.5231
118. Lorenzi T, Painter K J, Villa C. Phenotype structuring in collective cell migration: a tutorial of mathematical models and methods. *Journal of Mathematical Biology*. 2025;90(6):66. doi:10.1007/s00285-025-02223-y
119. Zhao H, Zhang J, Lü J. Local stabilization of the Burgers-Fisher equation via boundary control. *Asian Journal of Control*. 2024;26(2):1402-1413. doi:10.1002/asjc.3265
120. Alshammari F S, Akgül A, Jawaz M, Aqeel S A, Baber M Z, Ahmed N. Exact Solitary Wave Profiles for the Fisher Equation with Nonlinear Convection Term Arising in Physical Sciences. *Contemporary Mathematics*. 2026;7(1):200-219. doi:10.37256/cm.7120268526
121. Alimirzaluo E, Nadjafikhah M. Some exact solutions of KdV-Burgers-Kuramoto equation. *Journal of Physics Communications*. 2019;3(3):035025. doi:10.1088/2399-6528/ab103f



122. Raissi M, Perdikaris P, Karniadakis G E. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*. 2019;378:686-707. doi:10.1016/j.jcp.2018.10.045
123. Bezekçi B. The Refined Physics-Informed Neural Networks for Nonlinear Convection-Reaction-Diffusion Equations Using Exponential Schemes. *Black Sea Journal of Engineering and Science*. 2025;8(4):970-981. doi:10.34248/bsengineering.1645207
124. Wen Y, Chaolu T. Study of Burgers-Huxley Equation Using Neural Network Method. *Axioms*. 2023;12(5):429. doi:10.3390/axioms12050429
125. Kumar H, Yadav N. A Deep Learning Algorithm for Solving Generalized Burgers-Fisher and Burger's Equations. *International Journal of Applied and Computational Mathematics*. 2023;9(5):94. doi:10.1007/s40819-023-01557-9
126. Alavi J, Aminikhah H. Numerical Study of the Inverse Problem of Generalized Burgers-Fisher and Generalized Burgers-Huxley Equations. *Journal of Mathematics*. 2021;2021:1-15. doi:10.1155/2021/6652108
127. Chen Z, Liu Y, Sun H. Physics-informed learning of governing equations from scarce data. *Nature Communications*. 2021;12(1):6136. doi:10.1038/s41467-021-26434-1
128. Xiao P, Zheng M, Jiao A, Xiu Y, Lu L. Quantum DeepONet: Neural operators accelerated by quantum computing. *Quantum*. 2025;9:1761. doi:10.22331/q-2025-06-04-1761
129. Baber M Z, Mohammed W W, Ahmed N, Iqbal M S. Exact solitary wave propagations for the stochastic Burgers' equation under the influence of white noise and its comparison with computational scheme. *Scientific Reports*. 2024;14:10810. doi:10.1038/s41598-024-58553-2