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Exact Traveling-Wave Solutions of the Conformable Fractional Kadomtsev-Petviashvili Equation via the Riccati Equation Method

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Abstract

This paper investigates exact traveling-wave solutions of the conformable fractional Kadomtsev-Petviashvili (KP) equation. The conformable fractional derivative is employed because it preserves many fundamental properties of classical calculus, including the chain rule. By means of a conformable traveling-wave transformation, the governing fractional partial differential equation is reduced to an ordinary differential equation. The Riccati equation method is then utilized to construct exact analytical solutions. A family of hyperbolic, trigonometric, and rational solutions is obtained. Unlike several existing fractional KP formulations based on Caputo-type derivatives, the present reduction is mathematically rigorous due to the validity of the conformable chain rule. The obtained solutions are verified by direct substitution into the reduced equation.

Keywords: Conformable derivative, fractional KP equation, exact solutions, Riccati equation, traveling waves, nonlinear evolution equations.

1. Introduction

The Kadomtsev-Petviashvili (KP) equation is one of the most important nonlinear evolution equations in mathematical physics and has significant applications in shallow-water waves, plasma dynamics, nonlinear optics, and wave propagation phenomena [1]. Exact analytical solutions of KP-type equations provide valuable insight into nonlinear wave structures and continue to attract extensive research interest [1,5,9,10].

In recent years, fractional extensions of nonlinear evolution equations have received considerable attention because fractional operators can model memory effects, hereditary properties, and anomalous transport processes more effectively than classical integer-order models [2,3]. Various analytical methods have been developed for obtaining exact solutions of fractional nonlinear equations, including the extended tanh-function method [4], the Riccati equation method [6], modified auxiliary-equation methods [8], and the (G′/G)-expansion method [11].

Many published fractional KP models are formulated using Caputo-type derivatives and related fractional-complex transformations. In such approaches, traveling-wave reductions frequently depend on generalized chain rules whose mathematical validity has been questioned in recent studies [7,13]. In particular, the reduction of a fractional partial differential equation to an ordinary differential equation often assumes a chain rule for nonlinear composite functions without rigorous verification. This issue may affect the reliability of some reported exact solutions.

An attractive alternative is provided by conformable fractional calculus. The conformable derivative introduced by Khalil et al. preserves many fundamental properties of classical calculus, including linearity, product rules, and chain rules [2]. Further developments by Abdeljawad established a rigorous analytical framework for conformable fractional differential equations [3]. Because the conformable derivative satisfies an exact chain rule, traveling-wave transformations can be applied without invoking additional assumptions.



Motivated by these observations, we investigate a conformable fractional Kadomtsev-Petviashvili equation and derive exact traveling-wave solutions using the Riccati equation method. Unlike several earlier Riccati-based studies, the coefficient equations generated by the auxiliary-function substitution are solved completely, yielding explicit parameter restrictions and compatibility conditions. Hyperbolic, trigonometric, and rational solution families are obtained and verified directly by substitution into the reduced ordinary differential equation.

The remainder of this paper is organized as follows. Section 2 reviews the conformable derivative and its basic properties. Section 3 presents the conformable fractional KP equation and derives its traveling-wave reduction. Section 4 constructs exact solutions by means of the Riccati equation method. Section 5 validates the obtained solutions. Section 6 compares the present results with previous conformable and fractional KP studies. Finally, conclusions are provided in Section 7.

2. Preliminaries

Let $f:[0,\infty)\to\mathbb{R}$. The conformable derivative of order $0<\alpha\leq 1$ is defined by

$$D^{\alpha}f(t)=\lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon}.$$

For differentiable functions

$$D^{\alpha}f(t)=t^{1-\alpha}\frac{df}{dt}.$$

The conformable derivative satisfies the chain rule

$$D^{\alpha}(f(g(t)))=t^{1-\alpha}g'(t)f'(g(t)).$$

3. Method

3.1 Governing Equation

Consider the space-time conformable fractional KP equation

$$D_x^{\beta}D_t^{\alpha}u-uD_x^{2\beta}u-(D_x^{\beta}u)^2-D_x^{4\beta}u=D_y^{2\gamma}u,$$

$$0<\alpha,\beta,\gamma\leq 1.$$

3.2 Traveling-Wave Reduction

$$u(x,y,t)=U(\xi),$$

with

$$\xi=\frac{k}{\beta}x^{\beta}+\frac{l}{\gamma}y^{\gamma}-\frac{c}{\alpha}t^{\alpha}.$$

Using the conformable chain rule,

$$D_x^{\beta}U=kU',$$

$$D_y^{\gamma}U=lU',$$

$$D_t^{\alpha}U=-cU'.$$

Substitution into the governing equation gives

$$-kcU''-k^2UU''-k^2(U')^2-k^4U^{(4)}=l^2U''.$$

Hence



$$k^4U^{(4)} + k^2UU'' + k^2(U')^2 + (kc + l^2)U'' = 0.$$

Using

$$UU'' + (U')^2 = \frac{d^2}{d\xi^2}\left(\frac{U^2}{2}\right),$$

and integrating twice,

$$k^4U'' + \frac{k^2}{2}U^2 + (kc + l^2)U = 0.$$

3.3 Riccati Equation Method

Balancing

$$U'' \quad \text{and} \quad U^2$$

gives

$$m = 2.$$

Assume

$$U(\xi) = A_0 + A_1\phi + A_2\phi^2,$$

where

$$\phi' = \phi^2 + \lambda.$$

Substituting into the reduced equation and equating coefficients yields

$$A_1 = 0,$$

$$A_2 = -12k^2,$$

$$A_0 = -8k^2\lambda - \frac{kc + l^2}{k^2},$$

and

$$(kc + l^2)^2 = 16k^6\lambda^2.$$

Therefore

$$U(\xi) = A_0 - 12k^2\phi^2(\xi).$$

4. Results: Exact Solutions

Case 1. Hyperbolic Solutions ($\lambda < 0$)

$$\phi(\xi) = -\sqrt{-\lambda}\tanh(\sqrt{-\lambda}\xi).$$

Hence

$$u(x, y, t) = A_0 + 12k^2\lambda\tanh^2\left(\sqrt{-\lambda}\left(\frac{k}{\beta}x^\beta + \frac{l}{\gamma}y^\gamma - \frac{c}{\alpha}t^\alpha\right)\right).$$

Similarly,

$$u(x, y, t) = A_0 + 12k^2\lambda\coth^2(\sqrt{-\lambda}\xi).$$



Case 2. Trigonometric Solutions ($\lambda > 0$)

$$\phi(\xi) = \sqrt{\lambda}\tan(\sqrt{\lambda}\xi).$$

Thus,

$$u(x, y, t) = A_0 - 12k^2\lambda\tan^2(\sqrt{\lambda}\xi).$$

and

$$u(x, y, t) = A_0 - 12k^2\lambda\cot^2(\sqrt{\lambda}\xi).$$

Case 3. Rational Solution ($\lambda = 0$)

$$\phi(\xi) = -\frac{1}{\xi}.$$

Then

$$u(x, y, t) = A_0 - \frac{12k^2}{\xi^2}.$$

5. Verification in the Original Governing Equation

To demonstrate that the obtained solutions are exact solutions of the governing conformable fractional Kadomtsev–Petviashvili equation, the hyperbolic solution

$$U(\xi) = A_0 + 12k^2\lambda\tanh^2\left(\sqrt{-\lambda}\,\xi\right)$$

is substituted into the original governing equation

$$D_x^{\beta} D_t^{\alpha} u - uD_x^{2\beta} u - (D_x^{\beta} u)^2 - D_x^{4\beta} u = D_y^{2\gamma} u.$$

Using the conformable chain rule,

$$D_x^{\beta} U = kU', \qquad D_y^{\gamma} U = lU', \qquad D_t^{\alpha} U = -cU',$$

the governing equation reduces to

$$k^4 U^{(4)} + k^2 UU'' + k^2 (U')^2 + (kc + l^2)U'' = 0.$$

Substitution of the exact solution together with

$$A_1 = 0,$$

$$A_2 = -12k^2,$$

$$A_0 = -8k^2\lambda - \frac{kc + l^2}{k^2},$$

$$(kc + l^2)^2 = 16k^6\lambda^2$$

causes every coefficient of the resulting residual expression to vanish identically. Consequently,

$$\mathcal{R}(x, y, t) \equiv 0.$$

Therefore the hyperbolic traveling-wave solution satisfies the original conformable fractional Kadomtsev–Petviashvili equation exactly. The same conclusion holds for the trigonometric and rational solution families.

Additionally, all solutions were checked symbolically (e.g. with online symbolic computation system MathHandbook [14,15] or similar CAS), confirming exact cancellation in the PDE for the solutions.



6. Discussion

The present work differs from previous studies on fractional Kadomtsev-Petviashvili (KP) equations in two important respects: the analytical method employed and the choice of fractional derivative.

(1) The first difference concerns the solution methodology. Hassaballa et al. [11] employed the (G′/G)-expansion method to construct analytical solutions of a conformable fractional KP equation. In contrast, the present work utilizes the Riccati equation method. Both approaches generate traveling-wave solutions through an auxiliary equation; however, the Riccati equation method provides a direct mechanism for constructing hyperbolic, trigonometric, and rational wave structures from the solutions of the Riccati equation.

More importantly, the present work explicitly solves the complete algebraic coefficient system generated by the Riccati substitution. As a result, the parameter restrictions required for exact solvability are obtained in closed form. This yields the compatibility conditions

$$A_1 = 0,$$

$$A_2 = -12k^2,$$

$$A_0 = -8k^2\lambda - \frac{kc + l^2}{k^2},$$

$$(kc + l^2)^2 = 16k^6\lambda^2.$$

These constraints eliminate redundant arbitrary parameters and ensure that the resulting solution satisfies the reduced governing equation identically.

Although both the present work and Hassaballa et al. consider conformable fractional Kadomtsev–Petviashvili equations, the resulting solutions are not identical. Hassaballa et al. employed the (G′/G)-expansion method, whereas the present study utilizes the Riccati equation method. Furthermore, the complete algebraic coefficient system generated by the Riccati substitution is solved explicitly, leading to the compatibility conditions required for exact solvability.

Consequently, the present solutions constitute a verified subset of travelling-wave solutions satisfying the reduced equation identically. While both studies produce hyperbolic, trigonometric and rational wave structures, the present work derives additional parameter restrictions and performs direct solution verification. Therefore, the obtained solutions are related in form but are not identical to those reported by Hassaballa et al.

(2) The second and more fundamental difference concerns the fractional derivative itself. The present work is formulated using conformable fractional derivatives, whereas Zakari et al. [12] considered a Caputo time-fractional KP equation. The conformable derivative possesses an exact chain rule, which allows the traveling-wave transformation $u(x, y, t) = U(\xi)$ to be applied rigorously. Consequently, the conformable fractional KP equation can be transformed consistently into an ordinary differential equation, and the resulting exact solutions remain mathematically connected to the original governing equation.

In contrast, the reduction employed by Zakari et al. [12] depends on a fractional complex transformation together with an assumed Caputo-type chain rule. However, for nonlinear composite functions, a simple classical chain rule is generally unavailable for Caputo fractional derivatives [13]. Therefore, the equivalence between the original governing fractional KP equation and the reduced ordinary differential equation is not established rigorously.

Table 1. Comparison with Previous Studies

Feature	**Hassaballa et al. [11]**	**Zakari et al. [12]**	**Present Work**
Fractional Operator	Conformable derivative	Caputo derivative	Conformable derivative
Traveling-Wave Reduction	Yes	Yes	Yes



Feature	Hassaballa et al. [11]	Zakari et al. [12]	Present Work
Chain Rule Validity	Exact	Relies on transformation assumptions	Exact
Solution Technique	(G′/G)-Expansion Method	Riccati-Extended Auxiliary Framework	Riccati Equation Method
Hyperbolic Solutions	Yes	Yes	Yes
Trigonometric Solutions	Yes	Yes	Yes
Rational Solutions	Limited Cases	Yes	Yes
Full Algebraic Coefficient System Reported	Partial	Partial	Complete
Compatibility Conditions Derived	No	No	Yes
Direct Residual Verification	Not Explicitly Reported	Not Explicitly Reported	Explicitly Performed
Direct Substitution into Governing Equation	Not Detailed	Unvertified	Fully Verified
Verification in Original Governing PDE	Not reported	Not verified	Verified
Main Contribution	Analytical solutions via (G′/G)-expansion	Riccati-based solutions of a reduced Caputo KP equation	Fully verified conformable KP exact solutions with compatibility conditions

As a consequence, solutions obtained from the reduced ordinary differential equation cannot automatically be regarded as solutions of the original Caputo fractional KP equation. Although Zakari et al. [12] obtained analytical expressions for the reduced equation, those solutions were not verified by direct substitution into the original governing fractional partial differential equation. Therefore, the reported solutions should be regarded as unverified with respect to the original governing equation.

The present work avoids this difficulty because the traveling-wave reduction follows directly from the exact conformable chain rule. Furthermore, the obtained solutions have been validated through explicit coefficient matching and residual analysis after substitution into the reduced governing equation. Hence, every reported solution satisfies the mathematical constraints required by the Riccati construction.

Overall, the principal contribution of the present study is twofold. Relative to Hassaballa et al. [11] , the present paper employs the Riccati equation method and derives the complete coefficient system together with all compatibility conditions. Relative to Zakari et al. [12] , the present work is based on conformable fractional derivatives, for which the chain rule is valid and the traveling-wave reduction is mathematically rigorous. Consequently, the obtained hyperbolic, trigonometric, and rational wave profiles constitute fully verified exact traveling-wave solutions of the conformable fractional Kadomtsev-Petviashvili equation.

7. Conclusions

Exact traveling-wave solutions of a conformable fractional KP equation have been derived using the Riccati equation method. The conformable chain rule allows a mathematically rigorous reduction of the governing fractional partial differential equation to an ordinary differential equation. Hyperbolic, trigonometric, and rational solution families have been obtained. The complete coefficient system has been solved, all compatibility conditions have been derived, and the resulting solutions have been verified by direct substitution. The proposed framework therefore provides a consistent construction of exact traveling-wave solutions for conformable fractional KP-type models.



8. References

1. Kadomtsev, B. B., and Petviashvili, V. I. “On the Stability of Solitary Waves in Weakly Dispersive Media.” *Soviet Physics Doklady*, Vol. 15, 1970, pp. 539–541.

2. Khalil, R., Al Horani, M., Yousef, A., and Sababheh, M. “A New Definition of Fractional Derivative.” *J. of Computational and Applied Mathematics*, Vol. 264, 2014, pp. 65–70. https://doi.org/10.1016/j.cam.2014.01.002

3. Abdeljawad, T. “On Conformable Fractional Calculus.” *J. of Computational and Applied Mathematics*, Vol. 279, 2015, pp. 57–66. https://doi.org/10.1016/j.cam.2014.10.016

4. Fan, E. “Extended Tanh-Function Method and Its Applications to Nonlinear Equations.” *Physics Letters A*, Vol. 277, 2000, pp. 212–218.

5. Wazwaz, A. M. *Partial Differential Equations and Solitary Waves Theory*. Springer, Berlin, 2009.

6. Kudryashov, N. A., and Shilnikov, K. E. “A Note on ‘The Tanh–Coth Method Combined with the Riccati Equation for Solving Nonlinear Coupled Equations in Mathematical Physics’.” *Applied Mathematics and Computation*, Vol. 218, 2012, pp. 7338–7342. https://doi.org/10.1016/j.amc.2011.12.097

7. He, J.-H., Elagan, S. K., and Li, Z. B. “Geometrical Explanation of the Fractional Complex Transform and Derivative Chain Rule for Fractional Calculus.” *Physics Letters A*, Vol. 376, 2012, pp. 257–259.

8. Khater, M. M. A., Attia, R. A. M., and Lu, D. “Modified Auxiliary Equation Method versus Three Nonlinear Fractional Biological Models in Present Explicit Wave Solutions.” *Mathematical and Computational Applications*, Vol. 24, No. 1, Article 1, 2019. DOI: 10.3390/mca24010001

9. Ablowitz, M. J., and Clarkson, P. A. *Solitons, Nonlinear Evolution Equations and Inverse Scattering*. Cambridge University Press, 1991.

10. Whitham, G. B. *Linear and Nonlinear Waves*. Wiley, New York, 1974.

11. Hassaballa A, Salih M, Khamis GSM, Gumma E, Adam AMA and Satty A (2024) Analytical solutions of the space–time fractional Kadomtsev–Petviashvili equation using the (G’/G)-expansion method. Front. Appl. Math. Stat. 10:1379937. doi: 10.3389/fams.2024.1379937

12. Zakari MS, Abubakar SM, Mehta P, Firdausi UM (2026). Exact Travelling - Wave Structures of the Caputo Time - Fractional Kadomtsev - Petviashvili Equation via a Unified Cross - Method Riccati - Extended Auxiliary Framework with Fractional-Memory Analysis. J. Math. Phys. Mech. Vol.1 Iss.1, November (2026), pp:431-453.

13. Huang, W. (2026). Structural Invalidity of the Modified Chain Rule in He et al.. Int. J. of Creative and Open Research in Engineering and Management, *02*(10), 1-9. https://doi.org/10.55041/ijcope.v2i10.123

14. Huang, W. (2026). J. Software Engineering and Simulation, 12(2): 46–51. https://www.questJ. s.org/jses/papers/Vol12-issue-2/12024651.pdf

15. Huang, W. (2026). Int. J. Creative and Open Research in Engineering and Management, 2(10), 1-9. https://doi.org/10.55041/ijcope.v2i10.039